

# Choosing a Factoring Method for Polynomials

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 34

## Q Quick Review

Use this decision tree before you start any factoring problem.

**Step 1: Pull out a GCF first**, always. A common factor makes everything that follows easier.  $6x^3 - 12x$  becomes  $6x(x^2 - 2)$ , and now we only have to think about  $x^2 - 2$ .

**Step 2: Count the terms inside what's left.**

- **Two terms:** look for a difference of squares  $a^2 - b^2 = (a - b)(a + b)$ , or difference/sum of cubes  $a^3 \pm b^3$  which factor as  $(a \pm b)(a^2 \mp ab + b^2)$ . A sum of squares  $a^2 + b^2$  does not factor over the reals.

- **Three terms:** either a perfect-square trinomial  $a^2 \pm 2ab + b^2 = (a \pm b)^2$ , or a general trinomial. Use the  $x^2 + bx + c$  rule or AC method for  $ax^2 + bx + c$ .

- **Four terms:** try *grouping*. Split into two pairs, factor each pair, then look for a common binomial.

**Step 3: Re-check each new factor.** Sometimes a factor you produced is itself factorable. Quick check:  $3x^4 - 12x^2 = 3x^2(x^2 - 4) = 3x^2(x - 2)(x + 2)$ . Don't stop after one pass.

**Always verify:** FOIL or expand your factored answer and compare with the original. Not every polynomial factors over the integers —  $x^2 - 2$  and  $x^2 + 1$  are both prime over  $\mathbb{Z}$ .

## PRACTICE

Factor each polynomial completely over the integers.

- $6x^3 - 12x$  \_\_\_\_\_
- The table gives values of  $P(x) = x^2 - 49$ . Use its zeros to factor  $P(x)$ . \_\_\_\_\_

$x$	-7	-3	0	3	7
$P(x)$	0	-40	-49	-40	0

- $x^3 - 2x^2 + 5x - 10$  \_\_\_\_\_
- $x^2 - 10x + 25$  \_\_\_\_\_
- $3x^2 + 11x + 6$  \_\_\_\_\_
- $2x^3 - 8x$  \_\_\_\_\_
- $x^2 + 8x + 16$  \_\_\_\_\_
- $3x^4 - 12x^2$  \_\_\_\_\_
- $2x^3 + 8x^2 + 8x$  \_\_\_\_\_
- $x^3 + 27$  \_\_\_\_\_
- $x^3 - 8$  \_\_\_\_\_
- $4x^2 - 9$  \_\_\_\_\_
- $x^4 - 16$  \_\_\_\_\_



14. The table gives values of  $P(x) = 5x^2 - 45$ . Use its zeros to factor  $P(x)$  completely. \_\_\_\_\_

$x$	-3	-1	0	1	3
$P(x)$	0	-40	-45	-40	0

15. The table gives values of  $P(x) = x^2 + 5x + 6$ . Use its zeros to factor  $P(x)$ . \_\_\_\_\_

$x$	-3	-2	-1	0	1
$P(x)$	0	0	2	6	12

16.  $2x^2 + 5x - 3$  \_\_\_\_\_

17.  $x^3 - x^2 - 4x + 4$  \_\_\_\_\_

18. Factor completely:  $x^2 + 1$  \_\_\_\_\_

19. Factor completely:  $4x^2 + 12x + 9$  \_\_\_\_\_

20. Factor completely:  $12x^3 + 18x^2 - 12x$  \_\_\_\_\_

◆ Word Problems

21. A storage room's floor area is  $(3x^2 - 12)$  square feet. Factor this area completely to find possible dimensions. \_\_\_\_\_

22. A cubical box has volume  $(x^3 + 12x^2 + 48x + 64)$  cubic inches. Show that the volume is a perfect cube and find the side length. \_\_\_\_\_

23. A polynomial expression appears in a physics formula:  $2x^4 - 32$ . Factor it completely. \_\_\_\_\_

24. A trinomial factor in a model is  $9x^2 - 30x + 25$ . Identify the pattern and factor it. \_\_\_\_\_

Additional Practice

25. Write  $3x - 5 + x^3$  in standard form. \_\_\_\_\_

26. Find the degree of  $7x^4 - 2x^2 + 9$ . \_\_\_\_\_

27. Add  $(2x^2 + 3x - 1) + (x^2 - 5x + 4)$ . \_\_\_\_\_

28. Subtract  $(5x^2 - x + 6) - (2x^2 + 3x - 1)$ . \_\_\_\_\_

29. Multiply  $(x + 4)(x - 3)$ . \_\_\_\_\_

30. Factor  $x^2 + 9x + 20$ . \_\_\_\_\_

31. Factor  $6x^2 + 9x$ . \_\_\_\_\_

32. Find the GCF of  $12x^3$  and  $18x^2$ . \_\_\_\_\_

33. Divide  $(x^2 + 5x + 6)$  by  $(x + 2)$ . \_\_\_\_\_

34. Find the remainder when  $x^2 - 1$  is divided by  $x - 3$ . \_\_\_\_\_



## Answer Keys

1.  $6x(x^2 - 2)$
2.  $(x - 7)(x + 7)$
3.  $(x - 2)(x^2 + 5)$
4.  $(x - 5)^2$
5.  $(3x + 2)(x + 3)$
6.  $2x(x - 2)(x + 2)$
7.  $(x + 4)^2$
8.  $3x^2(x - 2)(x + 2)$
9.  $2x(x + 2)^2$
10.  $(x + 3)(x^2 - 3x + 9)$
11.  $(x - 2)(x^2 + 2x + 4)$
12.  $(2x - 3)(2x + 3)$
13.  $(x - 2)(x + 2)(x^2 + 4)$
14.  $5(x - 3)(x + 3)$
15.  $(x + 2)(x + 3)$
16.  $(2x - 1)(x + 3)$
17.  $(x - 1)(x - 2)(x + 2)$
18. *primeoverintegers*
19.  $(2x + 3)^2$
20.  $6x(2x - 1)(x + 2)$
21.  $3(x - 2)(x + 2)$
22.  $(x + 4)$  inches
23.  $2(x - 2)(x + 2)(x^2 + 4)$
24.  $(3x - 5)^2$

## Additional Practice Answers

25.  $x^3 + 3x - 5$
26. 4
27.  $3x^2 - 2x + 3$
28.  $3x^2 - 4x + 7$
29.  $x^2 + x - 12$
30.  $(x + 4)(x + 5)$
31.  $3x(2x + 3)$
32.  $6x^2$
33.  $x + 3$
34. 8

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

1. GCF is  $6x$ : pull out.  $6x^3 - 12x = 6x(x^2 - 2)$ . The piece  $x^2 - 2$  doesn't split over the integers (square roots of 2 are irrational).
2. The table reads 0 at  $x = -7$  and  $x = 7$ , so each gives a factor:  $(x + 7)$  and  $(x - 7)$ . That's the difference of squares  $(x - 7)(x + 7)$ .
3. Four terms: try grouping.  $(x^3 - 2x^2) + (5x - 10) = x^2(x - 2) + 5(x - 2) = (x - 2)(x^2 + 5)$ . The factor  $x^2 + 5$  doesn't split (sum, not difference, of squares).
4. Perfect-square trinomial:  $x^2 - 2(5)x + 5^2 = (x - 5)^2$ . Check: the first and last are squares, middle is  $-2 \cdot 1 \cdot 5 = -10 \checkmark$ .
5. AC method:  $ac = 18$ . Need product 18, sum 11: 9 and 2. Split:  $3x^2 + 9x + 2x + 6 = 3x(x + 3) + 2(x + 3) = (x + 3)(3x + 2)$ .
6. Keep the rule visible: GCF  $2x$ :  $2x(x^2 - 4)$ . Then difference of squares:  $2x(x - 2)(x + 2)$ . That gives a quick check on the answer.
7. Three terms with no GCF. First  $x^2$  and last  $16 = 4^2$  are squares; the middle  $8x = 2 \cdot 4 \cdot x$  is twice their roots' product, so it's a perfect square:  $(x + 4)^2$ .
8. Start with the key idea: GCF  $3x^2$ :  $3x^2(x^2 - 4)$ . Then difference of squares:  $3x^2(x - 2)(x + 2)$ . That gives a quick check on the answer.
9. GCF  $2x$ :  $2x(x^2 + 4x + 4)$ . The trinomial is a perfect square:  $(x + 2)^2$ . Full:  $2x(x + 2)^2$ .
10. Keep the rule visible: Sum of cubes:  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$  with  $a = x$ ,  $b = 3$ . So  $(x + 3)(x^2 - 3x + 9)$ . That gives a quick check on the answer.
11. Difference of cubes:  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$  with  $a = x$ ,  $b = 2$ . So  $(x - 2)(x^2 + 2x + 4)$ .
12. Two terms, both squares:  $4x^2 = (2x)^2$  and  $9 = 3^2$ . Difference of squares with  $a = 2x$ ,  $b = 3$  gives  $(2x - 3)(2x + 3)$ .
13. First pass:  $(x^2)^2 - 4^2 = (x^2 - 4)(x^2 + 4)$ . Then  $x^2 - 4 = (x - 2)(x + 2)$ . The factor  $x^2 + 4$  is a sum of squares — doesn't split.
14. The zeros are  $x = -3$  and  $x = 3$ . Pull the GCF 5:  $5(x^2 - 9)$ , then the difference of squares gives  $5(x - 3)(x + 3)$ .
15. The table hits 0 at  $x = -3$  and  $x = -2$ . Zeros at  $r$  give factors  $(x - r)$ , so  $(x + 3)$  and  $(x + 2)$ :  $P(x) = (x + 2)(x + 3)$ .
16. Start with the key idea:  $ac = -6$ . Need product  $-6$ , sum  $5$ : 6 and  $-1$ . Split:  $2x^2 + 6x - x - 3 = 2x(x + 3) - 1(x + 3) = (x + 3)(2x - 1)$ . That gives a quick check on the answer.
17. A careful way to see it: Group:  $x^2(x - 1) - 4(x - 1) = (x - 1)(x^2 - 4) = (x - 1)(x - 2)(x + 2)$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
18. This is a sum of squares with no common factor. Sum of squares does not factor over the reals — it's irreducible.
19. Perfect-square trinomial: first term  $(2x)^2$ , last term  $3^2$ , middle  $2(2x)(3) = 12x \checkmark$ . So  $(2x + 3)^2$ .
20. GCF  $6x$ :  $6x(2x^2 + 3x - 2)$ . AC method on  $2x^2 + 3x - 2$ :  $ac = -4$ , sum  $3$ : 4,  $-1$ . Split:  $2x^2 + 4x - x - 2 = 2x(x + 2) - 1(x + 2) = (x + 2)(2x - 1)$ . Full:  $6x(x + 2)(2x - 1)$ .
21. Pull GCF 3:  $3(x^2 - 4)$ . Then difference of squares:  $3(x - 2)(x + 2)$ . The room could be 3 feet by  $(x^2 - 4)$  feet, or  $(x - 2)$  by  $3(x + 2)$ , or  $(x + 2)$  by  $3(x - 2)$  — pick the factor grouping that matches the context.
22. Recognize the binomial-cube pattern  $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$  with  $a = x$ ,  $b = 4$ :  $x^3 + 12x^2 + 48x + 64 = (x + 4)^3$ . The side is  $(x + 4)$  inches. (For  $x = 2$ , the side is 6 and the volume is  $216 = 6^3$ . Sanity check passes.)
23. GCF 2:  $2(x^4 - 16)$ . Difference of squares:  $2(x^2 - 4)(x^2 + 4)$ . Factor again:  $x^2 - 4 = (x - 2)(x + 2)$ . The sum-of-squares factor  $x^2 + 4$  does not split over the integers.
24. Check the perfect-square pattern: first term  $(3x)^2 = 9x^2 \checkmark$ , last term  $5^2 = 25 \checkmark$ , middle  $2(3x)(5) = 30x$ , with a minus sign for the difference. So  $9x^2 - 30x + 25 = (3x - 5)^2$ . Quick check:  $(3x - 5)(3x - 5) = 9x^2 - 15x - 15x + 25 = 9x^2 - 30x + 25 \checkmark$ .



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