# Central Limit Theorem and Standard Error

Name: ______________ Date: ______________ Score: ______ / 35

## Quick Review

The **Central Limit Theorem** (CLT) is one of the most useful facts in statistics. Here's the punchline: take a population with *any* shape (skewed, uniform, bimodal – doesn't matter), draw a simple random sample of size $n$, and compute the sample mean $\bar{X}$. Now repeat that many times, collecting all the sample means. The *distribution of those sample means* is approximately normal – as long as $n$ is large enough (usually $n \geq 30$ does it).

**Three facts about the sampling distribution of** $\bar{X}$. (1) Its *mean* equals the population mean: $\mu_{\bar{X}} = \mu$. (2) Its *standard deviation*, called the **standard error**, is $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$. (3) Its *shape* is approximately normal for large $n$ regardless of the population's shape.

**Why** $\sqrt{n}$**, not** $n$**?** Bigger samples give more reliable means – but only as fast as $\sqrt{n}$, not as fast as $n$. To cut the standard error in *half*, you need to *quadruple* the sample size. This is exactly why polling organizations don't sample millions; the diminishing returns kick in fast.

**Computing a probability with the CLT.** If $\bar{X}$ is approximately normal with mean $\mu$ and standard error $\sigma/\sqrt{n}$, then $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ is approximately standard normal $N(0, 1)$. Use a $z$-table or calculator from there.

**Common slips.** Dividing by $n$ instead of $\sqrt{n}$ in the standard error. Forgetting the CLT needs a *large* sample (or a population that's already normal) before you can call $\bar{X}$ approximately normal. Confusing the standard deviation of the population ($\sigma$) with the standard error of the sample mean ($\sigma/\sqrt{n}$) – they're different quantities.

## PRACTICE

*Apply the CLT formulas and reason about how sample size changes the picture.*

1. Formula for standard error of $\bar{X}$ ________
2. Using row $1$ of the table, find the standard error $\sigma_{\bar{X}}$. ________

| $\sigma$ | $n$ |
|---|---|
| 20 | 100 |
| 15 | 25 |
| 12 | 36 |

3. Using row $2$ of the table above, find $\sigma_{\bar{X}}$. ________

| $\sigma$ | $n$ |
|---|---|
| 20 | 100 |
| 15 | 25 |
| 12 | 36 |

4. Using row $3$ of the table above, find $\sigma_{\bar{X}}$. ________

| $\sigma$ | $n$ |
|---|---|
| 20 | 100 |
| 15 | 25 |
| 12 | 36 |

5. If $n$ quadruples, standard error ________
6. If $n$ doubles, standard error scales by ________
7. What's the shape of $\bar{X}$ for large $n$? ________
8. $\mu_{\bar{X}} = ?$ ________
9. As $n \to \infty$, $\sigma_{\bar{X}} \to$ ________
10. If $\mu = 100$, $\sigma = 20$, $n = 400$, find $\sigma_{\bar{X}}$ ________

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11. Does the CLT require the population to be normal? ________
12. Sample size threshold for the CLT to apply (rule of thumb) ________
13. Compute $Z$ when $\bar{x} = 52, \mu = 50, \sigma/\sqrt{n} = 1$ ________
14. If sample size increases, what happens to the spread of $\bar{X}$? ________
15. $\sigma = 10,\ n = 64;\ \sigma_{\bar{X}}$ ________
16. True or false: every sample of size $n$ has $\bar{x} = \mu$ ________
17. To halve the standard error, sample size must ________
18. Standard deviation of population is $30,\ n = 9.\ \sigma_{\bar{X}} = ?$ ________
19. True or false: $\sigma_{\bar{X}} > \sigma$ ________
20. If $\mu = 70, \sigma = 14, n = 49$, what's $\sigma_{\bar{X}}$? ________

## ♦ Word Problems

21. A factory's lightbulbs have a mean lifespan of $\mu = 1200$ hours and a population standard deviation of $\sigma = 80$ hours. A quality-control sample of $n = 64$ bulbs is tested. What is the standard error of the sample mean lifespan? ____
22. A nationwide poll wants to cut its margin of error in half. By what factor must the polling sample size increase? ____
23. A teacher gives the same test every year. The population mean is $\mu = 72$ with $\sigma = 12$. This year's class of $n = 36$ students has a mean of $76$. How many standard errors above the long-run mean is this year's class? ____
24. A skeptic objects: "the salary data for this profession is heavily right-skewed, so we can't use a normal model." The data scientist replies that as long as $n$ is large enough, the *sample mean* is approximately normal. Which theorem is she invoking, and what's the rough sample-size threshold? ____

## Additional Practice

25. Find the mean of $4, 6, 8, 10$. ________
26. Find the median of $3, 9, 4, 10, 7$. ________
27. Find the range of $12, 5, 9, 20$. ________
28. Find the mode of $2, 3, 3, 5, 8$. ________
29. Find $z$ for $x = 72$, mean $60$, standard deviation $6$. ________
30. Interpret $z = -1.5$. ________
31. Predicted $y$ for $\hat{y} = 2x + 5$ at $x = 6$. ________
32. Residual if actual $= 20$ and predicted $= 17$. ________
33. Positive association: slope sign? ________
34. Margin of error $= 3\%$ around $58\%$. ________
35. Sample or census: survey every student. ________

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# Answer Keys

1. $\frac{\sigma}{\sqrt{n}}$
2. 2
3. 3
4. 2
5. halves
6. $\frac{1}{\sqrt{2}}$
7. approximately normal
8. $\mu$
9. 0
10. 1
11. No
12. $n \geq 30$
13. 2
14. decreases
15. 1.25
16. False
17. quadruple
18. 10
19. False
20. 2
21. $\sigma_{\bar{X}} = 10$ hours
22. 4
23. $Z = 2$
24. Central Limit Theorem; $n \geq 30$

### Additional Practice Answers

25. 7
26. 7
27. 15
28. 3
29. 2
30. 1.5 SD below mean
31. 17
32. 3
33. positive
34. 55% to 61%
35. census

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

**1.** A careful way to see it: Population std dev divided by the square root of the sample size. That gives a quick check on the answer.

**2.** Keep the rule visible: Row 1: $\frac{20}{\sqrt{100}} = \frac{20}{10} = 2$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**3.** One steady path is: Row 2: $\frac{15}{\sqrt{25}} = \frac{15}{5} = 3$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**4.** Start with the key idea: Row 3: $\frac{12}{\sqrt{36}} = \frac{12}{6} = 2$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**5.** A careful way to see it: $\sqrt{4n} = 2\sqrt{n}$, so dividing by $\sqrt{n}$ becomes dividing by $2\sqrt{n}$ – twice as much, half the standard error. That gives a quick check on the answer.

**6.** Keep the rule visible: $\sqrt{2n} = \sqrt{2}\sqrt{n}$, so standard error is divided by $\sqrt{2} \approx 1.414$ – it shrinks but not in half. That gives a quick check on the answer.

**7.** That's the CLT. The original population's shape doesn't matter once $n$ is big enough.

**8.** The mean of the sampling distribution always equals the population mean. The CLT changes the shape and the spread, not the center.

**9.** With infinite data, the sample mean equals the population mean exactly – no sampling error left.

**10.** Keep the rule visible: $\frac{20}{\sqrt{400}} = \frac{20}{20} = 1$. Big sample, tiny standard error. This is the part to check before moving on, because it keeps the answer tied to the original question.

**11.** That's the CLT's superpower: it works for almost any population shape, provided $n$ is large enough.

**12.** For most populations, $n = 30$ is good enough. Heavily skewed or odd populations may need larger $n$.

**13.** A careful way to see it: $Z = \frac{52 - 50}{1} = 2$. The sample mean is two standard errors above the population mean. That gives a quick check on the answer.

**14.** Larger $n$ shrinks the standard error – sample means cluster more tightly around $\mu$.

**15.** Standard error divides $\sigma$ by $\sqrt{n}$, not by $n$: $\frac{10}{\sqrt{64}} = \frac{10}{8} = 1.25$. Take the root of $n$ first ($\sqrt{64} = 8$), then divide.

**16.** Sample means vary around $\mu$ – that variability is exactly what standard error measures.

**17.** Since $\sigma_{\bar{X}} = \sigma/\sqrt{n}$, dividing the standard error by 2 means multiplying $\sqrt{n}$ by 2, which means multiplying $n$ by 4.

**18.** Keep the rule visible: $\frac{30}{\sqrt{9}} = \frac{30}{3} = 10$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**19.** One steady path is: $\sigma_{\bar{X}} = \sigma/\sqrt{n} < \sigma$ for any $n > 1$. Averaging shrinks variability. This is the part to check before moving on, because it keeps the answer tied to the original question.

**20.** Start with the key idea: $\frac{14}{\sqrt{49}} = \frac{14}{7} = 2$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**21.** Use the standard error formula: $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{80}{\sqrt{64}} = \frac{80}{8} = 10$ hours. So typical sample means of 64 bulbs land about 10 hours from the true mean of 1200 – considerably tighter than the population's 80-hour spread.

**22.** Margin of error scales like $\sigma/\sqrt{n}$. To halve it, $\sqrt{n}$ must double, which means $n$ must quadruple. So a poll that currently surveys 400 people would need to survey 1,600 to halve its margin of error – diminishing returns kick in fast, which is why polls rarely go past a few thousand.

**23.** Standard error: $\frac{12}{\sqrt{36}} = 2$. $Z = \frac{\bar{x} - \mu}{\sigma_{\bar{X}}} = \frac{76 - 72}{2} = 2$. The class is 2 standard errors above the typical mean – a noticeably strong performance (about the 97.5th percentile under a normal model). Of course, one good year doesn't necessarily mean lasting change.

**24.** The Central Limit Theorem says the sampling distribution of $\bar{X}$ is approximately normal for large $n$, *no matter what* the underlying population's shape looks like. A common rule of thumb is $n \geq 30$ – though heavily skewed data sometimes needs more. So the data scientist is right: skewed population $\neq$ skewed sampling distribution of $\bar{X}$ when $n$ is big.

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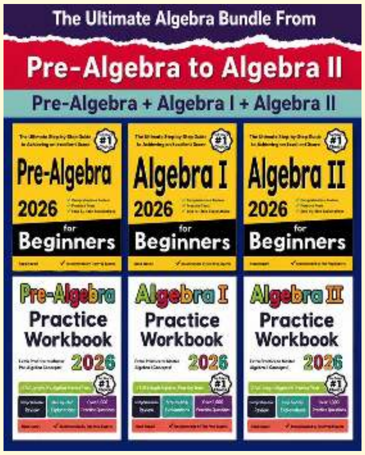


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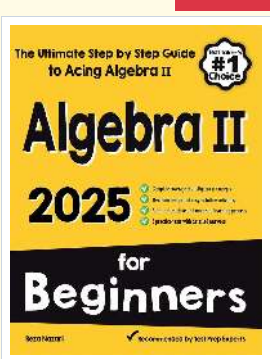


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