Center and Radius of Circles

Name: _____	Date: _____	Score: _____ / 30

Quick Review

A circle's standard form $(x-h)^2+(y-k)^2=r^2$ packs two key facts: the center (h,k) and the radius r. Pulling those out is sometimes a one-line read and sometimes asks for a completing-the-square pass first.

Reading from standard form. The center signs flip: $(x-1)^2+(y+4)^2=9$ has center $(1,-4)$. The right side is r^2, so take the square root: $r=\sqrt{9}=3$.

Reading from origin-centered form. $x^2+y^2=36$ has center $(0,0)$ and $r=\sqrt{36}=6$. Whenever there are no linear x or y terms, the center sits at the origin.

From a diameter's endpoints. The center is the midpoint of the two endpoints; the radius is half the distance between them (or, equivalently, the distance from the midpoint to either endpoint – both routes work).

From general form $x^2+y^2+Dx+Ey+F=0$. Complete the square. Group: $(x^2+Dx)+(y^2+Ey)=-F$. Add $(D/2)^2$ and $(E/2)^2$ to both sides. The result is $(x+D/2)^2+(y+E/2)^2=(D/2)^2+(E/2)^2-F$. Center $(-D/2,-E/2)$; r^2 is the right side. Quick check: $x^2+y^2-6x+8y=0$: center is $(-(-6)/2,-(8)/2)=(3,-4)$.

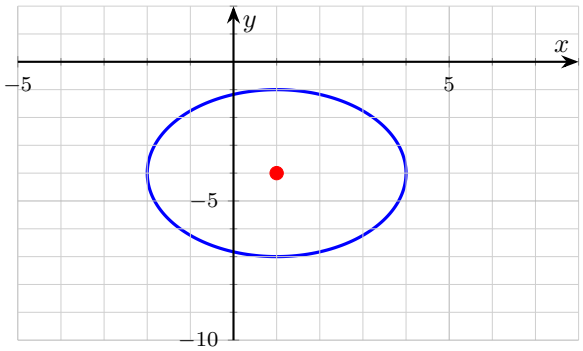
Always sanity-check $r^2>0$. If you complete the square and get $r^2\le 0$, the equation either describes a single point ($r^2=0$) or no real circle at all ($r^2<0$). Real, nondegenerate circles need $r^2>0$.

Common slips. Treating $r^2=9$ as if $r=9$ (it's 3). Reading $(x+4)$ as $h=+4$ (it's -4). Adding the completing-the-square constants to one side but not the other. Confusing diameter with radius (the radius is *half* the diameter).

PRACTICE

For each circle, find the center and radius. Use whichever method matches the given form: read directly from standard form, complete the square, or use the midpoint and distance formulas.

1. Find the center and radius of $(x-1)^2+(y+4)^2=9$. _____



2. Find the radius of $x^2+y^2=36$. _____

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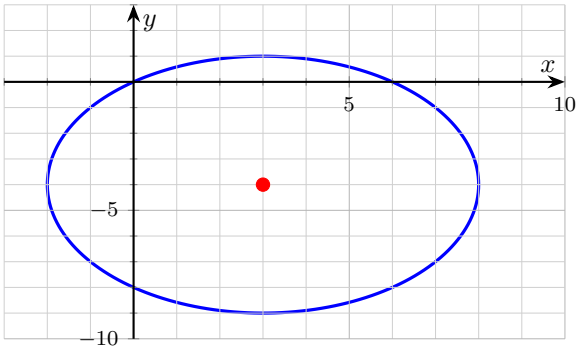
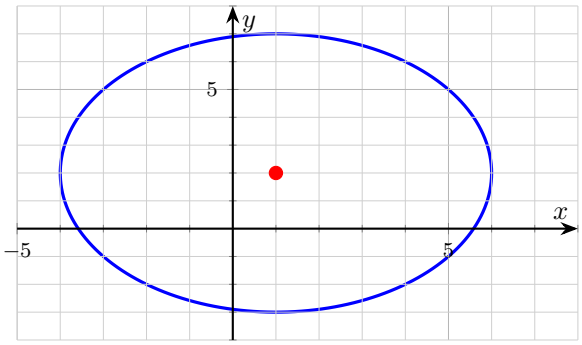
3. Find the center of $x^2 + y^2 - 6x + 8y = 0$. _____



4. Find the radius of $x^2 + y^2 + 4x - 6y + 9 = 0$. _____
5. A circle has a diameter with endpoints $(2, 1)$ and $(-4, 9)$. Find its center. _____
6. A circle has a diameter with endpoints $(0, 0)$ and $(6, 8)$. Find its radius. _____
7. Convert $x^2 + y^2 + 8x - 2y + 13 = 0$ to standard form. _____
8. What are the center and radius of $x^2 + y^2 - 10x - 2y + 17 = 0$? _____
9. A circle has a diameter with endpoints $(-6, 2)$ and $(2, 8)$. Find the center and radius. _____
10. Mark TRUE or FALSE: For a circle in standard form, the radius is the diameter divided by 2. _____
11. Find the center and radius of $x^2 + y^2 + 10x - 2y - 10 = 0$. _____
12. Find the radius of $3x^2 + 3y^2 - 12x + 18y - 30 = 0$. _____
13. What is the center of the circle with equation $x^2 + y^2 - 8x - 10y + 1 = 0$? _____
14. A circle has center $P(1, 2)$ and the point $Q(4, 6)$ lies on the circle. Find the radius. _____



15. Mark TRUE or FALSE: If $x^2 + y^2 + Dx + Ey + F = 0$ has $(D/2)^2 + (E/2)^2 - F < 0$, then the equation has no real solutions. _____
16. Find the center and radius of $(x + 7)^2 + (y - 3)^2 = 64$. _____
17. The segment $\overline{AB}$ is a diameter with $A(-3, 4)$ and $B(5, -2)$. Find the center and radius. _____
18. Find the radius of $2x^2 + 2y^2 + 4x - 8y + 2 = 0$. _____
19. A circle has equation $x^2 + y^2 = 64$. Find the diameter. _____
20. Find the center of a circle whose diameter has endpoints $(3, -5)$ and $(-3, 7)$. _____

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Word Problems

21. A circular swimming pool is described by the equation $x^2 + y^2 - 12x + 8y - 12 = 0$ (in meters, on a map of the backyard). Find the center, the radius, and the surface area of the pool. ____

22. A radar installation at the origin tracks aircraft within a circular range. Two extreme detections were recorded at $(20, 15)$ km and $(-20, -15)$ km, marking opposite points on the boundary of the range. Find the radar's maximum detection radius. ____

23. A circular garden has its center at $(-2, 3)$, and a sprinkler is mounted at the point $(4, 11)$ on the garden's edge. Find the radius of the garden, and write its equation in standard form. ____

24. A circle's general form is given as $x^2 + y^2 - 4x - 6y + 9 = 0$. A student claims the radius is 9 because 9 is the constant in the equation. Find the correct center and radius, and explain why the student's reasoning fails. ____

Additional Practice

25. Center and radius of $(x - 3)^2 + (y + 2)^2 = 25$. _____

26. Write a circle with center $(0, 0)$ and radius 7. _____

27. Find the radius of $x^2 + y^2 = 64$. _____

28. Find the center of $(x + 5)^2 + (y - 1)^2 = 9$. _____

29. Vertex of $y = (x - 4)^2 + 6$. _____

30. Axis of symmetry of $y = (x + 2)^2 - 3$. _____

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Answer Keys

1. center $(1, -4)$; $r = 3$
2. 6
3. $(3, -4)$
4. 2
5. $(-1, 5)$
6. 5
7. $(x + 4)^2 + (y - 1)^2 = 4$
8. center $(5, 1)$; $r = 3$
9. center $(-2, 5)$; $r = 5$
10. TRUE
11. center $(-5, 1)$; $r = 6$
12. $r = \sqrt{23}$
13. $(4, 5)$
14. $r = 5$
15. TRUE
16. center $(-7, 3)$; $r = 8$
17. center $(1, 1)$; $r = 5$
18. $r = \sqrt{4} = 2$
19. 16
20. $(0, 1)$
21. center $(6, -4)$; $r = 8$ m; $A = 64\pi \approx 201.06$ m^2
22. $r = 25$ km
23. $r = 10$; $(x + 2)^2 + (y - 3)^2 = 100$
24. center $(2, 3)$; $r = 2$

Additional Practice Answers

25. $(3, -2)$, $r = 5$
26. $x^2 + y^2 = 49$
27. 8
28. $(-5, 1)$
29. $(4, 6)$
30. $x = -2$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Standard form: $h = 1$, $k = -4$ (sign flip on $(y + 4)$), $r = \sqrt{9} = 3$. That gives a quick check on the answer.

2. With no linear x or y terms, the center is the origin and r^2 is the right side. Here $r^2 = 36$, so $r = \sqrt{36} = 6$ (not 36).

3. Shortcut: center is $(-D/2, -E/2) = (-(-6)/2, -(8)/2) = (3, -4)$. Or complete the square: $(x - 3)^2 + (y + 4)^2 = 9 + 16 = 25$. Center $(3, -4)$, $r = 5$.

4. Group and move the constant: $(x^2 + 4x) + (y^2 - 6y) = -9$. Half of 4 is 2 (4); half of -6 is -3 (9). Add both to each side: $(x+2)^2+(y-3)^2 = -9+4+9 = 4$. So $r^2 = 4$ and $r = \sqrt{4} = 2$.

5. The center is the midpoint of the diameter – average the two endpoints: $\left(\frac{2 + (-4)}{2}, \frac{1 + 9}{2}\right) = \left(\frac{-2}{2}, \frac{10}{2}\right) = (-1, 5)$.

6. The diameter is the distance between the endpoints: $\sqrt{(6 - 0)^2 + (8 - 0)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$. The radius is half the diameter: $r = \frac{10}{2} = 5$. (A 6-8-10 triangle, the 3-4-5 scaled by 2.)

7. Group and move the constant: $(x^2 + 8x) + (y^2 - 2y) = -13$. Half of 8 is 4 (16); half of -2 is -1 (1). Add both to each side: $(x + 4)^2 + (y - 1)^2 = -13 + 16 + 1 = 4$. (Center $(-4, 1)$, $r = 2$.)

8. Group and move the constant: $(x^2 - 10x) + (y^2 - 2y) = -17$. Half of -10 is -5 (25); half of -2 is -1 (1). Add both to each side: $(x - 5)^2 + (y - 1)^2 = -17 + 25 + 1 = 9$. Center $(5, 1)$, $r = \sqrt{9} = 3$.

9. Center is the midpoint: $\left(\frac{-6 + 2}{2}, \frac{2 + 8}{2}\right) = (-2, 5)$. Diameter length: $\sqrt{(2 - (-6))^2 + (8 - 2)^2} = \sqrt{64 + 36} = 10$, so the radius is half: $r = 5$. (Another 6-8-10 triple.)

10. Diameter is the longest chord, passing through the center; radius is half of it.

11. Group and move the constant: $(x^2 + 10x) + (y^2 - 2y) = 10$. Half of 10 is 5 (25); half of -2 is -1 (1). Add both to each side: $(x + 5)^2 + (y - 1)^2 = 10 + 25 + 1 = 36$. Center $(-5, 1)$, $r = \sqrt{36} = 6$.

12. Divide through by 3 first to get coefficient 1 on the squared terms: $x^2 + y^2 - 4x + 6y - 10 = 0$. Complete: $(x - 2)^2 + (y + 3)^2 = 10 + 4 + 9 = 23$. So $r = \sqrt{23} \approx 4.80$.

13. For $x^2 + y^2 + Dx + Ey + F = 0$, the center is $(-D/2, -E/2)$. Here $D = -8$ and $E = -10$, so the center is $\left(-\frac{-8}{2}, -\frac{-10}{2}\right) = (4, 5)$.

14. Keep the rule visible: $r =$ distance from P to $Q = \sqrt{(4 - 1)^2 + (6 - 2)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$. That gives a quick check on the answer.

15. That expression equals r^2 after completing the square. If it's negative, no real (x, y) pair satisfies the equation – there's no real circle.

16. Read directly from standard form. $(x + 7) = (x - (-7))$ gives $h = -7$, and $(y - 3)$ gives $k = 3$. The right side is $r^2 = 64$, so $r = \sqrt{64} = 8$.

17. Center is the midpoint of $\overline{AB}$: $\left(\frac{-3 + 5}{2}, \frac{4 + (-2)}{2}\right) = (1, 1)$. Radius is the distance from $(1, 1)$ to either endpoint, say $A(-3, 4)$: $\sqrt{(-3 - 1)^2 + (4 - 1)^2} = \sqrt{16 + 9} = 5$. (Another 3-4-5.)

18. First divide everything by 2 so the squared terms have coefficient 1: $x^2 + y^2 + 2x - 4y + 1 = 0$. Group: $(x^2 + 2x) + (y^2 - 4y) = -1$. Half of 2 is 1 (1); half of -4 is -2 (4). Add both: $(x+1)^2+(y-2)^2 = -1+1+4 = 4$, so $r = \sqrt{4} = 2$.

19. Centered at the origin, $r^2 = 64$, so $r = \sqrt{64} = 8$. The diameter is twice the radius: $2r = 16$.

20. The center is the midpoint of the diameter – average the endpoints: $\left(\frac{3 + (-3)}{2}, \frac{-5 + 7}{2}\right) = \left(\frac{0}{2}, \frac{2}{2}\right) = (0, 1)$.

21. Group: $(x^2 - 12x) + (y^2 + 8y) = 12$. Half of -12 is -6 (square: 36); half of 8 is 4 (square: 16). Add to both sides: $(x - 6)^2 + (y + 4)^2 = 12 + 36 + 16 = 64$. Center $(6, -4)$, radius $\sqrt{64} = 8$ m. Surface area: $\pi r^2 = 64\pi \approx 201.06$ m^2. (That's a generous home pool – about 2,164 square feet of water.)

22. The two detections are diametrically opposite (both at the boundary, on opposite sides). Their midpoint is $\left(\frac{20 + (-20)}{2}, \frac{15 + (-15)}{2}\right) = (0, 0)$, confirming the center sits at the radar installation. The diameter is the distance between them: $\sqrt{40^2 + 30^2} = \sqrt{2500} = 50$ km. Radius: half of that, 25 km. **Verify:** distance from $(0, 0)$ to $(20, 15)$ is $\sqrt{400 + 225} = \sqrt{625} = 25$ ✓. (Another scaled 3-4-5 triple: legs 20 and 15, hypotenuse 25.)

23. Radius is the distance from center $(-2, 3)$ to point $(4, 11)$: $r = \sqrt{(4 - (-2))^2 + (11 - 3)^2} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$. (A 6-8-10 right triangle, twice the 3-4-5.) Equation: $(x + 2)^2 + (y - 3)^2 = 100$. The sprinkler reaches exactly to the edge, so any point on the boundary satisfies equality.

24. Complete the square: $(x^2 - 4x) + (y^2 - 6y) = -9$. Half of -4 is -2 (square: 4); half of -6 is -3 (square: 9). Add to both sides: $(x - 2)^2 + (y - 3)^2 = -9 + 4 + 9 = 4$. Center $(2, 3)$, radius $\sqrt{4} = 2$. **The student's error:** reading the constant from the general form as if it were r^2. The general form mixes linear and constant terms in a way that hides the radius – you have to convert to standard form first. Even after completing the square, you square-root the right side. Lesson: never read the radius (or center) off the general form. Always go to standard form first.

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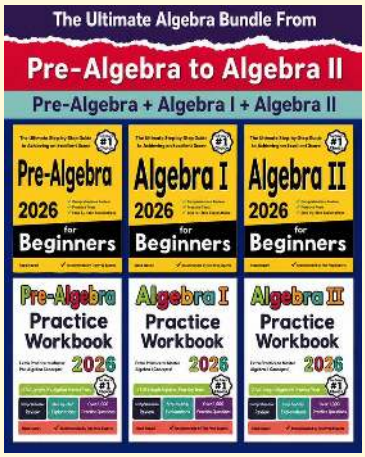

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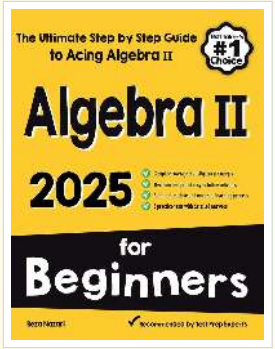

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