Binomial Theorem

Name:	Date:	Score: ______ / 34

Quick Review

The **Binomial Theorem** writes $(a+b)^n$ as a sum:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k.$$

What each piece does. Coefficient $\binom{n}{k}$ comes from row n of Pascal's triangle. Exponent on a decreases from n down to 0; exponent on b increases from 0 up to n; the two always sum to n. There are exactly $n+1$ terms.

Term-by-term. The kth term (zero-indexed) is $T_k = \binom{n}{k} a^{n-k} b^k$. So in $(a+b)^5$:

$$T_0 = a^5,\ T_1 = 5a^4b,\ T_2 = 10a^3b^2,\ T_3 = 10a^2b^3,\ T_4 = 5ab^4,\ T_5 = b^5.$$

Differences $(a-b)^n$**.** Treat as $(a+(-b))^n$. Signs alternate because each power of $-b$ supplies a ± 1. Odd k gives a negative term; even k keeps it positive.

Finding a specific term. If you only need the coefficient of (say) x^7 in $(2x+3)^{10}$, don't expand everything. The x^7 term comes from $\binom{10}{3}(2x)^7(3)^3 = 120 \cdot 128x^7 \cdot 27 = 414720x^7$. The exponent on a is what tells you which k to use: $a^{n-k} = x^7$, with $n = 10$, gives $k = 3$.

Common slips. Forgetting to raise the inner constants too: $(2x+3)^4$ has more than just row-4 coefficients – each term picks up powers of 2 and 3. Sign mistakes on $(a-b)^n$. Off-by-one between term index and exponent: term k (zero-indexed) has b^k, but if a problem says "the fourth term" it usually means $k = 3$ (one-indexed counting).

PRACTICE

Expand binomial powers or extract a single specified term. Use row n of Pascal's triangle for the coefficient; remember the powers of any inner constants.

1. Expand $(a+b)^2$. ________

2. Expand $(a+b)^3$. ________

3. Use the Pascal-triangle row shown to expand $(x+2)^4$. ________

$\binom{4}{k}$	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
value	1	4	6	4	1

4. Coefficient of x^3 in $(x+1)^5$. ________

5. The table gives row 4 of Pascal's triangle. Use it to find the coefficient of x^2 in $(x+3)^4$. ________

k	0	1	2	3	4
$\binom{4}{k}$	1	4	6	4	1

6. Expand $(2x-1)^3$. ________

7. The fourth term (one-indexed) in the expansion of $(x+2)^8$. ________

8. Coefficient of x^2y^4 in $(3x-y)^6$. ________

9. Term containing x^3 in $(2x-3)^5$. ________

10. Expand $(x-1)^4$. ________

11. Coefficient of x^3 in $(3x-1)^5$. ________

12. True or False: $(a+b)^n = a^n + b^n$ for $n \geq 2$. ________

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13. The table shows how many terms $(x+y)^n$ has after expanding. How many terms does $(x+y)^{12}$ have? ________

n	2	3	4	5
# terms	3	4	5	6

14. Coefficient of x^4 in $(x+5)^7$. ________

15. The constant term in $(x+\frac{1}{x})^4$. ________

16. Sum of all coefficients in $(2x+3)^4$. ________

17. Coefficient of x^5 in $(x+2)^7$. ________

18. Coefficient of x^3 in $(1+x)^{10}$. ________

19. Expand $(x+y)^4$. ________

20. Coefficient of a^3b^2 in $(a-2b)^5$. ________

♦ Word Problems

21. A flipped coin lands heads with probability $\frac{1}{2}$. The probability of exactly 3 heads in 5 flips comes from a single term of the binomial expansion of $(\frac{1}{2}+\frac{1}{2})^5=1$. Find that probability. ____

22. A polynomial $(x+a)^4$ contains the term $32x^3$ when expanded. Find a. ____

23. A textbook problem asks for the 5th term (one-indexed) in the expansion of $(x+y)^8$. Identify the term and its coefficient. ____

24. A genetics model says each offspring has a $\frac{1}{4}$ chance of showing a particular trait. For 6 offspring, the probability of exactly 2 showing the trait equals a term from the binomial expansion of $(\frac{3}{4}+\frac{1}{4})^6$. Find that probability as a fraction. ____

Additional Practice

25. Find the next term: $4, 9, 14, 19, \ldots$. ________

26. Find a_{10} if $a_1=3$ and $d=5$. ________

27. Find the next term: $2, 6, 18, 54, \ldots$. ________

28. Find a_6 if $a_1=5$ and $r=2$. ________

29. Sum $1+2+3+\cdots+20$. ________

30. Find S_5 for $3, 6, 12, 24, 48$. ________

31. Common difference of $12, 7, 2, -3, \ldots$. ________

32. Common ratio of $81, 27, 9, 3, \ldots$. ________

33. Evaluate $\sum_{k=1}^{4} 2k$. ________

34. Find $\binom{6}{2}$. ________

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Answer Keys

1. $\boxed{a^2 + 2ab + b^2}$
2. $\boxed{a^3 + 3a^2b + 3ab^2 + b^3}$
3. $\boxed{x^4 + 8x^3 + 24x^2 + 32x + 16}$
4. $\boxed{10}$
5. $\boxed{54}$
6. $\boxed{8x^3 - 12x^2 + 6x - 1}$
7. $\boxed{448x^5}$
8. $\boxed{135}$
9. $\boxed{720x^3}$
10. $\boxed{x^4 - 4x^3 + 6x^2 - 4x + 1}$
11. $\boxed{270}$
12. $\boxed{\text{False}}$
13. $\boxed{13}$
14. $\boxed{4375}$
15. $\boxed{6}$
16. $\boxed{625}$
17. $\boxed{84}$
18. $\boxed{120}$
19. $\boxed{x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4}$
20. $\boxed{40}$
21. $\boxed{\dfrac{5}{16}}$
22. $\boxed{a = 8}$
23. $\boxed{70x^4y^4}$
24. $\boxed{\dfrac{1215}{4096}}$

Additional Practice Answers

25. $\boxed{24}$
26. $\boxed{48}$
27. $\boxed{162}$
28. $\boxed{160}$
29. $\boxed{210}$
30. $\boxed{93}$
31. $\boxed{-5}$
32. $\boxed{\frac{1}{3}}$
33. $\boxed{20}$
34. $\boxed{15}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Row 2 of Pascal: $1, 2, 1$. So $(a+b)^2 = a^2 + 2ab + b^2$. That middle $2ab$ is the trap that "$(a+b)^2 = a^2 + b^2$" misses.

2. Row 3: $1, 3, 3, 1$. Exponents on a: $3, 2, 1, 0$; on b: $0, 1, 2, 3$. So $a^3 + 3a^2b + 3ab^2 + b^3$.

3. Row 4 supplies the coefficients $1, 4, 6, 4, 1$. Build each term with the matching power of 2: $x^4 + 4x^3(2) + 6x^2(4) + 4x(8) + (16) = x^4 + 8x^3 + 24x^2 + 32x + 16$.

4. In $(x+1)^5$, write $a = x$, $b = 1$. The x^3 term needs $a^{5-k} = x^3$, so $k = 2$, giving $\binom{5}{2}x^3(1)^2 = 10\,x^3$. So the coefficient is 10 (and the 1^2 doesn't change it).

5. The x^2 term uses $k = 2$, so $\binom{4}{2} = 6$ (read from the table). Don't forget the $(3)^2$: $\binom{4}{2}x^2(3)^2 = 6 \cdot 9\,x^2 = 54x^2$. So 54.

6. Row 3: $1, 3, 3, 1$. Build with $a = 2x$, $b = -1$: $(2x)^3 + 3(2x)^2(-1) + 3(2x)(-1)^2 + (-1)^3 = 8x^3 - 12x^2 + 6x - 1$. Signs flip on every odd power of -1.

7. "Fourth term" means $k = 3$ (zero-indexed). So $T_3 = \binom{8}{3}x^{8-3}(2)^3 = 56x^5 \cdot 8 = 448x^5$.

8. Start with the key idea: $\binom{6}{4}(3x)^2(-y)^4 = 15 \cdot 9x^2 \cdot y^4 = 135x^2y^4$. ($-1$ to an even power is $+1$.) That gives a quick check on the answer.

9. A careful way to see it: $\binom{5}{2}(2x)^3(-3)^2 = 10 \cdot 8x^3 \cdot 9 = 720x^3$. (Even power of -3 keeps it positive.) That gives a quick check on the answer.

10. Keep the rule visible: Row 4: $1, 4, 6, 4, 1$. With $b = -1$: $x^4 - 4x^3 + 6x^2 - 4x + 1$. Signs alternate. That gives a quick check on the answer.

11. In $(3x-1)^5$, the x^3 term comes from $(3x)^3$, so the power on $3x$ is 3 and $k = 2$. That gives $\binom{5}{2}(3x)^3(-1)^2 = 10 \cdot 27x^3 \cdot 1 = 270x^3$. Don't forget the $3^3 = 27$ from inside the binomial – that's where hand-calculations slip.

12. The expansion has middle terms with binomial coefficients and mixed powers. $(a+b)^2 = a^2 + 2ab + b^2 \neq a^2 + b^2$ as soon as both a, b are nonzero.

13. From the table, $(x+y)^n$ expands to $n+1$ terms (one for each k from 0 to n). So $(x+y)^{12}$ has 13 terms.

14. Keep the rule visible: $\binom{7}{3}x^4(5)^3 = 35 \cdot 125 \cdot x^4 = 4375x^4$. So 4375. (Carry the $(5)^3 = 125$ – this is where hand-calculations slip.) That gives a quick check on the answer.

15. Constant means x^0. From $(x + x^{-1})^4$: term $\binom{4}{k}x^{4-k}x^{-k} = \binom{4}{k}x^{4-2k}$. For x^0, $4 - 2k = 0 \Rightarrow k = 2$. So $\binom{4}{2} = 6$.

16. Plug in $x = 1$: $(2+3)^4 = 5^4 = 625$. (Substituting $x = 1$ into a polynomial gives the sum of all its coefficients.)

17. In $(x+2)^7$, the x^5 term needs $x^{7-k} = x^5$, so $k = 2$. That gives $\binom{7}{2}x^5(2)^2 = 21 \cdot 4\,x^5 = 84x^5$. Carry the $2^2 = 4$ from the constant piece – the coefficient is 84.

18. In $(1+x)^{10}$, the x^3 term is $\binom{10}{3}(1)^7x^3$. Since the 1's contribute nothing, the coefficient is just $\binom{10}{3} = \dfrac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$. (Same as row 10, position 3.)

19. Row 4: $1, 4, 6, 4, 1$. Use them in order: $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.

20. Start with the key idea: $\binom{5}{2}a^3(-2b)^2 = 10 \cdot 4 \cdot a^3b^2 = 40a^3b^2$. (Even power of -2 gives $+4$.) That gives a quick check on the answer.

21. The term that counts "3 heads in 5 flips" is $\binom{5}{3}(\frac{1}{2})^3(\frac{1}{2})^2 = 10 \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32} = \frac{5}{16}$. So probability $\frac{5}{16}$ (about 31.25%). (Reality check: a positive number between 0 and 1.)

22. The x^3 term in $(x+a)^4$ is $\binom{4}{1}x^3(a)^1 = 4ax^3$. Set $4a = 32$, so $a = 8$. (Reality check: $(x+8)^4$ has first two terms $x^4 + 32x^3$ ✓.)

23. Fifth term, one-indexed, means $k = 4$. $T_4 = \binom{8}{4}x^{8-4}y^4 = 70x^4y^4$. So the term is $70x^4y^4$, coefficient 70. (Sanity check: row 8 of Pascal's triangle is $1, 8, 28, 56, 70, 56, 28, 8, 1$ – position 4 is 70 ✓.)

24. The relevant term is $\binom{6}{2}(\frac{3}{4})^4(\frac{1}{4})^2 = 15 \cdot \dfrac{81}{256} \cdot \dfrac{1}{16} = \dfrac{15 \cdot 81}{4096} = \dfrac{1215}{4096}$. So about $\dfrac{1215}{4096} \approx 0.297$, or roughly 29.7% – which is a sensible probability.

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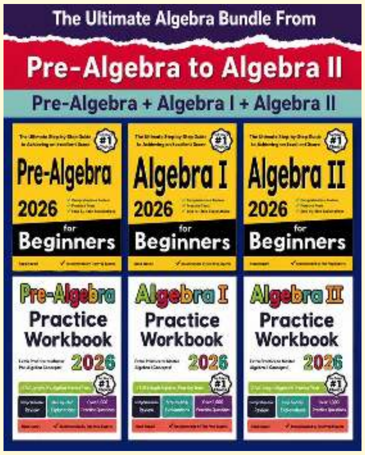

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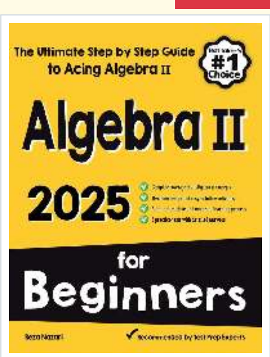

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