

# Average Rate of Change of a Function

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 37

## Q Quick Review

The **average rate of change** (ARC) of a function  $f$  on  $[a, b]$  is  $\frac{f(b) - f(a)}{b - a}$  — change in output over change in input. Geometrically, that's the **slope of the secant line** from  $(a, f(a))$  to  $(b, f(b))$ .

**Linear functions** have the same ARC on every interval, equal to the slope  $m$ . For nonlinear functions, ARC depends on which two points you pick. Quick check:  $f(x) = x^2$  on  $[1, 3]$ :  $\frac{9 - 1}{3 - 1} = 4$ . On  $[0, 2]$ :  $\frac{4 - 0}{2 - 0} = 2$ . Different intervals, different rates.

**Negative, zero, or positive.** ARC is positive when the function rose between  $a$  and  $b$ , negative when it fell, zero when  $f(a) = f(b)$  (such as when a ball returns to the same height).  $h(t) = -16t^2 + 64t + 5$  on  $[1, 3]$ :  $h(1) = 53, h(3) = 53$ , so  $ARC = 0$  ft/s — the ball went up and came back.

**Units.** Always carry the units. Distance vs. time gives speed (mi/h or m/s). Cost vs. items gives marginal cost (\$/item). A negative ARC for a population model means shrinking, not “invalid.”

**Trap.** ARC is *not* the same as the slope of the curve at a single point (that's calculus's derivative). It's an *average* over the chosen interval.

## PRACTICE

Compute the average rate of change. Include units when given.

1. ARC formula on  $[a, b]$  \_\_\_\_\_
2.  $f(x) = 3x + 1$  on  $[0, 4]$ . ARC? \_\_\_\_\_
3.  $f(x) = x^2$  on  $[1, 3]$ . ARC? \_\_\_\_\_
4.  $f(x) = -x^2 + 5$  on  $[0, 2]$ . ARC? \_\_\_\_\_
5. Use the table of  $g$  to find the average rate of change on  $[1, 4]$ . \_\_\_\_\_

$x$	0	1	2	3	4
$g(x)$	5	3	-1	-7	-15

6.  $f(x) = x^2 - 4x + 3$  on  $[0, 4]$ . ARC? \_\_\_\_\_
7.  $d(t) = 4t^2$  on  $[1, 3]$ . ARC? \_\_\_\_\_
8.  $h(t) = -16t^2 + 64t + 5$  on  $[1, 3]$ . ARC? \_\_\_\_\_
9.  $f(x) = x^2 - 2x$  on  $[2, 5]$ . ARC? \_\_\_\_\_
10. True or false: a linear function has the same ARC on every interval. \_\_\_\_\_
11.  $f(x) = 2^x$  on  $[0, 3]$ . ARC? \_\_\_\_\_
12.  $f(x) = \sqrt{x}$  on  $[1, 9]$ . ARC? \_\_\_\_\_
13. For data  $(x, y)$ :  $(1, 5), (4, 17)$ . ARC of  $y$  wrt  $x$ ? \_\_\_\_\_
14.  $f(x) = x^3$  on  $[-1, 1]$ . ARC? \_\_\_\_\_
15.  $f(x) = |x|$  on  $[-3, 2]$ . ARC? \_\_\_\_\_
16. Average rate is zero on  $[a, b]$ . What can you conclude about  $f(a)$  vs  $f(b)$ ? \_\_\_\_\_
17.  $P(t)$  models a town's population. ARC on  $[2010, 2020]$  is  $-150/\text{year}$ . Interpret. \_\_\_\_\_
18.  $f(x) = x^2 + 2x$  on  $[h, h + 1]$  in terms of  $h$ . ARC? \_\_\_\_\_



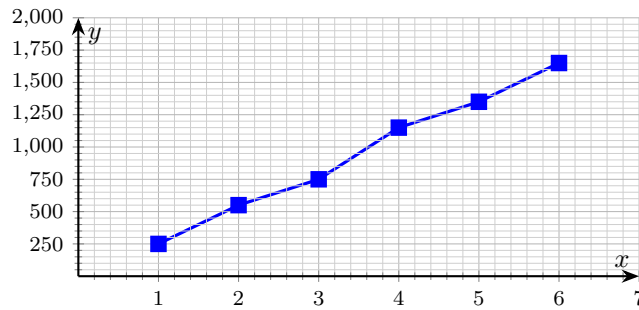
19.  $f$  is given by the table below. ARC on  $[2, 5]$ ? \_\_\_\_\_

$x$	0	1	2	3	4	5
$f(x)$	8	5	3	2	3	8

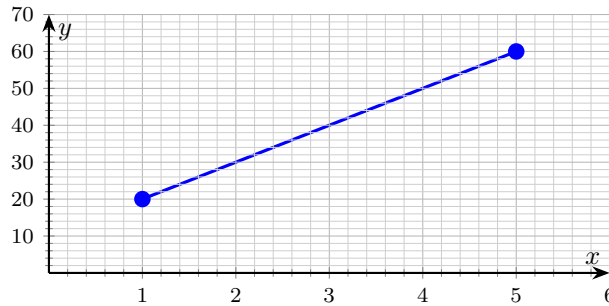
20.  $f(x) = 2x^2 - 3x$  on  $[-1, 2]$ . ARC? \_\_\_\_\_

◆ Word Problems

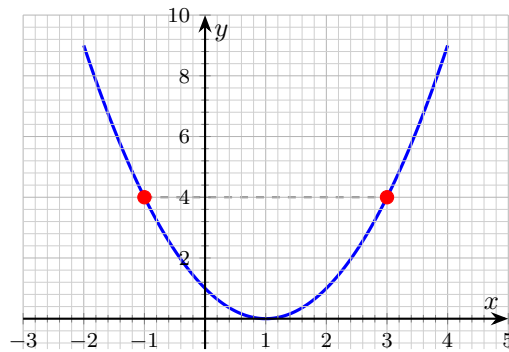
21. Liam invested \$250 in a small business. The graph shows the balance at the end of each month. Find the average rate of change of the balance from month 2 to month 6. \_\_\_\_\_



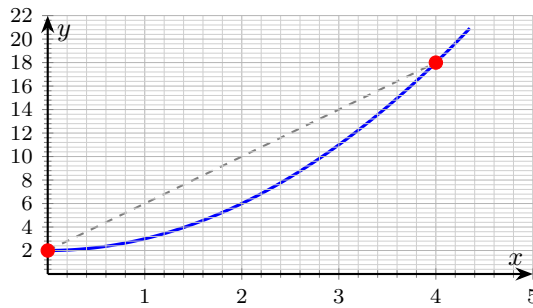
22. The graph shows a cyclist's distance from home. Find the average rate of change from hour 1 to hour 5. \_\_\_\_\_



23. The graph of  $f(x) = (x - 1)^2$  has two marked points. Find the ARC on  $[-1, 3]$ . \_\_\_\_\_



24. A tile pattern grows: after  $t$  minutes the count is  $N(t) = t^2 + 2$ . Find the ARC from  $t = 0$  to  $t = 4$ . \_\_\_\_\_



**Additional Practice**

- 25. If  $f(x) = 2x - 5$ , find  $f(4)$ . \_\_\_\_\_
- 26. If  $g(x) = x^2 + 1$ , find  $g(-3)$ . \_\_\_\_\_
- 27. For  $f(x) = 3x + 2$ , solve  $f(x) = 14$ . \_\_\_\_\_
- 28. Find  $(f + g)(x)$  if  $f = x + 1$ ,  $g = 2x - 5$ . \_\_\_\_\_
- 29. Find  $(fg)(x)$  if  $f = x - 2$ ,  $g = x + 3$ . \_\_\_\_\_
- 30. Find  $f(g(x))$  if  $f(x) = 2x$ ,  $g(x) = x + 7$ . \_\_\_\_\_
- 31. Find the inverse of  $f(x) = x - 9$ . \_\_\_\_\_
- 32. Find the inverse of  $f(x) = 3x + 1$ . \_\_\_\_\_
- 33. Domain of  $f(x) = \sqrt{x - 4}$ . \_\_\_\_\_
- 34. Domain of  $f(x) = \frac{1}{x + 6}$ . \_\_\_\_\_
- 35. Parent function for  $y = |x| + 3$ . \_\_\_\_\_
- 36. Shift  $y = x^2$  left 4. \_\_\_\_\_
- 37. Average rate from  $(1, 5)$  to  $(4, 17)$ . \_\_\_\_\_



## Answer Keys

1.  $\frac{f(b) - f(a)}{b - a}$

2. 3

3. 4

4. -2

5. -6

6. 0

7. 16

8. 0 ft/s

9. 5

10. true

11.  $\frac{7}{3}$

12.  $\frac{1}{4}$

## Additional Practice Answers

25. 3

26. 10

27.  $x = 4$

28.  $3x - 4$

29.  $x^2 + x - 6$

30.  $2x + 14$

31.  $f^{-1}(x) = x + 9$

13. 4

14. 1

15.  $-\frac{1}{5}$

16.  $f(a) = f(b)$

17. population shrank by about 150/yr on average

18.  $2h + 3$

19.  $\frac{5}{3}$

20. -1

21. \$275/month

22. 10 miles per hour

23. 0

24. 4 tiles per minute

32.  $f^{-1}(x) = \frac{x-1}{3}$

33.  $x \geq 4$

34.  $x \neq -6$

35.  $y = |x|$

36.  $y = (x + 4)^2$

37. 4

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

1. A careful way to see it: Difference of outputs over difference of inputs. Slope of the secant. That gives a quick check on the answer.

2. Keep the rule visible: Linear with slope 3; ARC equals slope on any interval. That gives a quick check on the answer.

3. Find both endpoint outputs:  $f(1) = 1^2 = 1$  and  $f(3) = 3^2 = 9$ . ARC is the change in output over the change in input:  $\frac{f(3) - f(1)}{3 - 1} = \frac{9 - 1}{2} = 4$ .

4. Evaluate the endpoints:  $f(0) = -0 + 5 = 5$  and  $f(2) = -4 + 5 = 1$ . Then  $\text{ARC} = \frac{f(2) - f(0)}{2 - 0} = \frac{1 - 5}{2} = \frac{-4}{2} = -2$ . The negative sign means the function fell on this interval.

5. A careful way to see it: Read the endpoints:  $g(1) = 3$  and  $g(4) = -15$ .  $\text{ARC} = \frac{g(4) - g(1)}{4 - 1} = \frac{-15 - 3}{3} = -6$ . That gives a quick check on the answer.

6. Keep the rule visible:  $f(0) = 3$ ,  $f(4) = 3$ .  $\text{ARC} = 0$ . (Endpoints equal — secant is horizontal.) That gives a quick check on the answer.

7. Endpoints:  $d(1) = 4(1)^2 = 4$  and  $d(3) = 4(3)^2 = 36$ .  $\text{ARC} = \frac{d(3) - d(1)}{3 - 1} = \frac{36 - 4}{2} = \frac{32}{2} = 16$  mi/h.

8. Start with the key idea:  $h(1) = 53$ ,  $h(3) = 53$ .  $\text{ARC} = 0$ . Ball came back to same height. That gives a quick check on the answer.

9. A careful way to see it: Endpoints:  $f(2) = 4 - 4 = 0$  and  $f(5) = 25 - 10 = 15$ .  $\text{ARC} = \frac{f(5) - f(2)}{5 - 2} = \frac{15 - 0}{3} = 5$ . That gives a quick check on the answer.

10. Keep the rule visible: Slope is constant for a line. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. Endpoints:  $f(0) = 2^0 = 1$  and  $f(3) = 2^3 = 8$ .  $\text{ARC} = \frac{f(3) - f(0)}{3 - 0} = \frac{8 - 1}{3} = \frac{7}{3} \approx 2.33$ .

12. Endpoints:  $f(1) = \sqrt{1} = 1$  and  $f(9) = \sqrt{9} = 3$ .  $\text{ARC} = \frac{f(9) - f(1)}{9 - 1} =$

$$\frac{3 - 1}{8} = \frac{2}{8} = \frac{1}{4}.$$

13. Use the two data points directly as  $(a, f(a))$  and  $(b, f(b))$ :  $\text{ARC} = \frac{17 - 5}{4 - 1} =$

$$\frac{12}{3} = 4.$$

14. Endpoints:  $f(-1) = (-1)^3 = -1$  and  $f(1) = 1^3 = 1$ .  $\text{ARC} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{1 - (-1)}{2} = \frac{2}{2} = 1$ . Mind the double negatives in both numerator and denominator.

15. One steady path is:  $f(-3) = 3$ ,  $f(2) = 2$ .  $\text{ARC} = \frac{2 - 3}{2 - (-3)} = \frac{-1}{5} = -0.2$ .

This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Start with the key idea: Numerator of ARC is 0, so endpoint outputs match. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: Negative ARC means decreasing on average over that decade. That gives a quick check on the answer.

18. Keep the rule visible:  $f(h+1) - f(h) = (h+1)^2 + 2(h+1) - (h^2 + 2h) = 2h + 1 + 2 = 2h + 3$ . Denominator is 1. That gives a quick check on the answer.

19. One steady path is:  $f(2) = 3$ ,  $f(5) = 8$ .  $\text{ARC} = \frac{8 - 3}{5 - 2} = \frac{5}{3}$ . This is the part to check before moving on, because it keeps the answer tied to the original question.

20. Start with the key idea:  $f(-1) = 2 + 3 = 5$ ,  $f(2) = 8 - 6 = 2$ .  $\text{ARC} = \frac{2 - 5}{3} = -1$ . This is the part to check before moving on, because it keeps the answer tied to the original question.

21. From the graph: balance at month 2 is \$550, and at month 6 is \$1,650.  $\text{ARC} = \frac{1650 - 550}{6 - 2} = \frac{1100}{4} = 275$  dollars per month. (The line of best fit through the data points has roughly that slope, and the secant matches.)



22. Distance changes from 20 to 60 miles ( $\Delta d = 40$ ). Time changes from 1 to 5 hours ( $\Delta t = 4$ ). ARC:  $\frac{40}{4} = 10$  mph. Steady cruise.

23. One steady path is:  $f(-1) = 4$ ,  $f(3) = 4$ . ARC:  $\frac{4 - 4}{3 - (-1)} = 0$ . Even though the parabola dipped between those two points, the endpoint outputs match,

so the average rate is zero. That gives a quick check on the answer.

24. Start with the key idea:  $N(0) = 2$  tiles,  $N(4) = 18$  tiles. ARC:  $\frac{18 - 2}{4 - 0} = \frac{16}{4} = 4$  tiles/min. The actual instantaneous growth rate is faster than this near  $t = 4$  and slower near  $t = 0$  — the secant just averages both behaviors. That gives a quick check on the answer.



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