Arithmetic Series

Name: ________________ Date: ____________ Score: ______ / 28

Quick Review

A **series** is a sum of sequence terms. An **arithmetic series** adds the terms of an arithmetic sequence: $S_n = a_1 + a_2 + \cdots + a_n$.

The pairing trick. Pair the first term with the last, the second with the second-to-last, and so on. Each pair sums to the same value $a_1 + a_n$. There are $n/2$ pairs (when n is even), so $S_n = \frac{n}{2}(a_1 + a_n)$. (The formula works for odd n too – the middle term equals the average of the ends, so the averaging argument still goes through.)

Two equivalent formulas. $S_n = \frac{n}{2}(a_1 + a_n)$ when you know the first and last terms. $S_n = \frac{n}{2}[2a_1 + (n-1)d]$ when you know a_1 and d but haven't found a_n yet. The second is just the first with $a_n = a_1 + (n-1)d$ plugged in. Use whichever needs less work.

Sigma form. An arithmetic series often shows up as $\sum_{k=1}^{n}(dk + c)$, where d is the common difference and $c = a_1 - d$. Compute a_1 and a_n, then plug in.

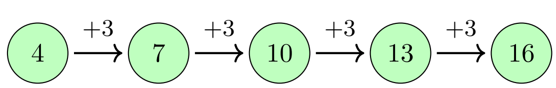
Solving for n. If you're given S_n and asked for n, plug into the formula and you'll get a quadratic in n. Take the positive integer root.

Common slips. Adding only the first n terms when the problem asks for a total *through* term n (those are the same – but don't slip and use $n+1$ terms by including a starting term you shouldn't). Forgetting to find a_n before plugging into the sum formula. Using the sequence formula when the question asks for the series total.

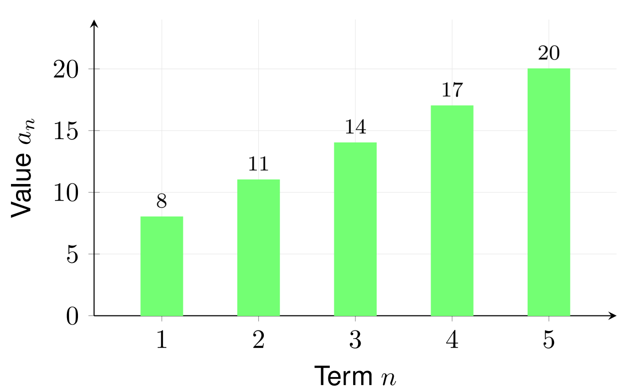
PRACTICE

Find sums of arithmetic series. Pick the formula that uses the data you've been given.

1. Find the sum $1 + 2 + 3 + \cdots + 10$. ________
2. Find the sum of the first 20 terms of $4, 7, 10, 13, \ldots$ ________



3. Find S_{15} for an arithmetic series with $a_1 = -2$ and $d = 4$. ________
4. Compute $\sum_{k=1}^{12}(3k + 2)$. ________
5. Find n such that the sum of $5 + 9 + 13 + \cdots$ to n terms equals 495. ________
6. Find the sum of the first 25 terms of $8, 11, 14, 17, \ldots$ ________



7. An arithmetic series has 24 terms with $a_1 = -7$ and $a_{24} = 85$. Find the sum. ________
8. Find the sum of all positive even integers less than 100. ________
9. Compute $\sum_{k=1}^{10}(2k - 5)$. ________

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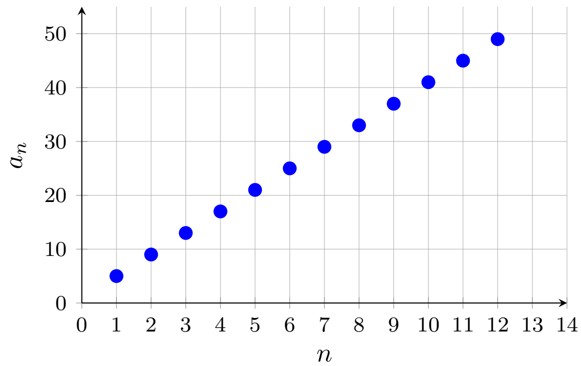
10. Find the sum: $5+9+13+\cdots+49$. ______



11. Find the sum of the first 30 terms of $-1, 2, 5, 8, \ldots$ ______

12. True or False: a finite arithmetic series with positive a_1 and positive d can have a negative sum. ______

13. Compute the sum $1+2+3+\cdots+50$. ______

14. Find the sum of the first 14 terms of $10, 7, 4, 1, \ldots$ ______

15. Compute $\sum_{k=1}^{8}(4k-3)$. ______

16. An arithmetic series has $a_1 = 3$, $d = 2$, and $S_n = 120$. Find n. ______

17. Find the sum $3+7+11+15+\cdots+99$. ______

18. Compute $1+3+5+\cdots+(2n-1)$ in closed form. ______

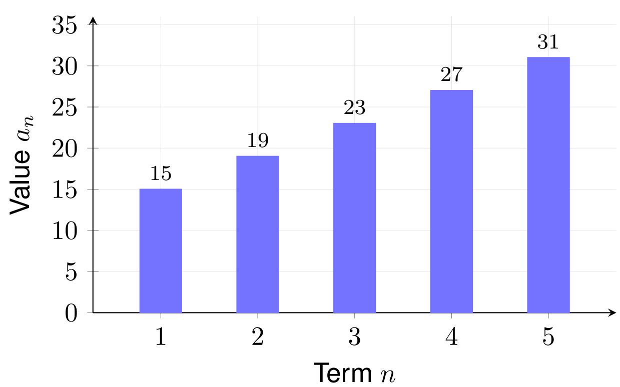
19. Find the sum of all multiples of 5 from 5 to 100. ______

20. Find S_{40} for the arithmetic sequence with $a_1 = -10$ and $d = 2$. ______

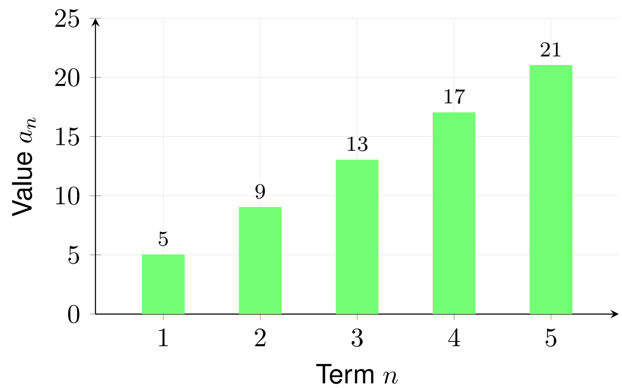
♦ Word Problems

21. A theater has 18 seats in the first row. Each row after that has 4 more seats than the row before. How many seats total are in the first 16 rows? ____

22. An amphitheater has 15 seats in row 1, 19 in row 2, 23 in row 3, and so on – adding 4 each row. How many seats are in the first 10 rows? ____



23. A student saves \$5 in week 1, \$9 in week 2, \$13 in week 3, continuing this arithmetic pattern (adding \$4 each week). After how many weeks does her *total* savings first exceed \$500? ____



24. A construction crew lays 12 feet of pipe on day 1. Each later day they lay 3 more feet than the day before. How many feet of pipe will they have laid in total after 20 days? ____

Additional Practice

25. Find the next term: $4, 9, 14, 19, \ldots$. ________

26. Find a_{10} if $a_1 = 3$ and $d = 5$. ________

27. Find the next term: $2, 6, 18, 54, \ldots$. ________

28. Find a_6 if $a_1 = 5$ and $r = 2$. ________

Answer Keys

1.	55	13.	1275
2.	650	14.	−133
3.	390	15.	120
4.	258	16.	$n = 10$
5.	$n = 15$	17.	1275
6.	1100	18.	n^2
7.	936	19.	1050
8.	2450	20.	1160
9.	60	21.	768 seats
10.	324	22.	330 seats
11.	1275	23.	16 weeks
12.	False	24.	810 feet

Additional Practice Answers

25.	24	27.	162
26.	48	28.	160

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Arithmetic series with $a_1 = 1$, $a_n = 10$, $n = 10$. $S_{10} = \frac{10}{2}(1 + 10) = 5(11) = 55$. (Gauss's trick, age 8 or so – pair $(1, 10), (2, 9), \ldots, (5, 6)$, five pairs of 11.)

2. Keep the rule visible: $a_1 = 4, d = 3, n = 20$. First get $a_{20} = 4 + 19(3) = 61$. Then $S_{20} = \frac{20}{2}(4 + 61) = 10(65) = 650$. (The diagram shows the first five terms feeding the series.) That gives a quick check on the answer.

3. First find the last term: $a_{15} = a_1 + (15 - 1)d = -2 + 14(4) = 54$. Then use $S_n = \frac{n}{2}(a_1 + a_n)$: $S_{15} = \frac{15}{2}(-2 + 54) = \frac{15}{2}(52) = 15(26) = 390$. Always find a_n before plugging into the sum formula.

4. The summand $3k + 2$ is linear in k, so this is an arithmetic series. Find the ends: $a_1 = 3(1) + 2 = 5$ and $a_{12} = 3(12) + 2 = 38$, with $n = 12$ terms. Then $S = \frac{n}{2}(a_1 + a_n) = \frac{12}{2}(5 + 38) = 6(43) = 258$.

5. A careful way to see it: $a_1 = 5$, $d = 4$. Use $S_n = \frac{n}{2}[2a_1 + (n - 1)d] = \frac{n}{2}(4n + 6) = n(2n + 3)$. Set $n(2n + 3) = 495$, so $2n^2 + 3n - 495 = 0$. Try $n = 15$: $15(33) = 495$ ✓. (Negative root is non-physical for term counts.) That gives a quick check on the answer.

6. Keep the rule visible: $a_1 = 8$, $d = 3$, $n = 25$. $a_{25} = 8 + 24(3) = 80$. $S_{25} = \frac{25}{2}(8 + 80) = \frac{25}{2}(88) = 25(44) = 1100$. (The bar chart visualizes the first five terms; each bar is 3 taller than the last.) That gives a quick check on the answer.

7. One steady path is: $S_{24} = \frac{24}{2}(-7 + 85) = 12(78) = 936$. (No need to find d – the first-and-last formula is faster when both ends are given.) That gives a quick check on the answer.

8. Even integers from 2 to 98: $a_1 = 2$, $a_n = 98$, $d = 2$. Count: $n = (98 - 2)/2 + 1 = 49$. $S_{49} = \frac{49}{2}(2 + 98) = \frac{49}{2}(100) = 2450$.

9. Arithmetic. $a_1 = 2(1) - 5 = -3$, $a_{10} = 2(10) - 5 = 15$, $n = 10$. $S = \frac{10}{2}(-3 + 15) = 5(12) = 60$.

10. Keep the rule visible: $a_1 = 5$, $d = 4$. Count: $n = (49 - 5)/4 + 1 = 12$ (matches the 12 plotted points). $S_{12} = \frac{12}{2}(5 + 49) = 6(54) = 324$. That gives a quick check on the answer.

11. Read $a_1 = -1$ and $d = 2 - (-1) = 3$, with $n = 30$. Find the last term: $a_{30} = -1 + (30 - 1)(3) = -1 + 87 = 86$. Then $S_{30} = \frac{30}{2}(a_1 + a_{30}) = 15(-1 + 86) = 15(85) = 1275$.

12. Both $a_1 > 0$ and all later terms (since $d > 0$) are positive, so every partial sum is positive. (Flip either sign and the answer can change.)

13. A careful way to see it: $a_1 = 1$, $a_{50} = 50$, $n = 50$. $S_{50} = \frac{50}{2}(1 + 50) = 25(51) = 1275$. (Sum of first n positive integers is $n(n + 1)/2$.) That gives a quick check on the answer.

14. Keep the rule visible: $a_1 = 10, d = -3, n = 14$. $a_{14} = 10+13(-3) = -29$. $S_{14} = \frac{14}{2}(10 + (-29)) = 7(-19) = -133$. (Negative d, negative sum – terms eventually go negative and pull the total down.) That gives a quick check on the answer.

15. The summand $4k - 3$ is linear, so this is arithmetic. Find the ends: $a_1 = 4(1) - 3 = 1$ and $a_8 = 4(8) - 3 = 29$, with $n = 8$. Then $S = \frac{n}{2}(a_1 + a_n) = \frac{8}{2}(1 + 29) = 4(30) = 120$.

16. Start with the key idea: $S_n = \frac{n}{2}[2(3)+(n-1)(2)] = \frac{n}{2}(2n+4) = n(n+2)$. Set $n(n + 2) = 120$: $n^2 + 2n - 120 = 0$. Factor: $(n - 10)(n + 12) = 0$, so $n = 10$ (the positive root). **Check:** $S_{10} = 10(12) = 120$ ✓. That gives a quick check on the answer.

17. A careful way to see it: $a_1 = 3$, $a_n = 99$, $d = 4$. Count terms: $n = (99 - 3)/4 + 1 = 25$. $S_{25} = \frac{25}{2}(3 + 99) = \frac{25}{2}(102) = 25(51) = 1275$. That gives a quick check on the answer.

18. Odd numbers from 1: $a_1 = 1, a_n = 2n - 1, d = 2$. $S = \frac{n}{2}(1 + (2n - 1)) = \frac{n}{2}(2n) = n^2$. (The sum of the first n odd numbers is a perfect square. Quick check: $1 + 3 + 5 + 7 = 16 = 4^2$ ✓.)

19. One steady path is: $a_1 = 5, a_n = 100, d = 5$. Count: $n = (100-5)/5+1 = 20$. $S_{20} = \frac{20}{2}(5 + 100) = 10(105) = 1050$. That gives a quick check on the answer.

20. Find the last term first: $a_{40} = a_1 + (40 - 1)d = -10 + 39(2) = 68$. Then $S_{40} = \frac{40}{2}(a_1 + a_{40}) = 20(-10 + 68) = 20(58) = 1160$.

21. Row counts form an arithmetic sequence with $a_1 = 18$, $d = 4$. Row 16: $a_{16} = 18 + 15(4) = 78$ seats. Total: $S_{16} = \frac{16}{2}(18 + 78) = 8(96) = 768$ seats. (Sanity check: average row has $(18 + 78)/2 = 48$ seats; 16 rows times $48 = 768$ ✓.)

22. Keep the rule visible: $a_1 = 15, d = 4, n = 10$. Row 10: $a_{10} = 15 + 9(4) = 51$. Total: $S_{10} = \frac{10}{2}(15 + 51) = 5(66) = 330$ seats. That gives a quick check on the answer.

23. One steady path is: $a_1 = 5, d = 4$. $S_n = \frac{n}{2}[2(5)+(n-1)(4)] = n(2n+3)$. Test $n = 15$: $15(33) = 495$ – still under. Test $n = 16$: $16(35) = 560 > 500$. So her total first exceeds \$500 at week 16. (Reality check: positive integer count of weeks, and the jump from \$495 to \$560 straddles \$500 neatly.) That gives a quick check on the answer.

24. Daily amounts form an arithmetic sequence with $a_1 = 12$, $d = 3$. Day 20: $a_{20} = 12 + 19(3) = 69$ feet. Total: $S_{20} = \frac{20}{2}(12 + 69) = 10(81) = 810$ feet. (Reality check: that's an average of $810/20 = 40.5$ feet/day, which is the midpoint of 12 and 69 – exactly what the pairing trick predicts.)

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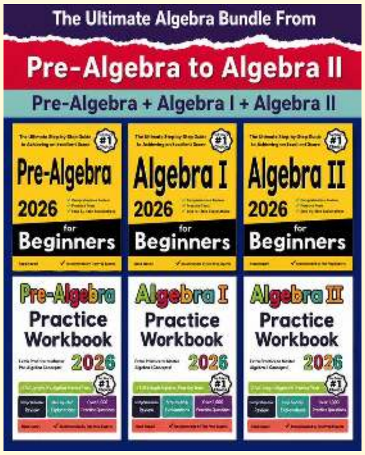

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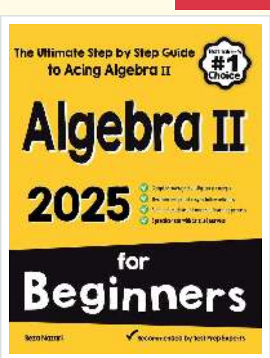

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