

Angles of Rotation

Name: _____ Date: _____ Score: _____ / 29

Q Quick Review

An angle in **standard position** has its vertex at the origin and its initial side on the positive x -axis. Rotate the terminal side counterclockwise for a **positive** angle, clockwise for a **negative** one. One full turn is 360° .

Coterminal angles. Two angles are coterminal when they share the same terminal side. That happens exactly when they differ by an integer multiple of 360° . To find a positive coterminal angle for a negative measure, add 360° (repeatedly if needed); to bring a huge angle into $[0, 360^\circ)$, subtract 360° as many times as you need.

Reference angles. The reference angle is the acute, positive angle between the terminal side and the x -axis – always between 0° and 90° . Once you've got the angle in $[0, 360^\circ)$, use the quadrant rule:

Q1 (0–90): reference = θ .

Q2 (90–180): reference = $180^\circ - \theta$.

Q3 (180–270): reference = $\theta - 180^\circ$.

Q4 (270–360): reference = $360^\circ - \theta$.

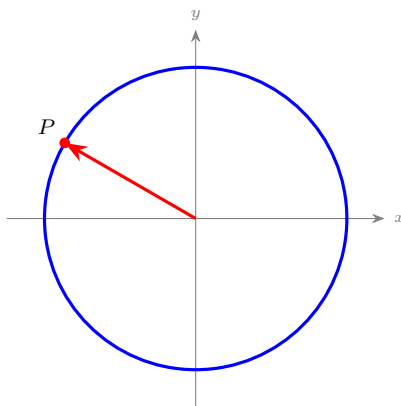
Watch out for. Angles like 90° , 180° , 270° sit on an axis – no quadrant. Their reference angles are the distance to the nearest x -axis: 90° , 0° , and 90° respectively.

Common slips. Forgetting to reduce a large or negative angle into $[0, 360^\circ)$ before picking a quadrant rule. Using $\theta - 90^\circ$ instead of $\theta - 180^\circ$ in Q3. Reporting a negative reference angle (they're always positive).

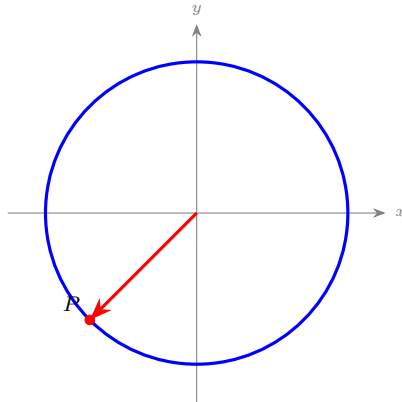
PRACTICE

Find a positive coterminal angle, then find the reference angle. Reduce to $[0, 360^\circ)$ first.

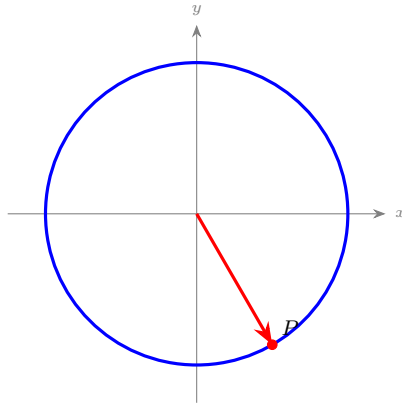
1. Find a positive coterminal angle for 400° . _____
2. Find a positive coterminal angle for -50° . _____
3. Reference angle for 30° . _____
4. Reference angle for the angle drawn below (150°). _____



5. Reference angle for the angle drawn below (225°). _____



6. Reference angle for the angle drawn below (300°). _____



7. Reference angle for -210° . _____

8. Reference angle for 570° . _____

9. Reference angle for 120° . _____

10. Reference angle for 240° . _____

11. Reference angle for -120° . _____

12. Reference angle for $\frac{11\pi}{6}$ (radians). _____

13. Reference angle for $\frac{5\pi}{4}$. _____

14. Find a positive coterminal angle for -300° . _____

15. Find a positive coterminal angle for 810° . _____

16. Reference angle for -45° . _____

17. Reference angle for 270° . _____

18. Reference angle for $\frac{7\pi}{6}$. _____

19. Find a positive coterminal angle in $[0, 360^\circ)$ for -720° . _____

20. Reference angle for $\frac{4\pi}{3}$. _____



◆ Word Problems

21. A wind turbine blade starts at the positive x -axis and rotates counterclockwise through 450° . Find a positive angle in $[0, 360^\circ)$ that ends in the same position, and identify the quadrant of its terminal side. _____
22. A figure skater spins clockwise through 780° starting from the positive x -axis. Where does the skater's facing direction end up? Give a positive angle in $[0, 360^\circ)$ and the reference angle. _____
23. An angle in standard position has measure 215° . Identify its quadrant and reference angle. _____
24. A Ferris wheel cabin starts level with the loading platform (positive x -axis direction from the center) and rotates counterclockwise. After turning through -410° (it ran backwards for safety), find a positive coterminal angle and the reference angle. _____

Additional Practice

25. Find $\sin \theta$ if opposite = 5, hypotenuse = 13. _____
26. Find $\cos \theta$ if adjacent = 12, hypotenuse = 13. _____
27. Find $\tan \theta$ if opposite = 7, adjacent = 4. _____
28. Find $\sin 30^\circ$. _____
29. Find $\cos 60^\circ$. _____



Answer Keys

1. 40°
2. 310°
3. 30°
4. 30°
5. 45°
6. 60°
7. 30°
8. 30°
9. 60°
10. 60°
11. 60°
12. $\frac{\pi}{6}$

Additional Practice Answers

25. $\frac{5}{13}$
26. $\frac{12}{13}$
27. $\frac{7}{4}$

13. $\frac{\pi}{4}$
14. 60°
15. 90°
16. 45°
17. 90°
18. $\frac{\pi}{6}$
19. 0°
20. $\frac{\pi}{3}$
21. 90° ; on the positive y -axis
22. 300° ; reference 60°
23. Quadrant III; 35°
24. 310° ; reference 50°

28. $\frac{1}{2}$
29. $\frac{1}{2}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Subtract one full rotation: $400^\circ - 360^\circ = 40^\circ$. Both angles land on the same terminal side.
2. Add 360° : $-50^\circ + 360^\circ = 310^\circ$. One full counterclockwise rotation lifts a clockwise 50° into the standard $[0, 360^\circ)$ range.
3. One steady path is: 30° is already in Q1, so the reference angle is the angle itself. (Q1 is the only quadrant where the angle and its reference match.) That gives a quick check on the answer.
4. The terminal side lands between 90° and 180° , so 150° is in Q2. In Q2 the reference angle is the swing back to the negative x -axis: $180^\circ - 150^\circ = 30^\circ$. Reference angles are always that acute gap to the x -axis, never to the y -axis.
5. Since 225° is between 180° and 270° , it sits in Q3. Q3's reference angle measures how far past the negative x -axis you've gone: $225^\circ - 180^\circ = 45^\circ$. (A common slip is subtracting 90° here – it's always 180° in Q3.)
6. Keep the rule visible: 300° falls between 270° and 360° , so it's in Q4. There the reference angle is the short hop forward to the positive x -axis: $360^\circ - 300^\circ = 60^\circ$. That gives a quick check on the answer.
7. Bring it positive first: $-210^\circ + 360^\circ = 150^\circ$, which is in Q2. Reference = $180^\circ - 150^\circ = 30^\circ$.
8. Subtract a full rotation: $570^\circ - 360^\circ = 210^\circ$, which is in Q3. Reference = $210^\circ - 180^\circ = 30^\circ$.
9. A careful way to see it: 120° is in Q2 (between 90° and 180°), so the reference angle is the distance back to the negative x -axis: $180^\circ - 120^\circ = 60^\circ$. That gives a quick check on the answer.
10. Keep the rule visible: 240° is in Q3 (between 180° and 270°), so subtract 180° to measure how far past the negative x -axis it lies: $240^\circ - 180^\circ = 60^\circ$. That gives a quick check on the answer.
11. One steady path is: Add 360° : $-120^\circ + 360^\circ = 240^\circ$ (Q3). Reference = $240^\circ - 180^\circ = 60^\circ$. That gives a quick check on the answer.
12. Start with the key idea: $\frac{11\pi}{6}$ is in Q4. Reference = $2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$. (Same idea as degrees, just in radians.) That gives a quick check on the answer.
13. A careful way to see it: $\frac{5\pi}{4}$ is in Q3. Reference = $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$. This is the part to check before moving on, because it keeps the answer tied to the original

question.

14. Keep the rule visible: $-300^\circ + 360^\circ = 60^\circ$. One full turn ahead of -300° lands at 60° in Q1. That gives a quick check on the answer.

15. One steady path is: $810^\circ - 2(360^\circ) = 810^\circ - 720^\circ = 90^\circ$. Two full rotations subtract cleanly. That gives a quick check on the answer.

16. Add 360° : $-45^\circ + 360^\circ = 315^\circ$ (Q4). Reference = $360^\circ - 315^\circ = 45^\circ$. (Quick way: the reference angle of $-\theta$ equals the reference angle of $+\theta$ – magnitude only.)

17. A careful way to see it: 270° sits on the negative y -axis – right between Q3 and Q4. The nearest x -axis is 90° away, so the reference angle is 90° . That gives a quick check on the answer.

18. Keep the rule visible: $\frac{7\pi}{6}$ is in Q3. Reference = $\frac{7\pi}{6} - \pi = \frac{\pi}{6}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

19. One steady path is: $-720^\circ + 2(360^\circ) = 0^\circ$. The terminal side is on the positive x -axis – back to the starting line. That gives a quick check on the answer.

20. Start with the key idea: $\frac{4\pi}{3}$ is in Q3. Reference = $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. A careful way to see it: $450^\circ - 360^\circ = 90^\circ$. The blade has gone one full turn plus an extra quarter turn, so its terminal side sits straight up on the positive y -axis (the boundary between Q1 and Q2). That gives a quick check on the answer.

22. Clockwise means negative: -780° . Add full rotations until positive: $-780^\circ + 3(360^\circ) = -780^\circ + 1080^\circ = 300^\circ$. The terminal side is in Q4, so the reference angle is $360^\circ - 300^\circ = 60^\circ$.

23. One steady path is: 215° is between 180° and 270° , so it sits in Q3. Reference = $215^\circ - 180^\circ = 35^\circ$. (Q3's rule: subtract 180° – the terminal side is that far past the negative x -axis.) That gives a quick check on the answer.

24. Coterminal: $-410^\circ + 2(360^\circ) = -410^\circ + 720^\circ = 310^\circ$. That's in Q4, so the reference angle is $360^\circ - 310^\circ = 50^\circ$. (Sanity check: 310° is 50° short of a full turn, matching the reference.)



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