Amplitude Period and Phase Shift

Name: _____	Date: _____	Score: _____ / 32

Quick Review

The general sinusoidal form packs four pieces of information into one equation:

$y = a\sin(b(x-c)) + d$ (or with $\cos$ in place of $\sin$).

What each letter does.

$|a|$ = **amplitude** – vertical stretch. Negative a flips the wave across the midline.

b = horizontal-stretch dial. The **period** is $\frac{2\pi}{|b|}$. Bigger $|b|$ means a faster (shorter) wave.

c = **phase shift** – the wave slides c units *right* when $c > 0$ and c units *left* when $c < 0$. (Counterintuitive but real: $y = \sin(x - \frac{\pi}{4})$ moves right by $\frac{\pi}{4}$.)

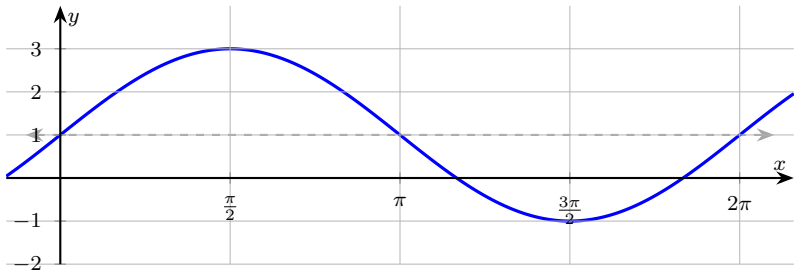
d = **midline** – the horizontal line $y = d$ that the wave oscillates around.

Range straight from the equation. The wave swings $|a|$ units above and below the midline, so range $= [d - |a|,\ d + |a|]$. Maximum $= d + |a|$; minimum $= d - |a|$.

Watch out for "b in front" tricks. For $y = \sin(bx - c)$ (with c not factored out), the phase shift is $\frac{c}{b}$, not c. Factor first: $\sin(bx - c) = \sin\left(b\left(x - \frac{c}{b}\right)\right)$.

Common slips. Mistaking b for the period (it isn't – divide 2π by it). Reading the phase shift in the wrong direction (a $-c$ inside the parentheses shifts $+c$ right). Forgetting that amplitude is always positive.

Here's a sinusoid with midline $y = 1$, amplitude 2, and no phase shift – range $[-1, 3]$:



PRACTICE

Read amplitude, period, phase shift, and midline from each equation or graph.

1. State the amplitude, period, and midline of $y = 4\sin(2x) + 1$. _____
2. State the phase shift of $y = \sin\left(x - \frac{\pi}{4}\right)$. _____
3. State the midline of $y = 3\cos x - 5$. _____
4. For $y = 2\cos\left(3\left(x - \frac{\pi}{6}\right)\right) + 4$, find amplitude, period, phase shift, and midline. _____
5. State the range of $y = 3\sin(4x) - 2$. _____
6. Find the phase shift of $y = \cos(2x - \pi)$. _____
7. State the maximum value of $y = 5\sin x + 2$. _____
8. State the minimum value of $y = -3\cos x + 4$. _____
9. Identify the four parameters of $y = -\sin\left(2\left(x + \frac{\pi}{4}\right)\right) - 1$. _____
10. What is the period of $y = \sin\left(\frac{x}{2}\right)$? _____

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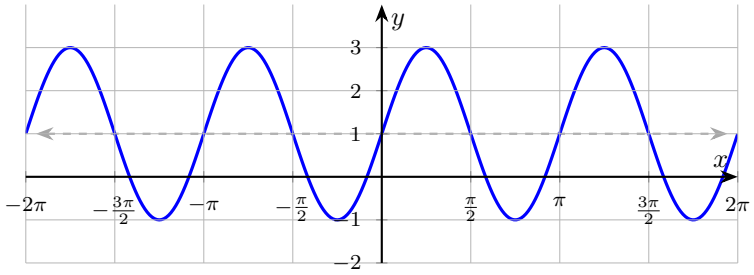
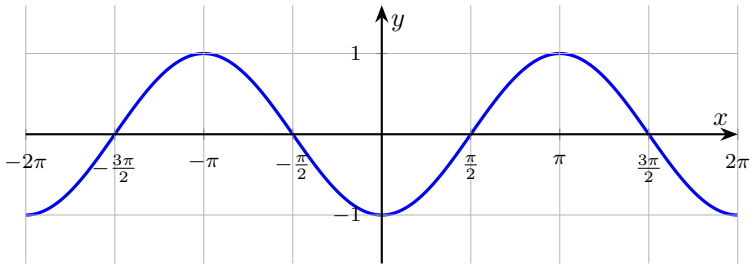
11. Read amplitude, period, and midline from the graph below. _____



12. Read the phase shift from the graph below (the parent sine has been shifted). _____



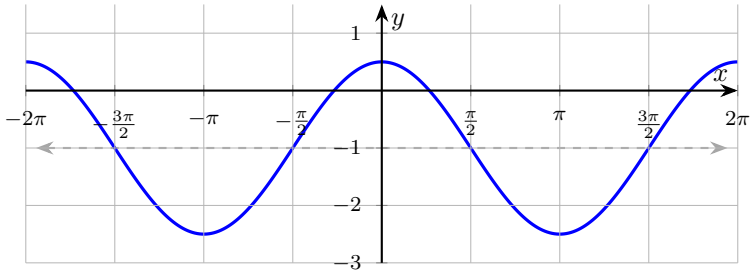
13. True or false: increasing $|b|$ in $y = a\sin(bx)$ stretches the graph horizontally. _____

14. State the amplitude of $y = -7\sin(3x)$. _____

15. State the range of $y = \cos\left(x - \frac{\pi}{3}\right) + 2$. _____

16. Find the phase shift of $y = \cos\left(2\left(x + \frac{\pi}{3}\right)\right)$. _____

17. Identify the midline from the graph below. _____



18. State the amplitude and period of $y = \frac{1}{2}\sin(4x)$. _____

19. Write a cosine equation with amplitude 3, period π, phase shift $\frac{\pi}{8}$ right, midline $y = -1$. _____

20. What is the maximum of $y = -2\sin(3x) + 5$? _____

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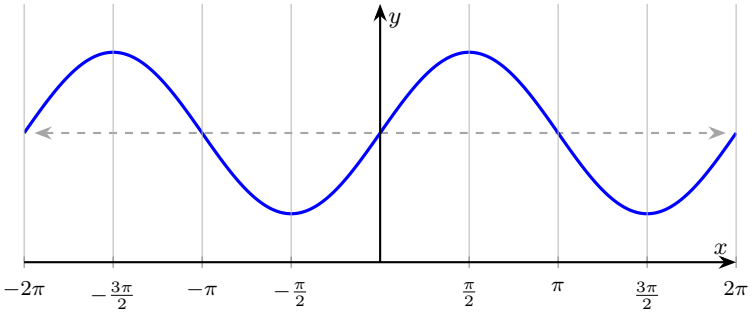
Word Problems

21. A tide-height model is $H(t) = 3\sin\left(\frac{\pi}{6}t\right) + 8$ feet, where t is hours after midnight. State the amplitude, period, and tide range. ____

22. A piston's position is $p(t) = 5\cos\left(2\left(t - \frac{\pi}{4}\right)\right)$ cm. Find the amplitude, period, and the first $t > 0$ at which the piston is at its maximum position. ____

23. A water wheel's lowest point is 1 m below the surface; its highest point is 5 m above. It completes one revolution every 4 s. Write a sinusoidal model $h(t) = a\sin(bt) + d$ for the height of a fixed point (starting at the midline, on the way up). ____

24. The graph below models a city's daylight hours over a year. Read amplitude, period, and midline. _____



Additional Practice

25. Amplitude of $y = 4\sin x$. _____

26. Period of $y = \sin(2x)$. _____

27. Amplitude of $y = -3\cos x$. _____

28. Period of $y = \tan(5x)$. _____

29. Midline of $y = 2\sin x - 7$. _____

30. Phase shift of $y = \sin(x - \pi/3)$. _____

31. Range of $y = 5\cos x$. _____

32. Range of $y = 2\sin x + 1$. _____

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Answer Keys

1. amplitude 4, period π, midline $y = 1$
2. $\frac{\pi}{4}$ right
3. $y = -5$
4. $|a| = 2$, $\frac{2\pi}{3}$, $\frac{\pi}{6}$ right, $y = 4$
5. $[-5, 1]$
6. $\frac{\pi}{2}$ right
7. 7
8. 1
9. $|a| = 1$, period π, shift $\frac{\pi}{4}$ left, $y = -1$
10. 4π
11. $|a| = 2$, period π, $y = 1$
12. $\frac{\pi}{2}$ right
13. False
14. 7
15. $[1, 3]$
16. $\frac{\pi}{3}$ left
17. $y = -1$
18. $|a| = \frac{1}{2}$, period $\frac{\pi}{2}$
19. $y = 3\cos\left(2\left(x - \frac{\pi}{8}\right)\right) - 1$
20. 7
21. amplitude 3, period 12 hr, $[5, 11]$ ft
22. 5; π; $t = \frac{\pi}{4}$
23. $h(t) = 3\sin\left(\frac{\pi}{2}t\right) + 2$
24. amplitude 2.5, period 2π, midline $y = 12$

Additional Practice Answers

25. 4
26. π
27. 3
28. $\frac{\pi}{5}$
29. $y = -7$
30. $\frac{\pi}{3}$ right
31. $[-5, 5]$
32. $[-1, 3]$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Read the parameters from $y = a\sin(bx) + d$: $a = 4$, $b = 2$, $d = 1$. Amplitude is $|a| = 4$; period is $\frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$; the midline is the outside constant $y = d = 1$. The wave oscillates between $1 - 4 = -3$ and $1 + 4 = 5$.

2. Compare to $\sin(x - c)$: here $c = \frac{\pi}{4}$. A *minus* sign inside means a shift to the *right* (this trips everyone up – the sign feels backwards). So the wave slides right by $\frac{\pi}{4}$.

3. The constant added (here subtracted) outside is the midline. $d = -5$, so the wave oscillates around $y = -5$.

4. This is already factored as $a\cos(b(x - c)) + d$: $a = 2$, $b = 3$, $c = \frac{\pi}{6}$, $d = 4$. Amplitude $|a| = 2$; period $\frac{2\pi}{|b|} = \frac{2\pi}{3}$; the $(x - \frac{\pi}{6})$ is a minus inside, so the shift is $\frac{\pi}{6}$ to the *right*; midline $y = 4$.

5. From $y = 3\sin(4x) - 2$: amplitude $|a| = 3$ and midline $d = -2$. The wave swings one amplitude each way from the midline, so range $= [d - |a|,\ d + |a|] = [-2 - 3,\ -2 + 3] = [-5, 1]$. The $b = 4$ only changes the period, not the range.

6. Factor first: $2x - \pi = 2\left(x - \frac{\pi}{2}\right)$. So $c = \frac{\pi}{2}$, a right shift of $\frac{\pi}{2}$. (Don't grab π directly – the $b = 2$ matters.)

7. From $y = 5\sin x + 2$: amplitude $|a| = 5$, midline $d = 2$. The highest the wave climbs is one amplitude above the midline, $\max = d + |a| = 2 + 5 = 7$.

8. From $y = -3\cos x + 4$: midline $d = 4$ and amplitude $|a| = |-3| = 3$ (the minus sign flips the wave but amplitude stays positive). The lowest point is one amplitude below the midline, $\min = d - |a| = 4 - 3 = 1$.

9. A careful way to see it: $a = -1$, $b = 2$, $c = -\frac{\pi}{4}$, $d = -1$. Period $= \frac{2\pi}{2} = \pi$. The $(x + \frac{\pi}{4})$ shifts left by $\frac{\pi}{4}$. Negative a flips the wave. That gives a quick check on the answer.

10. Here $b = \frac{1}{2}$, so period $= \frac{2\pi}{|b|} = \frac{2\pi}{1/2} = 4\pi$. Dividing by a fraction makes the result bigger – a small b stretches the wave wider.

11. The wave bounces between -1 and 3, so midline $= \frac{-1 + 3}{2} = 1$ and amplitude $= \frac{3 - (-1)}{2} = 2$. One cycle finishes in π units. Equation: $y = 2\sin(2x) + 1$.

12. The increasing midline crossing that lives at $x = 0$ for the parent sine has moved to $x = \frac{\pi}{2}$ – a right shift of $\frac{\pi}{2}$.

13. Bigger $|b|$ *compresses* the graph horizontally (shorter period). Smaller $|b|$ stretches it.

14. Amplitude is $|a| = |-7| = 7$. The minus sign flips the wave but doesn't change the amplitude.

15. Amplitude 1, midline 2. Range $= [2 - 1,\ 2 + 1] = [1, 3]$. The phase shift doesn't affect range.

16. Write it as $\cos(b(x - c))$: the inside is $x + \frac{\pi}{3} = x - \left(-\frac{\pi}{3}\right)$, so $c = -\frac{\pi}{3}$. A *plus* inside means a shift to the *left*, here by $\frac{\pi}{3}$.

17. The wave's max is 0.5 and min is -2.5. Midline $= \frac{0.5 + (-2.5)}{2} = -1$. (Amplitude $= 1.5$.)

18. Keep the rule visible: Read $a = \frac{1}{2}$ and $b = 4$. Amplitude is $|a| = \frac{1}{2}$. Period is $\frac{2\pi}{|b|} = \frac{2\pi}{4} = \frac{\pi}{2}$. That gives a quick check on the answer.

19. Build $y = a\cos(b(x - c)) + d$ piece by piece. Amplitude 3 gives $a = 3$. Period π gives $b = \frac{2\pi}{\pi} = 2$. A right shift of $\frac{\pi}{8}$ is a minus inside, so $c = \frac{\pi}{8}$. Midline $y = -1$ gives $d = -1$.

20. Start with the key idea: $|a| = 2$, $d = 5$. Maximum $= d + |a| = 5 + 2 = 7$. (The negative a flips the wave but max and min just swap which x they happen at – the values are still $d \pm |a|$.) That gives a quick check on the answer.

21. A careful way to see it: $a = 3$, $b = \frac{\pi}{6}$, $d = 8$. Period $= \frac{2\pi}{\pi/6} = 12$ hours – a high and low tide every twelve hours, which matches reality. Range $= [8 - 3,\ 8 + 3] = [5, 11]$ ft. That gives a quick check on the answer.

22. Keep the rule visible: $|a| = 5$ cm. Period $= \frac{2\pi}{2} = \pi$ seconds. Cosine maxes

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when its argument is 0: $2\left(t-\frac{\pi}{4}\right)=0 \Rightarrow t=\frac{\pi}{4}$. That gives a quick check on the answer.

23. Highest $=5$, lowest $=-1$, so midline $d=\frac{5+(-1)}{2}=2$ and amplitude $a=\frac{5-(-1)}{2}=3$. Period 4 gives $b=\frac{2\pi}{4}=\frac{\pi}{2}$. Starting at midline going up matches sine's default behavior, so no phase shift needed.

24. Crest at 14.5, trough at 9.5. Midline $=\frac{14.5+9.5}{2}=12$ hours of daylight on an average day, with amplitude $\frac{14.5-9.5}{2}=2.5$. The period is 2π in the graph's x-unit (which represents one full year).

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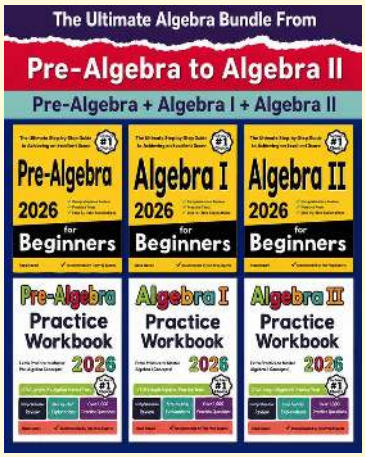

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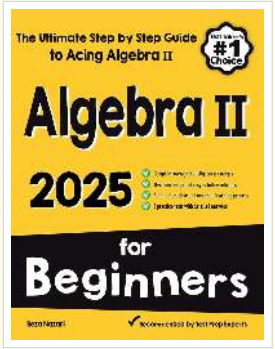

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