Alternating Series

Name: ____________	Date: ____________	Score: ______ / 30

Quick Review

An **alternating series** flips sign every term: positive, negative, positive, negative, and so on (or starting negative, depending on the setup). The factor doing the flipping is $(-1)^k$ or $(-1)^{k+1}$, and which one you use controls whether the first term ends up positive or negative.

Sign conventions. If k starts at 1:

$(-1)^k$ at $k = 1$ gives -1 – first term is *negative*.

$(-1)^{k+1}$ at $k = 1$ gives $+1$ – first term is *positive*.

Pick the one that matches the series you're writing. Always test $k = 1$ to make sure the sign comes out right.

Alternating geometric series. An infinite geometric series with negative ratio r (so $|r| < 1$ still) is alternating. Use the standard sum: $S = \frac{a_1}{1 - r}$. Don't be thrown by the negative ratio – the formula doesn't care about sign, only about $|r| < 1$.

Finite alternating sums. Substitute each k value and add, watching the signs. For something like $1 - 2 + 3 - 4 + \cdots + (-1)^{n+1}n$, the partial sums have a clean pattern: $1, -1, 2, -2, 3, -3, \ldots$ – alternating positive and negative integers.

Common slips. Using $(-1)^k$ when you meant $(-1)^{k+1}$ and getting the first sign wrong. Treating a negative-ratio geometric series as if it diverges – it converges as long as $|r| < 1$. Multiplying terms to get magnitudes when you should be keeping the signs.

PRACTICE

Compute alternating sums or write alternating series in sigma form. Test $k = 1$ to confirm the sign comes out right.

1. Which sequence is alternating? $A: 1, -2, 3, -4, 5, -6$ $B: 1, 2, 3, 4, 5, 6$ $C: -1, -2, -3, -4, -5$ ________
2. Which expression represents the alternating geometric series with $a_1 = 6$, $r = -\frac{1}{3}$? ________
3. The table lists the signed terms of $\sum_{k=1}^{4} (-1)^{k+1} \cdot k$. Find the sum. ________

k	1	2	3	4
$(-1)^{k+1}k$	1	-2	3	-4

4. $\sum_{k=0}^{\infty} \left(-\frac{1}{2}\right)^k$ ________
5. $3 - \frac{3}{4} + \frac{3}{16} - \frac{3}{64} + \cdots$ ________
6. Which formula makes the first term negative when $k = 1$? (a) $\sum(-1)^k a_k$ (b) $\sum(-1)^{k+1} a_k$ ________
7. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k}$ ________
8. True or False: every alternating series is arithmetic. ________
9. Which expression equals the finite sum $4 - 8 + 12 - 16 + 20$? ________
10. Find the infinite sum: $a_1 = 20$, $r = -\frac{1}{5}$. ________

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11. The table lists the signed terms of $\sum_{k=1}^{6}(-1)^k \cdot 2$. Find the sum. ________

k	1	2	3	4	5	6
$(-1)^k \cdot 2$	-2	2	-2	2	-2	2

12. $\sum_{k=1}^{5}(-1)^k \cdot 2$ ________

13. True or False: an infinite geometric series with $r = -0.9$ converges. ________

14. $\sum_{k=0}^{\infty}\left(-\frac{2}{3}\right)^k$ ________

15. The table lists the signed terms of $\sum_{k=1}^{5}(-1)^{k+1} \cdot k^2$. Find the sum. ________

k	1	2	3	4	5
$(-1)^{k+1}k^2$	1	-4	9	-16	25

16. Write $2 - 4 + 8 - 16 + 32 - \cdots$ in sigma form (infinite series). ________

17. Infinite alternating geometric series with $a_1 = 12$ and $r = -\frac{1}{2}$. Find S. ________

18. $\sum_{k=1}^{4}\frac{(-1)^k}{k}$ (partial sum). ________

19. Find: $\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k$. ________

20. True or False: multiplying every term of an alternating series by -1 changes whether it converges. ________

Word Problems

21. A ball bounces, each rebound reaching $\frac{3}{4}$ of the previous bounce's height. If the first bounce is 12 feet high, and we track signed motion (upward positive, downward negative), what does the alternating signed displacement series $12 - 12 + 9 - 9 + 6.75 - \cdots$ sum to as an infinite series? ____

22. Express the finite sum $5 - 10 + 15 - 20 + 25$ in sigma notation and evaluate it. ____

23. An infinite alternating geometric series describes oscillating tax revenue: the first year delivers \$80 million in revenue surplus, each later year's effect is $\frac{1}{2}$ of the previous year's effect but with opposite sign (so year 2 is a \$40M deficit, year 3 a \$20M surplus, etc.). Compute the cumulative long-run net surplus. ____

24. A drug's blood-level fluctuates: the day-1 concentration is 40 mg/L; day 2 drops to -20 (meaning 20 below baseline due to overcorrection); day 3 rises to $+10$; day 4 to -5; continuing with the same alternating-half pattern. Find the long-run total of all daily deviations from baseline. ____

Additional Practice

25. Find the next term: $4, 9, 14, 19, \ldots$. ________

26. Find a_{10} if $a_1 = 3$ and $d = 5$. ________

27. Find the next term: $2, 6, 18, 54, \ldots$. ________

28. Find a_6 if $a_1 = 5$ and $r = 2$. ________

29. Sum $1 + 2 + 3 + \cdots + 20$. ________

30. Find S_5 for $3, 6, 12, 24, 48$. ________

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Answer Keys

1. A
2. $\sum_{k=0}^{\infty} 6\left(-\frac{1}{3}\right)^k$
3. -2
4. $\frac{2}{3}$
5. $\frac{12}{5}$
6. (a)
7. $\frac{1}{3}$
8. False
9. $\sum_{k=1}^{5}(-1)^{k+1} \cdot 4k$
10. $\frac{50}{3}$
11. 0
12. -2
13. True
14. $\frac{3}{5}$
15. 15
16. $\sum_{k=0}^{\infty} 2(-2)^k$
17. 8
18. $-\frac{7}{12}$
19. $\frac{4}{5}$
20. False
21. 0 ft
22. $\sum_{k=1}^{5}(-1)^{k+1} \cdot 5k = 15$
23. $\frac{160}{3} \approx \$53.3$ million
24. $\frac{80}{3} \approx 26.7$ mg/L

Additional Practice Answers

25. 24
26. 48
27. 162
28. 160
29. 210
30. 93

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: A flips sign every term – that's alternating. B is all positive. C is all negative (same sign each term, no flip). That gives a quick check on the answer.

2. Geometric form $\sum a_1 r^k$ with $a_1 = 6$ and $r = -\frac{1}{3}$. Sigma starts at $k = 0$ so the $k = 0$ term is $6 \cdot 1 = 6$.

3. Add the signed terms straight from the table: $1-2+3-4 = -2$. (The $(-1)^{k+1}$ keeps odd k positive, even k negative.)

4. Start with the key idea: Geometric: $a_1 = 1$, $r = -\frac{1}{2}$. $|r| = \frac{1}{2} < 1$ ✓. $S = \dfrac{1}{1-(-1/2)} = \dfrac{1}{3/2} = \dfrac{2}{3}$. That gives a quick check on the answer.

5. Find the ratio: $-\frac{3}{4} \div 3 = -\frac{1}{4}$, so $a_1 = 3$ and $r = -\frac{1}{4}$. Since $|r| = \frac{1}{4} < 1$ it converges, and the negative sign doesn't change the formula: $S = \dfrac{a_1}{1-r} = \dfrac{3}{1-(-1/4)} = \dfrac{3}{5/4} = \dfrac{12}{5}$.

6. At $k = 1$: $(-1)^1 = -1$. So (a) gives $-a_1$ first. (b) gives $(-1)^2 = +1$, so positive first term.

7. Pull out $(-1)^{k+1}$ and recognize geometric structure: $a_1 = \dfrac{1}{2}$ (at $k = 1$), $r = -\frac{1}{2}$. $S = \dfrac{1/2}{1-(-1/2)} = \dfrac{1/2}{3/2} = \dfrac{1}{3}$.

8. Alternating just means signs flip; the magnitudes can do anything. Most alternating series in this chapter are geometric, not arithmetic. (Arithmetic alternating series are unusual.)

9. Magnitudes: $4, 8, 12, 16, 20$ – that's $4k$ for $k = 1, \ldots, 5$. First term positive, so use $(-1)^{k+1}$. Putting it together: $\sum_{k=1}^{5}(-1)^{k+1} \cdot 4k$.

10. Keep the rule visible: $|r| = \frac{1}{5} < 1$ ✓. $S = \dfrac{20}{1-(-1/5)} = \dfrac{20}{6/5} = \dfrac{100}{6} = \dfrac{50}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. The six terms pair up and cancel: $-2+2-2+2-2+2 = 0$. (Any even count of alternating $\pm c$ terms cancels out.)

12. Five terms: $-2, +2, -2, +2, -2$. Pair the first four to cancel; one -2 left over. Sum $= -2$. (Odd-numbered alternating sum leaves a leftover.)

13. A careful way to see it: $|r| = 0.9 < 1$, which satisfies the convergence condition. It converges (slowly, but it does). Sign of r doesn't matter for convergence – only $|r| < 1$. That gives a quick check on the answer.

14. Keep the rule visible: $a_1 = 1$, $r = -\frac{2}{3}$. $|r| = \frac{2}{3} < 1$ ✓. $S = \dfrac{1}{1-(-2/3)} = \dfrac{1}{5/3} = \dfrac{3}{5}$. That gives a quick check on the answer.

15. One steady path is: Add the signed squares from the table: $1-4+9-16+25 = 15$. That gives a quick check on the answer.

16. Magnitudes: $2, 4, 8, 16, 32, \ldots = 2 \cdot 2^k$ for $k = 0, 1, \ldots$. Signs alternate, starting positive: $(-1)^k \cdot 2^{k+1} = 2 \cdot (-2)^k$. So $\sum_{k=0}^{\infty} 2(-2)^k$. (Note: this series *diverges* since $|r| = 2 \geq 1$ – we wrote the sigma form, but the sum doesn't exist.)

17. Here $a_1 = 12$ and $r = -\frac{1}{2}$. Check convergence first: $|r| = \frac{1}{2} < 1$ ✓, so the infinite-sum formula applies. $S = \dfrac{a_1}{1-r} = \dfrac{12}{1-(-1/2)} = \dfrac{12}{3/2} = 8$. The negative ratio just makes the terms alternate – it never blocks convergence on its own.

18. Terms: $-1, +\frac{1}{2}, -\frac{1}{3}, +\frac{1}{4}$. Common denominator 12: $-\frac{12}{12}+\frac{6}{12}-\frac{4}{12}+\frac{3}{12} = \dfrac{-12+6-4+3}{12} = \dfrac{-7}{12}$.

19. One steady path is: $a_1 = 1$, $r = -\frac{1}{4}$. $S = \dfrac{1}{1-(-1/4)} = \dfrac{1}{5/4} = \dfrac{4}{5}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

20. Multiplying every term by -1 flips every sign but doesn't change magnitudes. Convergence depends on the magnitudes shrinking, so the series still converges (just to $-S$ instead of S).

21. Each bounce up is immediately followed by an identical bounce down, so the signed pairs cancel: $12-12 = 0$, $9-9 = 0$, and so on. Total signed displacement is 0 feet. (Reality check: a ball that keeps bouncing returns to its starting vertical position in the limit – net displacement zero, even though total distance traveled is positive and finite.)

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22. Magnitudes: $5, 10, 15, 20, 25 = 5k$ for $k = 1, \ldots, 5$. First term positive, so use $(-1)^{k+1}$. Sigma form: $\sum_{k=1}^{5}(-1)^{k+1} \cdot 5k$. Evaluate directly: $5 - 10 + 15 - 20 + 25 = 15$.

23. Alternating geometric with $a_1 = 80$, $r = -\frac{1}{2}$. $|r| = \frac{1}{2} < 1$ ✓. $S = \dfrac{80}{1-(-1/2)} = \dfrac{80}{3/2} = \dfrac{160}{3} \approx 53.33$, so about \$53.3 million net surplus. (Reality check: the first year's \$80M is partly offset by later swings, but the surplus wins because positive years are bigger in magnitude.)

24. Alternating geometric series with $a_1 = 40$ and $r = -\frac{1}{2}$ (each day is half the previous, with sign flipped). $|r| = \frac{1}{2} < 1$ ✓. $S = \dfrac{40}{1-(-1/2)} = \dfrac{40}{3/2} = \dfrac{80}{3} \approx 26.7$ mg/L total. (Reality check: positive total because the odd-day surpluses outweigh the even-day deficits.)