

# Alternating Series

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 30

## Q Quick Review

An **alternating series** flips sign every term: positive, negative, positive, negative, and so on (or starting negative, depending on the setup). The factor doing the flipping is  $(-1)^k$  or  $(-1)^{k+1}$ , and which one you use controls whether the first term ends up positive or negative.

**Sign conventions.** If  $k$  starts at 1:

$$(-1)^k \text{ at } k = 1 \text{ gives } -1 \text{ -- first term is } \textit{negative}.$$

$$(-1)^{k+1} \text{ at } k = 1 \text{ gives } +1 \text{ -- first term is } \textit{positive}.$$

Pick the one that matches the series you're writing. Always test  $k = 1$  to make sure the sign comes out right.

**Alternating geometric series.** An infinite geometric series with negative ratio  $r$  (so  $|r| < 1$  still) is alternating. Use the standard sum:  $S = \frac{a_1}{1-r}$ . Don't be thrown by the negative ratio – the formula doesn't care about sign, only about  $|r| < 1$ .

**Finite alternating sums.** Substitute each  $k$  value and add, watching the signs. For something like  $1 - 2 + 3 - 4 + \dots + (-1)^{n+1}n$ , the partial sums have a clean pattern: 1, -1, 2, -2, 3, -3, ... – alternating positive and negative integers.

**Common slips.** Using  $(-1)^k$  when you meant  $(-1)^{k+1}$  and getting the first sign wrong. Treating a negative-ratio geometric series as if it diverges – it converges as long as  $|r| < 1$ . Multiplying terms to get magnitudes when you should be keeping the signs.

## PRACTICE

Compute alternating sums or write alternating series in sigma form. Test  $k = 1$  to confirm the sign comes out right.

1. Which sequence is alternating? A : 1, -2, 3, -4, 5, -6 B : 1, 2, 3, 4, 5, 6 C : -1, -2, -3, -4, -5 \_\_\_\_\_

2. Which expression represents the alternating geometric series with  $a_1 = 6, r = -\frac{1}{3}$ ? \_\_\_\_\_

3. The table lists the signed terms of  $\sum_{k=1}^4 (-1)^{k+1} \cdot k$ . Find the sum. \_\_\_\_\_

$k$	1	2	3	4
$(-1)^{k+1}k$	1	-2	3	-4

4.  $\sum_{k=0}^{\infty} \left(-\frac{1}{2}\right)^k$  \_\_\_\_\_

5.  $3 - \frac{3}{4} + \frac{3}{16} - \frac{3}{64} + \dots$  \_\_\_\_\_

6. Which formula makes the first term negative when  $k = 1$ ? (a)  $\sum (-1)^k a_k$  (b)  $\sum (-1)^{k+1} a_k$  \_\_\_\_\_

7.  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k}$  \_\_\_\_\_

8. True or False: every alternating series is arithmetic. \_\_\_\_\_

9. Which expression equals the finite sum  $4 - 8 + 12 - 16 + 20$ ? \_\_\_\_\_

10. Find the infinite sum:  $a_1 = 20, r = -\frac{1}{5}$ . \_\_\_\_\_



11. The table lists the signed terms of  $\sum_{k=1}^6 (-1)^k \cdot 2$ . Find the sum. \_\_\_\_\_

$k$	1	2	3	4	5	6
$(-1)^k \cdot 2$	-2	2	-2	2	-2	2

12.  $\sum_{k=1}^5 (-1)^k \cdot 2$  \_\_\_\_\_

13. True or False: an infinite geometric series with  $r = -0.9$  converges. \_\_\_\_\_

14.  $\sum_{k=0}^{\infty} \left(-\frac{2}{3}\right)^k$  \_\_\_\_\_

15. The table lists the signed terms of  $\sum_{k=1}^5 (-1)^{k+1} \cdot k^2$ . Find the sum. \_\_\_\_\_

$k$	1	2	3	4	5
$(-1)^{k+1} k^2$	1	-4	9	-16	25

16. Write  $2 - 4 + 8 - 16 + 32 - \dots$  in sigma form (infinite series). \_\_\_\_\_

17. Infinite alternating geometric series with  $a_1 = 12$  and  $r = -\frac{1}{2}$ . Find  $S$ . \_\_\_\_\_

18.  $\sum_{k=1}^4 \frac{(-1)^k}{k}$  (partial sum). \_\_\_\_\_

19. Find:  $\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k$ . \_\_\_\_\_

20. True or False: multiplying every term of an alternating series by  $-1$  changes whether it converges. \_\_\_\_\_

◆ Word Problems

21. A ball bounces, each rebound reaching  $\frac{3}{4}$  of the previous bounce's height. If the first bounce is 12 feet high, and we track signed motion (upward positive, downward negative), what does the alternating signed displacement series  $12 - 12 + 9 - 9 + 6.75 - \dots$  sum to as an infinite series? \_\_\_\_\_

22. Express the finite sum  $5 - 10 + 15 - 20 + 25$  in sigma notation and evaluate it. \_\_\_\_\_

23. An infinite alternating geometric series describes oscillating tax revenue: the first year delivers \$80 million in revenue surplus, each later year's effect is  $\frac{1}{2}$  of the previous year's effect but with opposite sign (so year 2 is a \$40M deficit, year 3 a \$20M surplus, etc.). Compute the cumulative long-run net surplus. \_\_\_\_\_

24. A drug's blood-level fluctuates: the day-1 concentration is 40 mg/L; day 2 drops to  $-20$  (meaning 20 below baseline due to overcorrection); day 3 rises to  $+10$ ; day 4 to  $-5$ ; continuing with the same alternating-half pattern. Find the long-run total of all daily deviations from baseline. \_\_\_\_\_

Additional Practice

25. Find the next term: 4, 9, 14, 19, ... \_\_\_\_\_

26. Find  $a_{10}$  if  $a_1 = 3$  and  $d = 5$ . \_\_\_\_\_

27. Find the next term: 2, 6, 18, 54, ... \_\_\_\_\_

28. Find  $a_6$  if  $a_1 = 5$  and  $r = 2$ . \_\_\_\_\_

29. Sum  $1 + 2 + 3 + \dots + 20$ . \_\_\_\_\_

30. Find  $S_5$  for 3, 6, 12, 24, 48. \_\_\_\_\_



Answer Keys

<p>1. <math>A</math></p> <p>2. <math>\sum_{k=0}^{\infty} 6 \left(-\frac{1}{3}\right)^k</math></p> <p>3. <math>-2</math></p> <p>4. <math>\frac{2}{3}</math></p> <p>5. <math>\frac{12}{5}</math></p> <p>6. <math>(a)</math></p> <p>7. <math>\frac{1}{3}</math></p> <p>8. <math>False</math></p> <p>9. <math>\sum_{k=1}^5 (-1)^{k+1} \cdot 4k</math></p> <p>10. <math>\frac{50}{3}</math></p> <p>11. <math>0</math></p> <p>12. <math>-2</math></p>	<p>13. <math>True</math></p> <p>14. <math>\frac{3}{5}</math></p> <p>15. <math>15</math></p> <p>16. <math>\sum_{k=0}^{\infty} 2(-2)^k</math></p> <p>17. <math>8</math></p> <p>18. <math>-\frac{7}{12}</math></p> <p>19. <math>\frac{4}{5}</math></p> <p>20. <math>False</math></p> <p>21. <math>0 \text{ ft}</math></p> <p>22. <math>\sum_{k=1}^5 (-1)^{k+1} \cdot 5k = 15</math></p> <p>23. <math>\frac{160}{3} \approx \\$53.3 \text{ million}</math></p> <p>24. <math>\frac{80}{3} \approx 26.7 \text{ mg/L}</math></p>
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**Additional Practice Answers**

25. $24$	28. $160$
26. $48$	29. $210$
27. $162$	30. $93$

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- A careful way to see it: *A* flips sign every term – that’s alternating. *B* is all positive. *C* is all negative (same sign each term, no flip). That gives a quick check on the answer.
- Geometric form  $\sum a_1 r^k$  with  $a_1 = 6$  and  $r = -\frac{1}{3}$ . Sigma starts at  $k = 0$  so the  $k = 0$  term is  $6 \cdot 1 = 6$ .
- Add the signed terms straight from the table:  $1 - 2 + 3 - 4 = -2$ . (The  $(-1)^{k+1}$  keeps odd  $k$  positive, even  $k$  negative.)
- Start with the key idea: Geometric:  $a_1 = 1, r = -\frac{1}{2}$ .  $|r| = \frac{1}{2} < 1 \checkmark$ .  
 $S = \frac{1}{1 - (-1/2)} = \frac{1}{3/2} = \frac{2}{3}$ . That gives a quick check on the answer.
- Find the ratio:  $-\frac{3}{4} \div 3 = -\frac{1}{4}$ , so  $a_1 = 3$  and  $r = -\frac{1}{4}$ . Since  $|r| = \frac{1}{4} < 1$  it converges, and the negative sign doesn’t change the formula:  
 $S = \frac{a_1}{1 - r} = \frac{3}{1 - (-1/4)} = \frac{3}{5/4} = \frac{12}{5}$ .
- At  $k = 1$ :  $(-1)^1 = -1$ . So (a) gives  $-a_1$  first. (b) gives  $(-1)^2 = +1$ , so positive first term.
- Pull out  $(-1)^{k+1}$  and recognize geometric structure:  $a_1 = \frac{1}{2}$  (at  $k = 1$ ),  
 $r = -\frac{1}{2}$ .  $S = \frac{1/2}{1 - (-1/2)} = \frac{1/2}{3/2} = \frac{1}{3}$ .
- Alternating just means signs flip; the magnitudes can do anything. Most alternating series in this chapter are geometric, not arithmetic. (Arithmetic alternating series are unusual.)
- Magnitudes: 4, 8, 12, 16, 20 – that’s  $4k$  for  $k = 1, \dots, 5$ . First term positive, so use  $(-1)^{k+1}$ . Putting it together:  $\sum_{k=1}^5 (-1)^{k+1} \cdot 4k$ .
- Keep the rule visible:  $|r| = \frac{1}{5} < 1 \checkmark$ .  $S = \frac{20}{1 - (-1/5)} = \frac{20}{6/5} = \frac{100}{6} = \frac{50}{3}$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- The six terms pair up and cancel:  $-2 + 2 - 2 + 2 - 2 + 2 = 0$ . (Any even count of alternating  $\pm c$  terms cancels out.)

- Five terms:  $-2, +2, -2, +2, -2$ . Pair the first four to cancel; one  $-2$  left over. Sum =  $-2$ . (Odd-numbered alternating sum leaves a leftover.)
- A careful way to see it:  $|r| = 0.9 < 1$ , which satisfies the convergence condition. It converges (slowly, but it does). Sign of  $r$  doesn’t matter for convergence – only  $|r| < 1$ . That gives a quick check on the answer.
- Keep the rule visible:  $a_1 = 1, r = -\frac{2}{3}$ .  $|r| = \frac{2}{3} < 1 \checkmark$ .  $S = \frac{1}{1 - (-2/3)} = \frac{1}{5/3} = \frac{3}{5}$ . That gives a quick check on the answer.
- One steady path is: Add the signed squares from the table:  $1 - 4 + 9 - 16 + 25 = 15$ . That gives a quick check on the answer.
- Magnitudes:  $2, 4, 8, 16, 32, \dots = 2 \cdot 2^k$  for  $k = 0, 1, \dots$ . Signs alternate, starting positive:  $(-1)^k \cdot 2^{k+1} = 2 \cdot (-2)^k$ . So  $\sum_{k=0}^{\infty} 2(-2)^k$ . (Note: this series *diverges* since  $|r| = 2 \geq 1$  – we wrote the sigma form, but the sum doesn’t exist.)
- Here  $a_1 = 12$  and  $r = -\frac{1}{2}$ . Check convergence first:  $|r| = \frac{1}{2} < 1 \checkmark$ , so the infinite-sum formula applies.  $S = \frac{a_1}{1 - r} = \frac{12}{1 - (-1/2)} = \frac{12}{3/2} = 8$ . The negative ratio just makes the terms alternate – it never blocks convergence on its own.
- Terms:  $-1, +\frac{1}{2}, -\frac{1}{3}, +\frac{1}{4}$ . Common denominator 12:  $-\frac{12}{12} + \frac{6}{12} - \frac{4}{12} + \frac{3}{12} = \frac{-12 + 6 - 4 + 3}{12} = \frac{-7}{12}$ .
- One steady path is:  $a_1 = 1, r = -\frac{1}{4}$ .  $S = \frac{1}{1 - (-1/4)} = \frac{1}{5/4} = \frac{4}{5}$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- Multiplying every term by  $-1$  flips every sign but doesn’t change magnitudes. Convergence depends on the magnitudes shrinking, so the series still converges (just to  $-S$  instead of  $S$ ).
- Each bounce up is immediately followed by an identical bounce down, so the signed pairs cancel:  $12 - 12 = 0, 9 - 9 = 0$ , and so on. Total signed displacement is 0 feet. (Reality check: a ball that keeps bouncing returns to its starting vertical position in the limit – net displacement zero, even though total distance traveled is positive and finite.)



**22.** Magnitudes:  $5, 10, 15, 20, 25 = 5k$  for  $k = 1, \dots, 5$ . First term positive, so use  $(-1)^{k+1}$ . Sigma form:  $\sum_{k=1}^5 (-1)^{k+1} \cdot 5k$ . Evaluate directly:  $5 - 10 + 15 - 20 + 25 = 15$ .

**23.** Alternating geometric with  $a_1 = 80$ ,  $r = -\frac{1}{2}$ .  $|r| = \frac{1}{2} < 1$  ✓.  

$$S = \frac{80}{1 - (-1/2)} = \frac{80}{3/2} = \frac{160}{3} \approx 53.33$$
, so about \$53.3 million net surplus.

(Reality check: the first year's \$80M is partly offset by later swings, but the surplus wins because positive years are bigger in magnitude.)

**24.** Alternating geometric series with  $a_1 = 40$  and  $r = -\frac{1}{2}$  (each day is half the previous, with sign flipped).  $|r| = \frac{1}{2} < 1$  ✓. 
$$S = \frac{40}{1 - (-1/2)} = \frac{40}{3/2} = \frac{80}{3} \approx 26.7$$
 mg/L total. (Reality check: positive total because the odd-day surpluses outweigh the even-day deficits.)



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