Adding and Subtracting Matrices

Name: ____________________	Date: ____________	Score: _____ / 29

Quick Review

Adding or subtracting matrices is done **entry by entry**: match positions, then add (or subtract) the matching values. For this to even make sense, both matrices must have the *same dimensions* — a 2×3 matrix and a 2×2 matrix cannot be added. There's no clever workaround.

Addition: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$. Each entry pairs up with its twin in the same position. **Subtraction:** flip every sign in the second matrix, then add. The *minus-sign trap* is the most common slip — when you subtract a negative entry, you add. So $5 - (-3) = 8$, not $5 - 3 = 2$. Write out the signs explicitly before computing.

A **scalar** (just a number, not a matrix) multiplies *every* entry: $2 \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -4 & 6 \end{bmatrix}$. Combine with addition / subtraction the way you would for a linear expression: distribute the scalar first, then add or subtract the resulting matrices.

Matrix addition is **commutative** ($A + B = B + A$) and **associative** ($(A + B) + C = A + (B + C)$), and the **zero matrix** $\mathbf{0}$ acts like the number 0: $A + \mathbf{0} = A$. Subtraction is *not* commutative — $A - B$ and $B - A$ are opposites, not equals.

PRACTICE

Add or subtract the matrices. State "undefined" if dimensions don't match.

1. Two food trucks log sales of tacos and burritos over two days (rows are days, columns are tacos then burritos). Truck A's counts are $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ and Truck B's are $\begin{bmatrix} 5 & 7 \\ 0 & 2 \end{bmatrix}$. Add the two matrices to find the combined sales. ________

Truck	Day	Tacos	Burritos
A	1	2	1
A	2	4	3
B	1	5	7
B	2	0	2

2. $\begin{bmatrix} 8 & 5 \\ 6 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ ________

3. $\begin{bmatrix} 4 & -3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$ ________

4. $\begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 2 & -1 \\ 0 & 6 & 3 \end{bmatrix}$ ________

5. A recipe's base ingredient grid is A; matrix B is the extra amount added per doubled batch. Compute $A + 2B$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$. ________

Matrix	Row	Col 1	Col 2
A	1	1	2
A	2	3	4
B	1	0	1
B	2	-2	3

6. $\begin{bmatrix} 6 & -2 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} -3 & 4 \\ 7 & -1 \end{bmatrix}$ ________

7. $3 \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$ ________

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8. A gym logs reps for two athletes across two lifts in matrices G_1 (week 1) and G_2 (week 2), rows are athletes and columns are lifts. With $G_1 = \begin{bmatrix} 12 & 8 \\ 15 & 10 \end{bmatrix}$ and $G_2 = \begin{bmatrix} 9 & 11 \\ 14 & 7 \end{bmatrix}$, compute $2G_1 - G_2$. ______

Matrix	Athlete	Lift 1	Lift 2
G_1	1	12	8
G_1	2	15	10
G_2	1	9	11
G_2	2	14	7

9. $\begin{bmatrix} 5 & -3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ______

10. $\begin{bmatrix} 3 & 7 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 7 \\ -2 & 4 \end{bmatrix}$ ______

11. $-2\begin{bmatrix} 1 & -4 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -1 & 6 \end{bmatrix}$ ______

12. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ ______

13. $\begin{bmatrix} 1 & 0 & -2 \\ 3 & 5 & 1 \\ 4 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ -3 & 2 & 1 \\ 4 & 0 & 1 \end{bmatrix}$ ______

14. $4\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix} - 3\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$ ______

15. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ ______

16. $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$ ______

17. $\frac{1}{2}\begin{bmatrix} 4 & -6 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ ______

18. X if $X + \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 9 \end{bmatrix}$ ______

19. $3A - 2A$ for any matrix A ______

20. $\begin{bmatrix} 2.5 & 1.5 \\ 3.0 & -1.0 \end{bmatrix} - \begin{bmatrix} 0.5 & 0.5 \\ 1.0 & -1.5 \end{bmatrix}$ ______

♦ Word Problems

21. Two stores track inventory of three shirt sizes. Store X's matrix is $\begin{bmatrix} 12 & 8 & 5 \\ 7 & 11 & 4 \end{bmatrix}$ and Store Y's is $\begin{bmatrix} 9 & 6 & 3 \\ 4 & 5 & 2 \end{bmatrix}$ (rows are seasons, columns are sizes S, M, L). What is the combined inventory matrix? ____

22. A team's points per quarter across two games are $G_1 = \begin{bmatrix} 18 & 22 \\ 19 & 24 \end{bmatrix}$ and $G_2 = \begin{bmatrix} 21 & 17 \\ 23 & 20 \end{bmatrix}$ (rows are teams A and B; columns are quarters 1 and 2). Find $G_1 + G_2$ — the team-by-team, quarter-by-quarter total. ____

23. A bakery tracks pastries baked vs. pastries sold each day. Tuesday baked $B = \begin{bmatrix} 40 & 30 & 25 \end{bmatrix}$ and sold $S = \begin{bmatrix} 32 & 28 & 18 \end{bmatrix}$ (columns: croissants, muffins, scones). Compute $B - S$, the leftover inventory by category. ____

24. Three classrooms order supplies monthly. The order matrix for January is $J = \begin{bmatrix} 20 & 15 \\ 25 & 10 \\ 18 & 22 \end{bmatrix}$ (rows: classrooms; columns: notebooks, pens). In February the school plans to triple notebooks and cut pens in half. Write the February matrix F if it's modeled by $F = \begin{bmatrix} 3 & 0 \\ 0 & 0.5 \end{bmatrix}$ applied entrywise to each row of J. (Compute F directly — multiply the notebook column by 3 and the pen column by 0.5.) ______

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Additional Practice

25. State the dimensions of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. ______

26. Add $\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 5 & 6 \end{bmatrix}$. ______

27. Subtract $\begin{bmatrix} 7 & 2 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix}$. ______

28. Find $\det \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$. ______

29. Find entry a_{21} in $\begin{bmatrix} 8 & 9 \\ -3 & 4 \end{bmatrix}$. ______

Answer Keys

1. $\begin{bmatrix} 7 & 8 \\ 4 & 5 \end{bmatrix}$
2. $\begin{bmatrix} 5 & 3 \\ 5 & 5 \end{bmatrix}$
3. $\begin{bmatrix} 5 & -5 \\ -5 & 9 \end{bmatrix}$
4. $\begin{bmatrix} 6 & 2 & 1 \\ 3 & 5 & 7 \end{bmatrix}$
5. $\begin{bmatrix} 1 & 4 \\ -1 & 10 \end{bmatrix}$
6. $\begin{bmatrix} 9 & -6 \\ -6 & 6 \end{bmatrix}$
7. $\begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix}$
8. $\begin{bmatrix} 15 & 5 \\ 16 & 13 \end{bmatrix}$
9. $\begin{bmatrix} 5 & -3 \\ 1 & 2 \end{bmatrix}$
10. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
11. $\begin{bmatrix} 3 & 10 \\ -7 & 6 \end{bmatrix}$
12. undefined
13. $\begin{bmatrix} 1 & -1 & -4 \\ 6 & 3 & 0 \\ 0 & -1 & 1 \end{bmatrix}$
14. $\begin{bmatrix} -10 & 11 \\ 12 & -32 \end{bmatrix}$
15. $\begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$
16. $\begin{bmatrix} -3 & -3 & -3 \end{bmatrix}$
17. $\begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$
18. $X = \begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix}$
19. A
20. $\begin{bmatrix} 2 & 1 \\ 2 & 0.5 \end{bmatrix}$
21. $\begin{bmatrix} 21 & 14 & 8 \\ 11 & 16 & 6 \end{bmatrix}$
22. $\begin{bmatrix} 39 & 39 \\ 42 & 44 \end{bmatrix}$
23. $\begin{bmatrix} 8 & 2 & 7 \end{bmatrix}$
24. $F = \begin{bmatrix} 60 & 7.5 \\ 75 & 5 \\ 54 & 11 \end{bmatrix}$

Additional Practice Answers

25. 2×3
26. $\begin{bmatrix} 4 & 3 \\ 7 & 6 \end{bmatrix}$
27. $\begin{bmatrix} 4 & -2 \\ 3 & 4 \end{bmatrix}$
28. 2
29. -3

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Add matching positions, since both matrices are 2×2. Top row: $2 + 5 = 7$ and $1 + 7 = 8$; bottom row: $4 + 0 = 4$ and $3 + 2 = 5$. Stack in the same shape $\left[\begin{smallmatrix} 7 & 8 \\ 4 & 5 \end{smallmatrix}\right]$. Each combined entry is one truck's count plus the other's at the same day and item — never row-with-column, just same-spot-with-same-spot.

2. Subtract each position from its twin: $8-3 = 5, 5-2 = 3, 6-1 = 5, 9-4 = 5$. The result keeps the same 2×2 shape. Subtraction is just adding the opposite, done entry by entry — no row-column mixing here.

3. Watch the double negatives: $4 - (-1) = 5$ and $5 - (-4) = 9$. The other two are $-3 - 2 = -5$ and $-2 - 3 = -5$. Whenever you see *minus a negative*, write it out as $+$ before computing.

4. Add the matching entries position by position. Row 1: $1 + 5 = 6, 0 + 2 = 2, 2 + (-1) = 1$. Row 2: $3 + 0 = 3, -1 + 6 = 5, 4 + 3 = 7$. Both inputs are 2×3, so the sum is 2×3 too — watch that $2 + (-1) = 1$, where adding a negative is the same as subtracting.

5. Scale first: the 2 multiplies every entry of B, so $2B = \left[\begin{smallmatrix} 0 & 2 \\ -4 & 6 \end{smallmatrix}\right]$. Now add to A position by position: $1 + 0 = 1, 2 + 2 = 4, 3 + (-4) = -1, 4 + 6 = 10$. The trick is to do the scalar pass completely before adding, so the -4 is already in place when you reach $3 + (-4)$.

6. Go position by position: $6 - (-3) = 6 + 3 = 9, -2 - 4 = -6, 1 - 7 = -6, 5 - (-1) = 5 + 1 = 6$. The two minus-a-negative spots (9 and 6) are where students slip — rewrite the double minus as a plus before computing, every time.

7. A scalar multiplies every entry, so distribute the 3 to all four: $3(2) = 6, 3(-1) = -3, 3(0) = 0, 3(4) = 12$. The positions stay fixed; only the values scale. Watch the sign on $3(-1) = -3$ — a positive scalar keeps each entry's sign.

8. Scale first: $2G_1$ doubles each entry, giving $\left[\begin{smallmatrix} 24 & 16 \\ 30 & 20 \end{smallmatrix}\right]$. Now subtract G_2 position by position: $24 - 9 = 15, 16 - 11 = 5, 30 - 14 = 16, 20 - 7 = 13$. Order matters — $G_2 - 2G_1$ would flip every sign, so keep the scaled G_1 first.

9. The zero matrix adds nothing to any entry — it's the additive identity. (Same role 0 plays for ordinary addition.)

10. Subtracting any matrix from itself gives the zero matrix of matching shape. Every entry minus itself is 0.

11. Distribute the -2 to every entry first: $-2(1) = -2, -2(-4) = 8, -2(3) = -6, -2(0) = 0$, giving $\left[\begin{smallmatrix} -2 & 8 \\ -6 & 0 \end{smallmatrix}\right]$. Now add the second matrix: $-2 + 5 = 3, 8 + 2 = 10, -6 + (-1) = -7, 0 + 6 = 6$. A negative scalar flips the sign of every entry — the $-2(-4) = 8$ is the easy one to fumble.

12. Dimensions don't match: a 2×2 cannot be added to a 2×3. Addition requires identical shape — there's no way to pair up positions.

13. Subtract each entry from its twin. Row 1: $1-0 = 1, 0-1 = -1, -2-2 = -4$. Row 2: $3 - (-3) = 6, 5 - 2 = 3, 1 - 1 = 0$. Row 3: $4 - 4 = 0, -1 - 0 = -1, 2 - 1 = 1$. The $3 - (-3) = 6$ spot is the trap — subtracting a negative adds, so it jumps to 6, not 0.

14. Scale each matrix first. $4 \left[\begin{smallmatrix} -1 & 2 \\ 3 & -5 \end{smallmatrix}\right] = \left[\begin{smallmatrix} -4 & 8 \\ 12 & -20 \end{smallmatrix}\right]$ and $3 \left[\begin{smallmatrix} 2 & -1 \\ 0 & 4 \end{smallmatrix}\right] = \left[\begin{smallmatrix} 6 & -3 \\ 0 & 12 \end{smallmatrix}\right]$. Now subtract position by position: $-4 - 6 = -10, 8 - (-3) = 11, 12 - 0 = 12, -20 - 12 = -32$. Finish both scalings before you subtract, so each entry is ready to pair off.

15. Column vectors add like any other matrix — one entry per row: $1 + 4 = 5, 2 + 5 = 7, 3 + 6 = 9$, stacked vertically. Two 3×1 inputs give a 3×1 result.

16. Row vectors subtract entry by entry: $1 - 4 = -3, 2 - 5 = -3, 3 - 6 = -3$. The result is a 1×3 row vector, the same shape as both inputs.

17. A fractional scalar works just like any other: halve every entry, so $\frac{1}{2} \left[\begin{smallmatrix} 4 & -6 \\ 8 & 2 \end{smallmatrix}\right] = \left[\begin{smallmatrix} 2 & -3 \\ 4 & 1 \end{smallmatrix}\right]$. Now add position by position: $2 + 0 = 2, -3 + 1 = -2, 4 + (-1) = 3,$

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$1 + 0 = 1$. Scale first, then add the matching entries.

18. Treat X like an unknown and subtract the known matrix from both sides: $X = \vec{B} - \vec{A}$ position by position. $5 - 2 = 3$, $0 - (-1) = 1$, $1 - 3 = -2$, $9 - 4 = 5$, so $X = \left[\begin{smallmatrix} 3 & 1 \\ -2 & 5 \end{smallmatrix}\right]$. Watch $0 - (-1) = 1$ — matrix equations behave like number equations when only addition is involved.

19. Scalar arithmetic distributes over a single matrix: $3A - 2A = (3 - 2)A = 1 \cdot A = A$. The numerical scalars combine first, just like with regular algebra.

20. Decimal entries subtract position by position, no different from whole numbers: $2.5 - 0.5 = 2, 1.5 - 0.5 = 1, 3.0 - 1.0 = 2, -1.0 - (-1.5) = -1.0 + 1.5 = 0.5$. That last one is the trick spot — subtracting -1.5 adds 1.5, landing on 0.5.

21. Add entry by entry. Row 1: $(12 + 9, 8 + 6, 5 + 3) = (21, 14, 8)$. Row 2: $(7 + 4, 11 + 5, 4 + 2) = (11, 16, 6)$. The combined matrix preserves the same shape. (Notice $T_{1,2} = 14$ — that's the total medium-shirt inventory in the first season across both stores.)

22. Sum entry by entry: $(18 + 21, 22 + 17, 19 + 23, 24 + 20) = (39, 39, 42, 44)$. The result is the total points each team scored in each quarter across both games — a useful summary matrix in a single object.

23. Subtract entry by entry: $(40 - 32, 30 - 28, 25 - 18) = (8, 2, 7)$. So 8 croissants, 2 muffins, and 7 scones went unsold. (Treating category totals as a row vector lets you do the whole accounting in one line.)

24. Apply the column scalings: column 1 (notebooks) $\times 3$ gives $60, 75, 54$. Column 2 (pens) $\times 0.5$ gives $7.5, 5, 11$. Stack: $F = [60, 7.5; 75, 5; 54, 11]$. Multiplying each column by its own scalar is what diagonal matrices do in the next section — a preview.

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