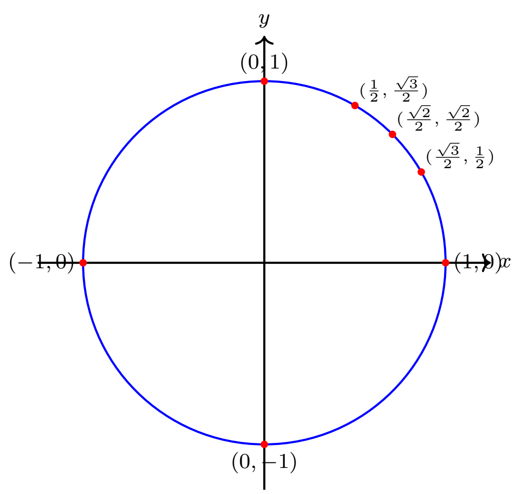
Unit Circle

Name: ____________ Date: ____________ Score: ______ / 37

Quick Review

The **unit circle** is the circle of radius 1 centered at the origin: $x^2 + y^2 = 1$. It does something magical – it turns the abstract sine and cosine functions into honest geometric coordinates.

The core fact. For any angle θ in standard position, the terminal side hits the unit circle at the point $(\cos\theta, \sin\theta)$. The x-coordinate *is* cosine; the y-coordinate *is* sine.



Special-angle coordinates to memorize (Q1).

$\theta = 0$: $(1, 0)$

$\theta = \frac{\pi}{6} = 30°$: $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\theta = \frac{\pi}{4} = 45°$: $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\theta = \frac{\pi}{3} = 60°$: $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\theta = \frac{\pi}{2} = 90°$: $(0, 1)$

Sign by quadrant. Coordinates inherit the quadrant's signs. Q2 has $x < 0, y > 0$ – so cosine negative, sine positive. Q3 has both negative. Q4 has $x > 0, y < 0$.

Coterminal angles share the same point. Adding $360°$ (or 2π) returns you to the same spot on the circle, so all six trig functions repeat. That's the origin of periodicity.

Identifying angles from a point. Given a point on the unit circle, the angle is whichever standard-position angle has that point on its terminal side. Use the quadrant and the reference angle to pick the right value.

Common slips. Swapping x and y: $(\cos\theta, \sin\theta)$ – cosine first. Saying the unit circle has equation $x + y = 1$ (it's $x^2 + y^2 = 1$). Treating "$(\frac{3}{5}, \frac{4}{5})$" as a point not on the unit circle (it is – check: $\frac{9}{25} + \frac{16}{25} = 1$).

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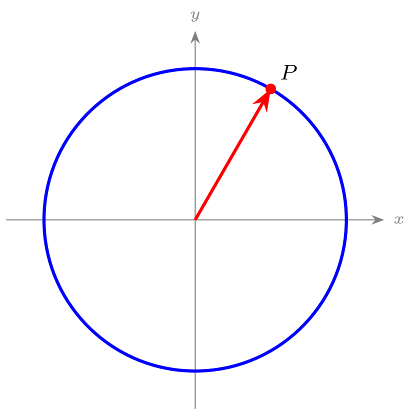
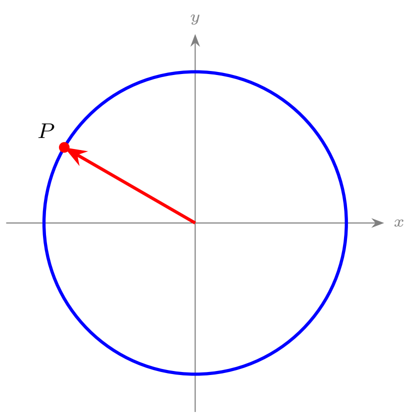
PRACTICE

Read coordinates and angles off the unit circle. Coordinates are $(\cos\theta, \sin\theta)$.

1. Point on unit circle at $\theta = 0$. ________
2. Point at $\theta = \frac{\pi}{2}$. ________
3. Point at $\theta = \pi$. ________
4. Point at $\theta = \frac{3\pi}{2}$. ________
5. Give the coordinates of the point at $\theta = \frac{\pi}{3}$ drawn below. ________



6. Give the coordinates of the point at $\theta = \frac{5\pi}{6}$ drawn below. ________



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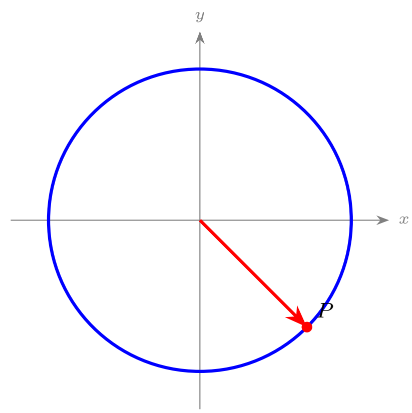
7. Give the coordinates of the point at $\theta = \dfrac{7\pi}{4}$ drawn below. ______



8. Point at $\theta = \dfrac{4\pi}{3}$. ______

9. Point at $\theta = \dfrac{11\pi}{6}$. ______

10. Does $\left(\dfrac{3}{5}, \dfrac{4}{5}\right)$ lie on the unit circle? ______

11. Does $(1, 1)$ lie on the unit circle? ______

12. $\sin\theta$ at $\theta = \dfrac{4\pi}{3}$. ______

13. $\cos\theta$ at $\theta = \dfrac{5\pi}{6}$. ______

14. Find the angle in $[0, 2\pi)$ with unit-circle point $\left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$. ______

15. Find the angle in $[0, 2\pi)$ with unit-circle point $(0, -1)$. ______

16. $\tan\theta$ at $\theta = \dfrac{2\pi}{3}$. ______

17. Equation of the unit circle. ______

18. True or False: coterminal angles correspond to the same point on the unit circle. ______

19. Point at $\theta = -\dfrac{\pi}{4}$. ______

20. Find $\cos\theta$ when the unit-circle point is $\left(\dfrac{5}{13}, -\dfrac{12}{13}\right)$. ______

Word Problems

21. An ant walks counterclockwise on the unit circle from $(1, 0)$, sweeping out an angle of $\dfrac{5\pi}{3}$. Where does it end up? ______

22. A point $\left(-\dfrac{4}{5}, \dfrac{3}{5}\right)$ lies on the unit circle. Find $\sin\theta$, $\cos\theta$, and $\tan\theta$ at the corresponding angle. ______

23. For what angle θ in $[0, 2\pi)$ does the unit circle pass through $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$? ______

24. A particle on the unit circle moves so that at time t seconds, its angle from the positive x-axis is $\theta(t) = \dfrac{\pi}{6}t$ radians. Find the particle's coordinates at $t = 2$ seconds. ______

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Additional Practice

25. Find $\sin\theta$ if opposite $= 5$, hypotenuse $= 13$. ________

26. Find $\cos\theta$ if adjacent $= 12$, hypotenuse $= 13$. ________

27. Find $\tan\theta$ if opposite $= 7$, adjacent $= 4$. ________

28. Find $\sin 30°$. ________

29. Find $\cos 60°$. ________

30. Find $\tan 45°$. ________

31. Convert $180°$ to radians. ________

32. Convert $\frac{\pi}{3}$ radians to degrees. ________

33. Find a coterminal angle with $70°$. ________

34. Reference angle for $150°$. ________

35. Use $\sin^2\theta + \cos^2\theta$. ________

36. If $\sin\theta = \frac{3}{5}$, θ in QI, find $\cos\theta$. ________

37. Find $\sec\theta$ if $\cos\theta = \frac{2}{5}$. ________

Answer Keys

1. $(1, 0)$
2. $(0, 1)$
3. $(-1, 0)$
4. $(0, -1)$
5. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
6. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
7. $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
8. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
9. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
10. Yes
11. No
12. $-\frac{\sqrt{3}}{2}$
13. $-\frac{\sqrt{3}}{2}$
14. $\frac{4\pi}{3}$
15. $\frac{3\pi}{2}$
16. $-\sqrt{3}$
17. $x^2 + y^2 = 1$
18. True
19. $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
20. $\frac{5}{13}$
21. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
22. $\sin\theta = \frac{3}{5},\ \cos\theta = -\frac{4}{5},\ \tan\theta = -\frac{3}{4}$
23. $\theta = \frac{\pi}{4}$
24. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Additional Practice Answers

25. $\frac{5}{13}$
26. $\frac{12}{13}$
27. $\frac{7}{4}$
28. $\frac{1}{2}$
29. $\frac{1}{2}$
30. 1
31. π
32. 60°
33. 430°
34. 30°
35. 1
36. $\frac{4}{5}$
37. $\frac{5}{2}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. The point is $(\cos\theta, \sin\theta)$. At $\theta = 0$ the terminal side lies on the positive x-axis, where $\cos 0 = 1$ and $\sin 0 = 0$, giving $(1, 0)$.

2. At $\frac{\pi}{2} = 90^\circ$ you're at the top of the circle: $\cos\frac{\pi}{2} = 0$ and $\sin\frac{\pi}{2} = 1$, so the point is $(0, 1)$.

3. At $\pi = 180^\circ$ the terminal side points along the negative x-axis: $\cos\pi = -1$, $\sin\pi = 0$, so $(-1, 0)$.

4. At $\frac{3\pi}{2} = 270^\circ$ you're at the bottom of the circle on the negative y-axis: $\cos = 0$, $\sin = -1$, so $(0, -1)$.

5. The drawn ray is the Q1 special angle $\frac{\pi}{3} = 60^\circ$. Coordinates are $(\cos\theta, \sin\theta)$, so $\left(\cos\frac{\pi}{3}, \sin\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. (Cosine first – a common spot to swap x and y.)

6. Keep the rule visible: $\frac{5\pi}{6} = 150^\circ$, Q2, reference $\frac{\pi}{6}$. Q2: $x < 0, y > 0$. This is the part to check before moving on, because it keeps the answer tied to the original question.

7. One steady path is: $\frac{7\pi}{4} = 315^\circ$ is in Q4 with reference angle $\frac{\pi}{4}$. The reference-angle coordinates are both $\frac{\sqrt{2}}{2}$; in Q4 the x stays positive and the y turns negative, giving $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$. That gives a quick check on the answer.

8. Start with the key idea: $\frac{4\pi}{3} = 240^\circ$ is in Q3 with reference angle $\frac{\pi}{3}$, whose coordinates are $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$. Q3 makes both negative: $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. That gives a quick check on the answer.

9. A careful way to see it: $\frac{11\pi}{6} = 330^\circ$ is in Q4 with reference angle $\frac{\pi}{6}$ (coordinates $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$). In Q4, $x > 0$ and $y < 0$, so the point is $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$. That gives a quick check on the answer.

10. Keep the rule visible: Check: $\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = 1$ ✓. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: $1^2 + 1^2 = 2 \neq 1$. (Its distance from the origin is $\sqrt{2}$, not 1.) That gives a quick check on the answer.

12. Sine is the y-coordinate of the unit-circle point. At $\frac{4\pi}{3} = 240^\circ$ (Q3) the point

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is $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, so $\sin\theta = -\frac{\sqrt{3}}{2}$.

13. Cosine is the x-coordinate. At $\frac{5\pi}{6} = 150^\circ$ (Q2) the point is $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, so $\cos\theta = -\frac{\sqrt{3}}{2}$ (negative, as cosine is throughout Q2).

14. Both coordinates negative -> Q3. Reference angle (from x-axis): $\frac{\pi}{3}$ (since $|x| = \frac{1}{2} = \cos\frac{\pi}{3}$). Q3 angle: $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$.

15. One steady path is: $(0, -1)$ is the bottom of the circle, straight down the negative y-axis. The standard-position angle pointing there is $\frac{3\pi}{2} = 270^\circ$. That gives a quick check on the answer.

16. Start with the key idea: Point is $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. $\tan = \frac{y}{x} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$.
This is the part to check before moving on, because it keeps the answer tied to the original question.

17. It's the circle of radius 1 centered at the origin, so by the distance formula every point (x, y) on it satisfies $x^2 + y^2 = 1$. (Note the squares – not $x + y = 1$.)

18. Coterminal angles share a terminal side, so they hit the unit circle at the same point.

19. One steady path is: Coterminal with $\frac{7\pi}{4}$ (in Q4). Coordinates: $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$. That gives a quick check on the answer.

20. Cosine is the x-coordinate: $\frac{5}{13}$. (Quick check the point is on the circle: $\frac{25+144}{169} = 1$ ✓.)

21. A careful way to see it: $\frac{5\pi}{3} = 300^\circ$, Q4, reference $\frac{\pi}{3}$. Q4 coordinates: $x > 0$, $y < 0$. At reference $\frac{\pi}{3}$: $|x| = \cos\frac{\pi}{3} = \frac{1}{2}$ and $|y| = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$. With Q4 signs, the point is $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. That gives a quick check on the answer.

22. On the unit circle, $\cos\theta = x$ and $\sin\theta = y$. So $\cos\theta = -\frac{4}{5}$ and $\sin\theta = \frac{3}{5}$. Then $\tan\theta = \frac{y}{x} = \frac{3/5}{-4/5} = -\frac{3}{4}$. (Q2 – sine positive, cosine negative – consistent with ASTC.)

23. Both coordinates positive and equal point to a 45° angle in Q1. $\frac{\pi}{4} = 45^\circ$ with $\cos = \sin = \frac{\sqrt{2}}{2}$.

24. At $t = 2$: $\theta = \frac{\pi}{6} \cdot 2 = \frac{\pi}{3} = 60^\circ$. Unit-circle coordinates at $\frac{\pi}{3}$: $\left(\cos\frac{\pi}{3}, \sin\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. (After 6 seconds, $\theta = \pi$ and the particle is at $(-1, 0)$.)

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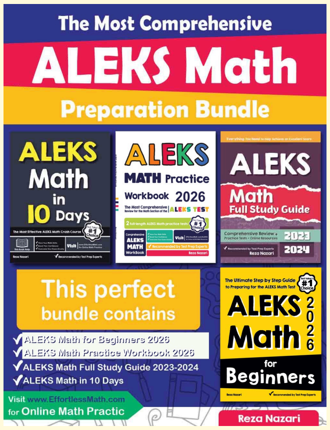

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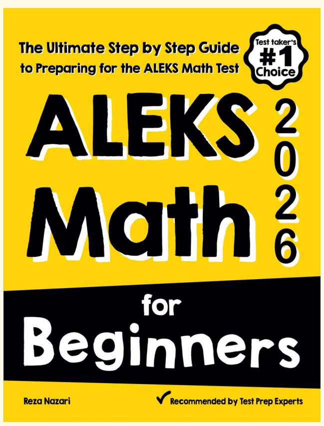

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