Function Inverses

Name: ______________ Date: ______________ Score: ______ / 31

Quick Review

An **inverse function** undoes the original. If f takes x to y, then f^{-1} takes y back to x. Officially: $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1}, and $f^{-1}(f(x)) = x$ for every x in the domain of f. (Heads up: $f^{-1}(x)$ means the inverse function, *not* the reciprocal $\frac{1}{f(x)}$.)

To find an inverse algebraically: write $y = f(x)$, swap x and y, then solve for y. Quick check: $f(x) = 3x - 5 \Rightarrow y = 3x - 5 \Rightarrow x = 3y - 5 \Rightarrow y = \dfrac{x+5}{3}$. So $f^{-1}(x) = \dfrac{x+5}{3}$.

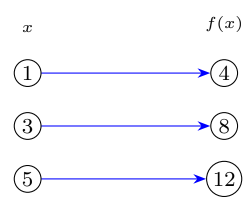
Existence. Only *one-to-one* functions have inverse functions (those passing the *horizontal-line test*). $f(x) = x^2$ on all reals isn't one-to-one (2 and -2 both map to 4), so its inverse isn't a function unless we restrict the domain. Take $f(x) = (x-4)^2$ with $x \geq 4$: now it's one-to-one, and $f^{-1}(x) = 4 + \sqrt{x}$ (positive root only).

Graphs. f and f^{-1} are reflections across the line $y = x$. If (a, b) is on f, then (b, a) is on f^{-1}. The domain of f^{-1} equals the range of f, and vice versa. **Double inverse** returns the original: $(f^{-1})^{-1} = f$.

PRACTICE

Find the inverse or use inverse-function facts as asked.

1. Simplify $f(f^{-1}(x))$ for x in the domain of f^{-1}. ________
2. $f(x) = x + 7;\ f^{-1}(x)$ ________
3. $g(x) = 3x - 5;\ g^{-1}(x)$ ________
4. $h(x) = \dfrac{x-2}{4};\ h^{-1}(x)$ ________
5. $f(x) = 2x + 8;\ g$ with $g(f(x)) = x$ ________
6. $f(x) = 5x - 1$. Is its inverse a function? ________
7. $f(x) = \dfrac{2x+1}{3};\ f^{-1}(x)$ ________
8. The mapping diagram shows f. Give one point on f^{-1}. ________



9. $f(x) = (x-4)^2,\ x \geq 4;\ f^{-1}(x)$ ________
10. $f(x) = \frac{1}{2}x + 1;\ f^{-1}(x)$ ________
11. $(f^{-1})^{-1}(x)$ ________
12. If (a, b) is on f, what point must be on f^{-1}? ________
13. $f(x) = x^2$. Inverse a function (no restriction)? ________
14. $f(x) = 3x - 2;\ f^{-1}(x)$ ________

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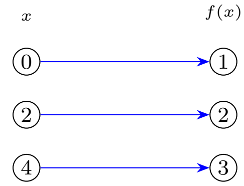
15. The mapping diagram shows f. Which point lies on f^{-1}, $(0,1)$ or $(1,0)$? ______



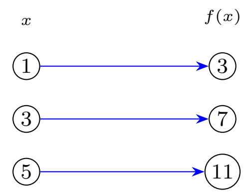
16. $f(x) = \sqrt{x},\ x \geq 0;\ f^{-1}(x)$ ______

17. $f(x) = \dfrac{x}{4} - 3;\ f^{-1}(x)$ ______

18. $f^{-1}(x) = \dfrac{1}{f(x)}$? ______

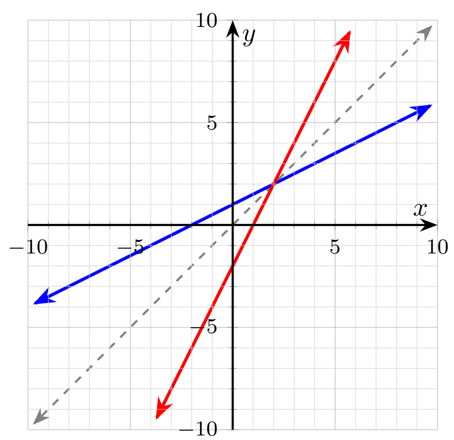
19. $f(x) = x^3 + 1;\ f^{-1}(x)$ ______

20. The mapping diagram shows a one-to-one function f. Find $f^{-1}(7)$. ______

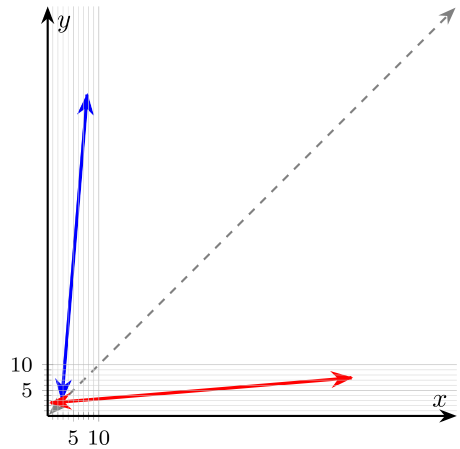


♦ Word Problems

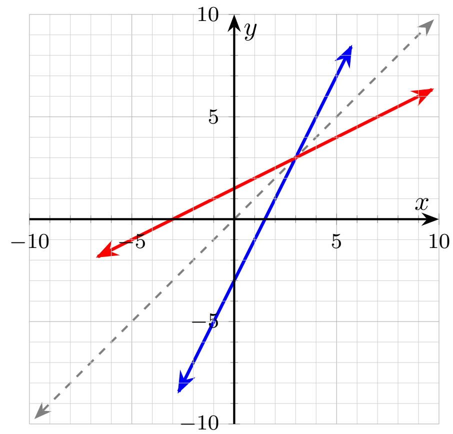
21. The function $C(F) = \frac{5}{9}(F - 32)$ converts Fahrenheit to Celsius. Find $C^{-1}(c)$, then convert $25°$ Celsius back to Fahrenheit. ______



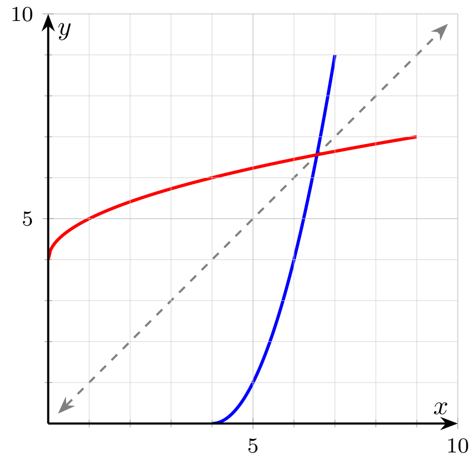
22. A ticket-pricing function is $P(n) = 12n - 30$, where n is the number of tickets bought (group discount factored in) and $P(n)$ is the total bill in dollars. Find $P^{-1}(b)$ and interpret it. Sketch P and P^{-1} on the plane below. ____



23. Show that $f(x) = 2x - 3$ and $g(x) = \dfrac{x+3}{2}$ are inverses. Then use the plane below to sketch both and the line $y = x$. ____



24. Let $f(x) = (x-4)^2$ with restricted domain $x \geq 4$. Find $f^{-1}(x)$ and its domain, then verify with one example. Sketch f and f^{-1} over the visible window. ____



Additional Practice

25. If $f(x) = 2x - 5$, find $f(4)$. ________

26. If $g(x) = x^2 + 1$, find $g(-3)$. ________

27. For $f(x) = 3x + 2$, solve $f(x) = 14$. ________

28. Find $(f+g)(x)$ if $f = x + 1$, $g = 2x - 5$. ________

29. Find $(fg)(x)$ if $f = x - 2$, $g = x + 3$. ________

30. Find $f(g(x))$ if $f(x) = 2x$, $g(x) = x + 7$. ________

31. Find the inverse of $f(x) = x - 9$. ________

Answer Keys

1. $\boxed{x}$
2. $\boxed{x-7}$
3. $\boxed{\dfrac{x+5}{3}}$
4. $\boxed{4x+2}$
5. $\boxed{g(x)=\dfrac{x-8}{2}}$
6. yes
7. $\boxed{\dfrac{3x-1}{2}}$
8. $(8,3)$ (or $(4,1)$ or $(12,5)$)
9. $\boxed{4+\sqrt{x}}$
10. $\boxed{2x-2}$
11. $\boxed{f(x)}$
12. $\boxed{(b,a)}$
13. no
14. $\boxed{\dfrac{x+2}{3}}$
15. $\boxed{(1,0)}$
16. $\boxed{x^2,\ x\ge 0}$
17. $\boxed{4(x+3)}$
18. no
19. $\boxed{\sqrt[3]{x-1}}$
20. $\boxed{3}$
21. $\boxed{C^{-1}(c)=\frac{9}{5}c+32,\ C^{-1}(25)=77^\circ F}$
22. $P^{-1}(b)=\dfrac{b+30}{12}$; tickets bought for a bill of b
23. $\boxed{f\circ g=g\circ f=x}$
24. $f^{-1}(x)=4+\sqrt{x}$, domain $x\ge 0$

Additional Practice Answers

25. $\boxed{3}$
26. $\boxed{10}$
27. $\boxed{x=4}$
28. $\boxed{3x-4}$
29. $\boxed{x^2+x-6}$
30. $\boxed{2x+14}$
31. $\boxed{f^{-1}(x)=x+9}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: By definition, inverses undo each other: $f\circ f^{-1}=I$. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. Write $y=x+7$, swap to get $x=y+7$, then solve for y: $y=x-7$. The inverse undoes the function, and subtracting 7 reverses adding 7. So $f^{-1}(x)=x-7$.

3. Write $y=3x-5$ and swap x and y: $x=3y-5$. Solve for y — add 5 to get $x+5=3y$, then divide by 3: $y=\dfrac{x+5}{3}$. So $g^{-1}(x)=\dfrac{x+5}{3}$.

4. Write $y=\dfrac{x-2}{4}$ and swap: $x=\dfrac{y-2}{4}$. Clear the fraction by multiplying by 4: $4x=y-2$, then add 2: $y=4x+2$. So $h^{-1}(x)=4x+2$.

5. The function g that undoes f is its inverse. Write $y=2x+8$ and swap: $x=2y+8$. Subtract 8 ($x-8=2y$) and divide by 2: $g(x)=\dfrac{x-8}{2}$.

6. Keep the rule visible: Linear with nonzero slope is one-to-one. Passes horizontal-line test. That gives a quick check on the answer.

7. Write $y=\dfrac{2x+1}{3}$ and swap: $x=\dfrac{2y+1}{3}$. Multiply by 3: $3x=2y+1$. Subtract 1 and divide by 2: $y=\dfrac{3x-1}{2}$. So $f^{-1}(x)=\dfrac{3x-1}{2}$.

8. The inverse reverses every arrow. Reading the $3\to 8$ arrow backward gives $8\to 3$, so $(8,3)$ is on f^{-1}. Any reversed pair works.

9. A careful way to see it: With $x\ge 4$, $x-4\ge 0$. So $\sqrt{(x-4)^2}=x-4$, giving $y=4+\sqrt{x}$ (positive branch). That gives a quick check on the answer.

10. Keep the rule visible: $x=\frac{1}{2}y+1\Rightarrow 2x-2=y$. Check: $f(2x-2)=\frac{1}{2}(2x-2)+1=x-1+1=x$. That gives a quick check on the answer.

11. One steady path is: Inverting twice returns the original function. This is the part to check before moving on, because it keeps the answer tied to the original question.

12. Start with the key idea: Inverse swaps each input-output pair. This is the part to check before moving on, because it keeps the answer tied to the original question.

13. A careful way to see it: f isn't one-to-one ($f(2)=f(-2)=4$). Inverse relation fails vertical-line test. That gives a quick check on the answer.

14. Write $y=3x-2$ and swap: $x=3y-2$. Add 2 ($x+2=3y$), then divide by 3: $y=\dfrac{x+2}{3}$. So $f^{-1}(x)=\dfrac{x+2}{3}$.

15. The diagram shows $f(0)=1$, so $(0,1)$ is on f. The inverse swaps the pair to $(1,0)$. So $(1,0)$ is the point on f^{-1}.

16. Start with the key idea: Swap: $x=\sqrt{y}\Rightarrow y=x^2$. Restrict to $x\ge 0$ to match f's range. That gives a quick check on the answer.

17. A careful way to see it: $x=\dfrac{y}{4}-3\Rightarrow x+3=\dfrac{y}{4}\Rightarrow y=4(x+3)$, i.e. $4x+12$. This is the part to check before moving on, because it keeps the answer tied to the original question.

18. Notation trap: f^{-1} is the inverse function, not the reciprocal. E.g. for $f(x)=2x$, $f^{-1}(x)=\frac{x}{2}\ne\dfrac{1}{2x}$.

19. One steady path is: $x=y^3+1\Rightarrow x-1=y^3\Rightarrow y=\sqrt[3]{x-1}$. Cube roots are one-to-one on all reals, so no domain restriction needed. That gives a quick check on the answer.

20. Start with the key idea: f^{-1} reverses the arrows. The arrow into 7 comes from 3, so $f^{-1}(7)=3$. (Reading the diagram backward is exactly what an inverse does.) That gives a quick check on the answer.

21. Set $c=\frac{5}{9}(F-32)$ and solve for F. Multiply both sides by $\frac{9}{5}$: $\frac{9}{5}c=F-32$, so $F=\frac{9}{5}c+32$. At $c=25$: $\frac{9}{5}(25)+32=45+32=77$. So 25°C $=77^\circ$F. (Quick sanity-check: freezing 0°C $\to$ 32°F. Formula works.) The plot at right shows a generic function-inverse pair $f(x)=\frac{1}{2}x+1$ and $f^{-1}(x)=2x-2$ reflecting across the dashed line $y=x$ — the universal picture of an inverse pair.

22. Solve $b=12n-30$ for n: $b+30=12n$, so $n=\dfrac{b+30}{12}$. The inverse takes a bill b and returns the number of tickets. At $b=90$: $\dfrac{120}{12}=10$ tickets. (Bills divisible by 12 after adding 30 give whole-ticket counts, which is what the real context demands.)

Answer graph

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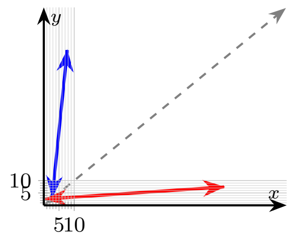
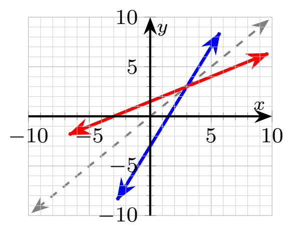


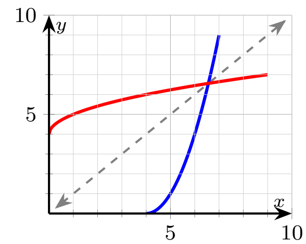
23. One steady path is: $f(g(x)) = 2 \cdot \dfrac{x+3}{2} - 3 = (x+3) - 3 = x$. $g(f(x)) = \dfrac{(2x-3)+3}{2} = \dfrac{2x}{2} = x$. Both compositions return the input, so they are inverses. Visually, the two lines reflect across $y = x$ — a $45°$ flip. The slopes are reciprocals: 2 for f, $\frac{1}{2}$ for g. That's always true of linear inverses. That gives a quick check on the answer.

Answer graph



24. Swap variables: $x = (y-4)^2$. Since the original $x \geq 4$ gives $y - 4 \geq 0$, take the positive square root: $\sqrt{x} = y - 4$, so $y = 4 + \sqrt{x}$. Domain of f^{-1} is the range of f, namely $[0, \infty)$. Check at $x = 5$: $f(5) = 1$ and $f^{-1}(1) = 4 + 1 = 5$. Round trip returns the starting input. The restriction $x \geq 4$ on f was essential — without it, f wouldn't be one-to-one and the inverse wouldn't be a function.

Answer graph



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