Asymptotes of Rational Functions

Name: ____________ Date: ____________ Score: ______ / 35

Quick Review

For $f(x) = \frac{p(x)}{q(x)}$ in lowest terms, three kinds of asymptotes can appear.

Vertical asymptote. Set the reduced denominator to zero. Each solution is an x-value where the graph runs toward $\pm\infty$.

Horizontal asymptote. Compare the degree of the numerator (n) to the denominator (d). $n < d$: $y = 0$. $n = d$: $y = \frac{\text{leading coeff of } p}{\text{leading coeff of } q}$. $n > d$: no horizontal asymptote (the curve grows without bound in the long run).

Slant (oblique) asymptote. When $n = d + 1$, polynomial long division gives $f(x) = mx + b + \frac{r(x)}{q(x)}$. The remainder term vanishes at large $|x|$, leaving the line $y = mx + b$ as the asymptote.

Holes – not asymptotes. If a factor cancels from top and bottom, that x-value gives a *hole*, not a vertical asymptote. To find the hole's y, plug into the simplified expression.

Can a graph cross a horizontal asymptote? Yes. The horizontal asymptote describes end behavior, not a fence. The curve may dip across it for moderate x. Vertical asymptotes are different – the function is undefined there, so the curve cannot cross.

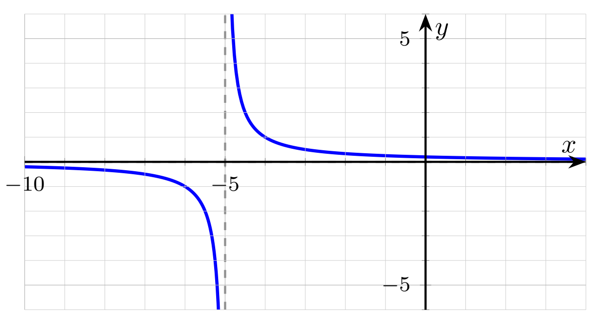
Common slips. Confusing leading-coefficient ratio with constant-term ratio. Forgetting to simplify before looking for asymptotes (a factor that cancels gives a hole). Reporting an asymptote in $y =$ form when the answer should be $x =$ (or vice versa).

Reading the figures. On each plot below, dashed gray lines are asymptotes – vertical (a vertical line at $x = a$) or horizontal (a horizontal line at $y = k$).

PRACTICE

For each function, find every vertical, horizontal, or slant asymptote, and identify any holes.

1. Find the vertical asymptote of $f(x) = \frac{1}{x+5}$. ________

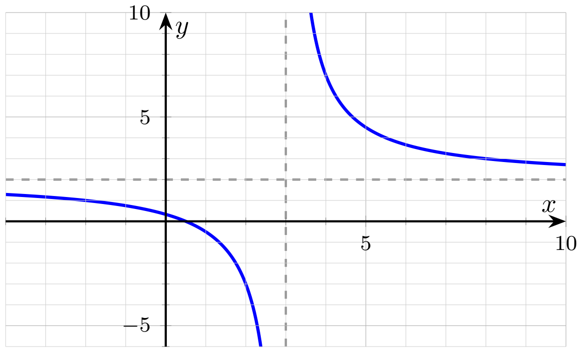


2. Find the horizontal asymptote of $f(x) = \frac{3}{x}$. ________

3. Find the horizontal asymptote of $f(x) = \frac{3x+1}{2x+5}$. ________

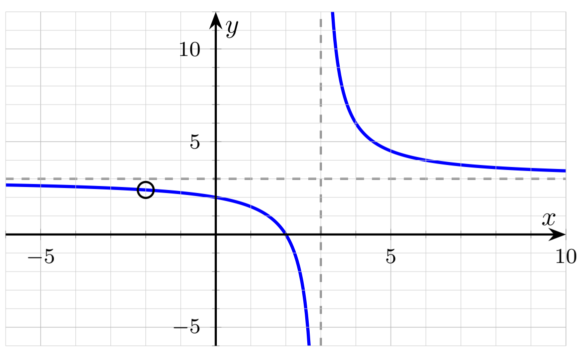
4. Does $f(x) = \frac{x^2+1}{x}$ have a horizontal asymptote? If not, find the slant asymptote. ________

5. Identify both asymptotes of $f(x) = \dfrac{2x-1}{x-3}$. Confirm from the graph. ________



6. Identify the asymptotes and holes of $f(x) = \dfrac{x^2-9}{x-3}$. ________

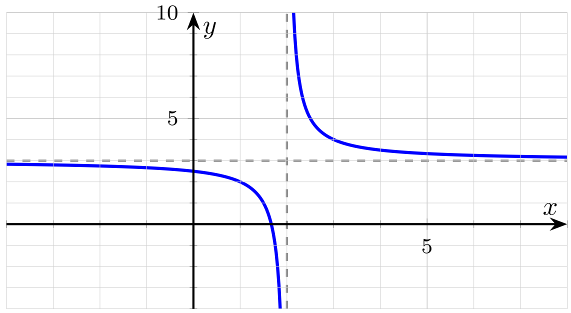
7. For $f(x) = \dfrac{3x^2-12}{x^2-x-6}$, find the vertical asymptote and the horizontal asymptote. ________



8. For $f(x) = \dfrac{x^2+2x+3}{x+1}$, identify the slant and the vertical asymptotes. ________

9. For $g(x) = \dfrac{x^2-4}{x^2-5x+6}$, identify the hole and the vertical asymptote. ________

10. For $f(x) = \dfrac{1}{x-2} + 3$, identify both asymptotes. Confirm from the graph. ______



11. For $f(x) = \dfrac{3x-2}{x+1}$, identify both asymptotes. ______

12. Mark TRUE or FALSE: A rational function can cross a vertical asymptote. ______

13. For $f(x) = \dfrac{3x^2-12}{x^2-4}$, identify all asymptotes and holes. ______

14. For $f(x) = \dfrac{x^2+1}{x^2-4}$, find the horizontal asymptote. ______

15. For $f(x) = \dfrac{x+3}{(x-1)^2}$, find all asymptotes. ______

16. For $f(x) = \dfrac{2x^2+5x-3}{x+3}$, find any slant asymptote or hole. ______

17. For $f(x) = \dfrac{x^2-1}{x+1}$, find the asymptote or hole. ______

18. For $f(x) = \dfrac{4}{x^2+1}$, find any asymptote. ______

19. For $f(x) = \dfrac{x^3}{x^2-1}$, find any slant asymptote. ______

20. For $f(x) = \dfrac{5x^2}{x^2-4}$, find the horizontal asymptote. ______

♦ Word Problems

21. A drug-concentration model is $C(t) = \dfrac{100t}{t^2+25}$ mg/L for $t \geq 0$ minutes. Identify the horizontal asymptote and explain its meaning in context. ____

22. A long-term population model is $P(t) = \dfrac{500t+1000}{t+5}$ for $t \geq 0$ years. Find the horizontal asymptote and explain what it means in context. ____

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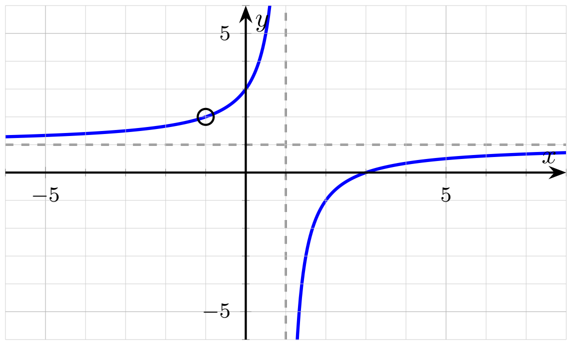
23. For the rational function $f(x) = \dfrac{x^2 - 2x - 3}{x^2 - 1}$, identify every asymptote and hole. Then sketch the key features. The graph below confirms. ____



24. For $f(x) = \dfrac{x^2 + 2}{x - 1}$, identify the slant asymptote and the vertical asymptote, then evaluate $f(10)$ to confirm the slant prediction approximately. ____

Additional Practice

25. Simplify $\dfrac{x^2 - 9}{x - 3}$. ________

26. Excluded value of $\dfrac{1}{x + 4}$. ________

27. Domain of $f(x) = \dfrac{x}{x - 5}$. ________

28. Multiply $\dfrac{x}{3} \cdot \dfrac{6}{x}$. ________

29. Divide $\dfrac{x^2}{5} \div \dfrac{x}{10}$. ________

30. Add $\dfrac{3}{x} + \dfrac{5}{x}$. ________

31. Subtract $\dfrac{7}{x - 1} - \dfrac{2}{x - 1}$. ________

32. Solve $\dfrac{1}{x} = 4$. ________

33. Solve $\dfrac{x + 2}{x - 1} = 3$. ________

34. Vertical asymptote of $y = \dfrac{4}{x + 8}$. ________

35. Horizontal asymptote of $y = \dfrac{3x + 1}{x - 2}$. ________

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Answer Keys

1. $x = -5$
2. $y = 0$
3. $y = \frac{3}{2}$
4. slant: $y = x$
5. VA: $x = 3$, HA: $y = 2$
6. hole at $(3, 6)$, no VA, no HA
7. VA: $x = 3$, HA: $y = 3$, hole at $(-2, \frac{12}{5})$
8. VA: $x = -1$, slant: $y = x + 1$
9. hole at $(2, -4)$, VA: $x = 3$
10. VA: $x = 2$, HA: $y = 3$
11. VA: $x = -1$, HA: $y = 3$
12. FALSE
13. HA: $y = 3$; holes at $x = \pm 2$
14. $y = 1$
15. VA: $x = 1$, HA: $y = 0$
16. hole at $(-3, -7)$, no asymptotes
17. hole at $(-1, -2)$, no VA, no HA
18. HA: $y = 0$, no VA
19. $y = x$
20. $y = 5$
21. $y = 0$; concentration approaches 0 long-term
22. $y = 500$; population approaches 500 long-term
23. VA: $x = 1$; HA: $y = 1$; hole at $(-1, 2)$
24. VA: $x = 1$; slant: $y = x + 1$; $f(10) \approx 11.33$

Additional Practice Answers

25. $x + 3,\ x \neq 3$
26. $x = -4$
27. $x \neq 5$
28. 2
29. $2x$
30. $\frac{8}{x}$
31. $\frac{5}{x-1}$
32. $x = \frac{1}{4}$
33. $x = \frac{5}{2}$
34. $x = -8$
35. $y = 3$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Set $x+5=0$: $x=-5$. (Horizontal asymptote $y=0$ also visible – numerator's degree is less than denominator's.)

2. Keep the rule visible: Degree of numerator (0) is less than denominator's (1), so HA is $y=0$. That gives a quick check on the answer.

3. Numerator and denominator both have degree 1. When the degrees match, the horizontal asymptote is the ratio of the leading coefficients: $\frac{3}{2}$. (At large $|x|$ the constants $+1$ and $+5$ barely matter, so $f \to \frac{3x}{2x} = \frac{3}{2}$.)

4. Numerator degree exceeds denominator's by 1: slant asymptote. Long division: $\frac{x^2+1}{x} = x + \frac{1}{x}$. Slant: $y=x$.

5. A careful way to see it: VA: $x=3$ (denom zero). HA: equal degrees, ratio $\frac{2}{1}=2$. That gives a quick check on the answer.

6. Factor and cancel: $\frac{(x-3)(x+3)}{x-3} = x+3$ for $x \neq 3$. The simplified form is linear, so no asymptotes. The canceled $(x-3)$ produces a hole at $x=3$, $y=3+3=6$.

7. Factor: top $= 3(x-2)(x+2)$; bottom $=(x-3)(x+2)$. Cancel $(x+2)$: simplified is $\frac{3(x-2)}{x-3}$. Hole at $x=-2$: $y=\frac{3(-4)}{-5}=\frac{12}{5}$. Remaining denominator gives VA $x=3$; equal degrees give HA $y=3$.

8. Long division: $\frac{x^2+2x+3}{x+1} = x+1+\frac{2}{x+1}$. Slant $y=x+1$. Denominator zero at $x=-1$: VA.

9. Factor: top $=(x-2)(x+2)$; bottom $=(x-2)(x-3)$. Cancel $(x-2)$. Hole at $x=2$: $y=\frac{2+2}{2-3}=-4$. Remaining denominator: VA at $x=3$.

10. Keep the rule visible: $\frac{1}{x-2}+3$ is the parent $\frac{1}{x}$ shifted right 2 and up 3. VA: $x=2$. HA: $y=3$ (the term $\frac{1}{x-2} \to 0$ at large $|x|$, leaving $y \to 3$). That gives a quick check on the answer.

11. The fraction is already in lowest terms (no common factor). Set the denominator to zero for the vertical asymptote: $x+1=0 \Rightarrow x=-1$. Degrees are equal (1 and 1), so the horizontal asymptote is the leading-coefficient ratio $\frac{3}{1}=3$.

12. The function is undefined at a vertical asymptote – the curve cannot touch it. (Horizontal asymptotes are different and *can* be crossed.)

13. Factor: top $=3(x-2)(x+2)$; bottom $=(x-2)(x+2)$. Both factors cancel. Simplified is the constant 3 – horizontal line $y=3$, no asymptotes other than that, but holes at $x=2$ and $x=-2$ where the original was undefined.

14. Both top and bottom have degree 2. Equal degrees means the horizontal asymptote is the ratio of leading coefficients, $\frac{1}{1}=1$. (The denominator x^2-4 does zero out at $x=\pm 2$, giving vertical asymptotes there, but the question only asks for the horizontal one.)

15. VA at $x=1$ (denominator zero, doesn't cancel). HA: numerator degree 1, denominator degree 2, so $y=0$.

16. Factor: $2x^2+5x-3=(2x-1)(x+3)$. Cancel $(x+3)$: simplified is $2x-1$. Linear, so no asymptotes. Hole at $x=-3$: $y=2(-3)-1=-7$.

17. Factor: $x^2-1=(x-1)(x+1)$. Cancel $(x+1)$: simplified $=x-1$. Linear – no asymptotes. Hole at $x=-1$: $y=-1-1=-2$.

18. Denominator x^2+1 is never zero for real x: no VA. Numerator degree $<$ denominator: HA $y=0$.

19. One steady path is: Long division: $\frac{x^3}{x^2-1} = x + \frac{x}{x^2-1}$. Slant: $y=x$. This is the part to check before moving on, because it keeps the answer tied to the original question.

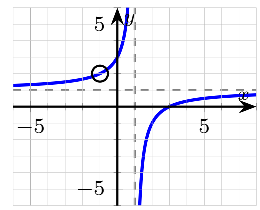
20. Top and bottom are both degree 2, so the horizontal asymptote is the ratio of leading coefficients: $\frac{5}{1}=5$. At large $|x|$ the -4 is negligible and $f \to \frac{5x^2}{x^2}=5$.

21. Numerator degree 1, denominator degree 2, so the horizontal asymptote is $y=0$. In context: as time grows large, the drug's concentration drops toward zero. The model rises from $C(0)=0$, peaks somewhere, and decays. (The peak is at $t=5$: $C(5)=\frac{500}{50}=10$ mg/L – a useful number to know but not required for the asymptote question.)

22. Equal degrees; ratio of leading coefficients $\frac{500}{1} = 500$. In context: the model predicts the population approaches 500 but never quite reaches it. (At $t = 0$: $P = \frac{1000}{5} = 200$ – the starting population. At $t = 20$: $\frac{11000}{25} = 440$. At $t = 100$: $\frac{51000}{105} \approx 486$ – closer to 500. The asymptote is the long-run capacity.)

23. Factor: top $= (x-3)(x+1)$; bottom $= (x-1)(x+1)$. Cancel $(x+1)$: simplified $= \frac{x-3}{x-1}$. Hole at $x = -1$: $y = \frac{-1-3}{-1-1} = \frac{-4}{-2} = 2$, so $(-1, 2)$. Remaining denominator $\Rightarrow$ VA at $x = 1$. Equal degrees $\Rightarrow$ HA at $y = \frac{1}{1} = 1$. All three features show up on the plot.

Answer graph



24. VA: $x - 1 = 0 \Rightarrow x = 1$. Slant: numerator's degree exceeds denominator's by 1. Long division: $\frac{x^2+2}{x-1} = x + 1 + \frac{3}{x-1}$. Slant: $y = x + 1$. **Verify at** $x = 10$**:** $f(10) = \frac{102}{9} \approx 11.33$. Slant line predicts $y = 11$. Difference: $\frac{3}{9} = \frac{1}{3} \approx 0.33$, matching the remainder term $\frac{3}{x-1} = \frac{3}{9}$ ✓. At very large x the difference shrinks toward zero – the definition of an asymptote.

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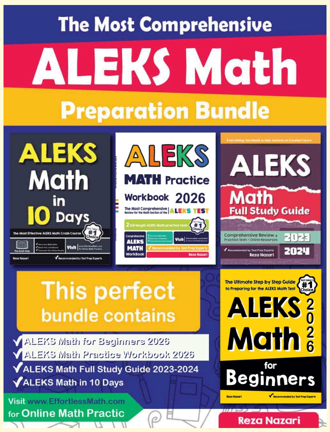

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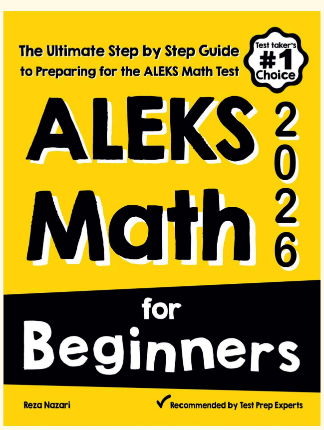

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