Rational Equations

Name: _____ Date: _____ Score: _____ / 34

Quick Review

A **rational equation** has variables in at least one denominator. There's exactly one safe recipe and one non-negotiable final step.

The recipe. (1) Note the domain restrictions – any x that makes a denominator zero. (2) Multiply both sides by the LCD of all denominators. The denominators vanish; you're left with a polynomial equation. (3) Solve. (4) **Check every candidate** in the original equation.

Why the check is mandatory. Multiplying both sides by the LCD can introduce *extraneous* solutions – values that satisfy the polynomial equation but make a denominator zero in the original. A common warning case is $\frac{x}{x-2} = \frac{2}{x-2} + 3$ multiplies out to $1 = 3$ (a contradiction), so the equation has **no solution** – even though, without the check, naive algebra might produce $x = 2$, which is forbidden.

Cross-multiplication. For two single fractions set equal, $\frac{a}{b} = \frac{c}{d}$, cross-multiply: $ad = bc$. This is just multiplying by the LCD (bd) in disguise – and the same extraneous-root check still applies.

Same-denominator shortcut. If both sides already share the same nonzero denominator, set the numerators equal. $\frac{5}{x-1} = \frac{x+3}{x-1} \Rightarrow 5 = x + 3 \Rightarrow x = 2$. Check: denominator at $x = 2$ is $1 \neq 0$, so the solution is valid.

Common slips. Skipping the check. Multiplying by an expression that could be zero and not noticing it might wipe out a solution. Distributing a negative sign incorrectly when the LCD has multiple factors. Treating a contradiction ($1 = 3$ etc.) as if it gave an answer.

PRACTICE

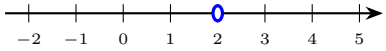
Identify the domain, multiply both sides by the LCD, solve, and CHECK every candidate in the original equation. Reject anything that makes a denominator zero.

1. Solve $\frac{x}{4} = \frac{3}{2}$. _____

2. Solve $\frac{x+1}{3} = \frac{4}{x}$. _____

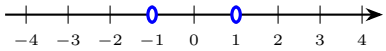
3. Solve $\frac{1}{x} + \frac{1}{2} = \frac{3}{x}$. _____

4. Solve $\frac{x}{x-2} = \frac{2}{x-2} + 3$. The number line marks the value the domain forbids (open dot). _____

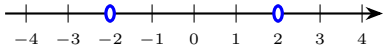


5. Solve $\frac{2}{x-1} = \frac{6}{x+5}$. _____

6. Solve $\frac{x}{x-1} = \frac{2}{x+1} + 1$. The number line shows the two forbidden values (open dots). _____



7. Solve $\frac{3}{x-2} - \frac{1}{x+2} = \frac{8}{x^2-4}$. The number line marks the values that make a denominator zero (open dots). _____



8. Solve $\frac{5}{x-1} = \frac{x+3}{x-1}$. _____

9. Solve $\frac{2}{x+1} + \frac{3}{x-1} = 0$. _____

10. Solve $\frac{3}{x} + \frac{1}{2} = 2$. _____

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11. Solve $\frac{4}{x-3} = \frac{2}{x-1}$. _____

12. Solve $\frac{x}{x+2} = \frac{3}{x+2}$. _____

13. Solve $\frac{1}{x-1} = \frac{2}{x+1}$. _____

14. Solve $\frac{2x}{x+1} - \frac{1}{x} = 1$. _____

15. Solve $\frac{x}{x-3} + \frac{3}{x-3} = 2$. _____

16. Solve $\frac{1}{x-2} + \frac{1}{x+2} = \frac{4}{x^2-4}$. _____

17. Solve $\frac{6}{x} - \frac{4}{x-1} = 0$. _____

18. Solve $\frac{x-2}{x+3} = \frac{1}{2}$. _____

19. Solve $\frac{x+1}{x-2} = \frac{x-1}{x+2}$. _____

20. Solve $\frac{2}{x} + 1 = \frac{4}{x}$. _____

♦ Word Problems

21. A swimmer can cross a river of width 1 km in time T satisfying $\frac{1}{T} = \frac{1}{3} + \frac{1}{6}$ (where the two terms model two simultaneous speed contributions). Find T, in hours, and verify by substituting back. ____

22. A motorboat travels 24 miles downstream and 24 miles back upstream in a total of 5 hours. The current's speed is 2 mph. Let x be the boat's speed in still water (mph). Set up and solve a rational equation for x. Discard any extraneous or non-physical roots. ____

23. A student claims that $x = 2$ solves $\frac{x}{x-2} = \frac{2}{x-2} + 3$ because cross-manipulation gives $x = 2$. Explain why the claim is wrong, and state the correct conclusion. ____

24. Two friends bike to a campsite 60 miles away. Anna rides 5 mph faster than Ben. Anna arrives 1 hour earlier. Set up and solve a rational equation for Ben's speed. Check the answer. ____

Additional Practice

25. Simplify $\frac{x^2-9}{x-3}$. _____

26. Excluded value of $\frac{1}{x+4}$. _____

27. Domain of $f(x) = \frac{x}{x-5}$. _____

28. Multiply $\frac{x}{3} \cdot \frac{6}{x}$. _____

29. Divide $\frac{x^2}{5} \div \frac{x}{10}$. _____

30. Add $\frac{3}{x} + \frac{5}{x}$. _____

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31. Subtract $\frac{7}{x-1} - \frac{2}{x-1}$. _____

32. Solve $\frac{1}{x} = 4$. _____

33. Solve $\frac{x+2}{x-1} = 3$. _____

34. Vertical asymptote of $y = \frac{4}{x+8}$. _____

Answer Keys

1. $\boxed{x = 6}$
2. $\boxed{x = 3 \text{ or } x = -4}$
3. $\boxed{x = 4}$
4. $\boxed{\text{no solution}}$
5. $\boxed{x = 4}$
6. $\boxed{x = 3}$
7. $\boxed{x = 0}$
8. $\boxed{x = 2}$
9. $\boxed{x = -\frac{1}{5}}$
10. $\boxed{x = 2}$
11. $\boxed{x = -1}$
12. $\boxed{x = 3}$
13. $\boxed{x = 3}$
14. $\boxed{x = 1 \pm \sqrt{2}}$
15. $\boxed{x = 9}$
16. $\boxed{\text{no solution}}$
17. $\boxed{x = 3}$
18. $\boxed{x = 7}$
19. $\boxed{x = 0}$
20. $\boxed{x = 2}$
21. $\boxed{T = 2 \text{ hours}}$
22. $\boxed{x = 10 \text{ mph}}$
23. $\boxed{\text{No solution: } x = 2 \text{ is extraneous.}}$
24. $\boxed{\text{Ben: } 15 \text{ mph; Anna: } 20 \text{ mph}}$

Additional Practice Answers

25. $\boxed{x + 3,\ x \neq 3}$
26. $\boxed{x = -4}$
27. $\boxed{x \neq 5}$
28. $\boxed{2}$
29. $\boxed{2x}$
30. $\boxed{\frac{8}{x}}$
31. $\boxed{\frac{5}{x-1}}$
32. $\boxed{x = \frac{1}{4}}$
33. $\boxed{x = \frac{5}{2}}$
34. $\boxed{x = -8}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Cross-multiply: $2x = 12$, so $x = 6$. **Check:** $\frac{6}{4} = \frac{3}{2}$. ✓. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. Cross-multiply: $x(x+1) = 12$, so $x^2 + x - 12 = 0$ and $(x+4)(x-3) = 0$. Candidates: $x = 3,\ -4$. **Check** $x = 3$: $\frac{4}{3} = \frac{4}{3}$ ✓. **Check** $x = -4$: $\frac{-3}{3} = \frac{4}{-4} \Rightarrow -1 = -1$ ✓. Both valid (neither makes a denominator zero – the only restriction was $x \neq 0$).

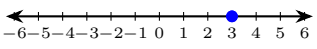
3. Domain: $x \neq 0$. LCD $= 2x$. Multiply: $2 + x = 6$, so $x = 4$. **Check:** $\frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$, and $\frac{3}{4} = \frac{3}{4}$ ✓. Valid.

4. Domain: $x \neq 2$. Subtract $\frac{2}{x-2}$ from both sides: $\frac{x-2}{x-2} = 3 \Rightarrow 1 = 3$. Contradiction – **no solution**. (If you tried to clear denominators by multiplying by $x-2$, you'd get $x = 2+3(x-2) = 3x-4$, so $-2x = -4$ and $x = 2$. But $x = 2$ is **extraneous** – forbidden by the domain. So the equation truly has no solution.)

5. Cross-multiply: $2(x+5) = 6(x-1)$. Expand: $2x + 10 = 6x - 6$. Collect: $16 = 4x$, so $x = 4$. **Check:** domain requires $x \neq 1, -5$; both are fine at $x = 4$. $\frac{2}{3} = \frac{6}{9} = \frac{2}{3}$ ✓.

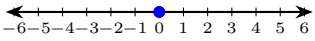
6. Domain: $x \neq \pm 1$. LCD $= (x-1)(x+1)$. Multiply both sides: $x(x+1) = 2(x-1) + (x-1)(x+1) = 2x - 2 + x^2 - 1 = x^2 + 2x - 3$. Simplify: $x^2 + x = x^2 + 2x - 3$, so $-x = -3$ and $x = 3$. **Check:** $\frac{3}{2} = \frac{2}{4} + 1 = \frac{1}{2} + 1 = \frac{3}{2}$ ✓. Valid (and $3 \neq \pm 1$, so the domain check passes).

Answer graph



7. Domain: $x \neq \pm 2$. Factor: $x^2 - 4 = (x-2)(x+2)$, so LCD $= (x-2)(x+2)$. Multiply through: $3(x+2) - (x-2) = 8$. Expand: $3x + 6 - x + 2 = 8$, so $2x + 8 = 8$ and $x = 0$. **Check:** $\frac{3}{-2} - \frac{1}{2} = -\frac{3}{2} - \frac{1}{2} = -2$, and $\frac{8}{-4} = -2$ ✓. Valid ($0 \neq \pm 2$).

Answer graph



8. Domain: $x \neq 1$. Same nonzero denominator – set numerators equal: $5 = x + 3$, so $x = 2$. **Check:** $\frac{5}{1} = \frac{5}{1}$ ✓.

9. Domain: $x \neq \pm 1$. LCD $= (x+1)(x-1)$. Multiply: $2(x-1) + 3(x+1) = 0 \Rightarrow 2x - 2 + 3x + 3 = 0 \Rightarrow 5x + 1 = 0$, so $x = -\frac{1}{5}$. **Check:** $\frac{2}{4/5} + \frac{3}{-6/5} = \frac{5}{2} - \frac{5}{2} = 0$ ✓.

10. Domain: $x \neq 0$. LCD $= 2x$. Multiply: $6 + x = 4x$, so $3x = 6$ and $x = 2$. **Check:** $\frac{3}{2} + \frac{1}{2} = 2$ ✓.

11. Cross-multiply: $4(x-1) = 2(x-3)$. Expand: $4x - 4 = 2x - 6$, so $2x = -2$ and $x = -1$. **Check:** domain $x \neq 1, 3$; $x = -1$ passes. $\frac{4}{-4} = \frac{2}{-2} = -1$ ✓.

12. Domain: $x \neq -2$. Same denominator: set numerators equal: $x = 3$. **Check:** $\frac{3}{5} = \frac{3}{5}$ ✓.

13. Cross-multiply: $x + 1 = 2(x-1)$. Expand: $x + 1 = 2x - 2$, so $x = 3$. **Check:** $\frac{1}{2} = \frac{2}{4} = \frac{1}{2}$ ✓.

14. Domain: $x \neq 0, -1$. LCD $= x(x+1)$. Multiply: $2x \cdot x - (x+1) = x(x+1)$. Expand: $2x^2 - x - 1 = x^2 + x$, so $x^2 - 2x - 1 = 0$. Quadratic: $x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$. **Check** $x = 1 + \sqrt{2} \approx 2.41$: $\frac{2(2.41)}{3.41} - \frac{1}{2.41} \approx 1.41 - 0.41 = 1$ ✓. **Check** $x = 1 - \sqrt{2} \approx -0.414$: $x + 1 \approx 0.586$, so $\frac{2x}{x+1} \approx \frac{-0.828}{0.586} \approx -1.414$, and $\frac{1}{x} \approx -2.414$. Difference: $-1.414 - (-2.414) = 1$ ✓. Both valid.

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15. Domain: $x \neq 3$. Same denominator: combine to get $\frac{x+3}{x-3} = 2$. Cross-multiply: $x + 3 = 2(x - 3)$, so $x + 3 = 2x - 6$ and $x = 9$. **Check:** $\frac{9}{6} + \frac{3}{6} = \frac{12}{6} = 2$ ✓. Valid (and $9 \neq 3$, so the domain check passes).

16. Domain: $x \neq \pm 2$. $x^2 - 4 = (x - 2)(x + 2)$. LCD $= (x - 2)(x + 2)$. Multiply: $(x + 2) + (x - 2) = 4 \Rightarrow 2x = 4 \Rightarrow x = 2$. **Check:** $x = 2$ makes the denominator zero – **extraneous**. No solution.

17. Domain: $x \neq 0, 1$. LCD $= x(x - 1)$. Multiply: $6(x - 1) - 4x = 0 \Rightarrow 6x - 6 - 4x = 0 \Rightarrow 2x = 6$, so $x = 3$. **Check:** $\frac{6}{3} - \frac{4}{2} = 2 - 2 = 0$ ✓.

18. Keep the rule visible: Cross-multiply: $2(x - 2) = x + 3$. Expand: $2x - 4 = x + 3$, so $x = 7$. **Check:** $\frac{5}{10} = \frac{1}{2}$ ✓. That gives a quick check on the answer.

19. Domain: $x \neq \pm 2$. Cross-multiply: $(x + 1)(x + 2) = (x - 1)(x - 2)$. Expand: $x^2 + 3x + 2 = x^2 - 3x + 2$, so $6x = 0$ and $x = 0$. **Check:** $\frac{1}{-2} = \frac{-1}{2} \Rightarrow -\frac{1}{2} = -\frac{1}{2}$ ✓.

20. Start with the key idea: Domain: $x \neq 0$. Multiply by x: $2 + x = 4$, so $x = 2$. **Check:** $\frac{2}{2} + 1 = 2$, and $\frac{4}{2} = 2$ ✓. That gives a quick check on the answer.

21. Combine the right side: $\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$. So $\frac{1}{T} = \frac{1}{2}$, giving $T = 2$ hours. **Check:** the right side at $T = 2$ is $\frac{1}{2}$, matching $\frac{1}{T} = \frac{1}{2}$ ✓. Domain OK ($T \neq 0$). (The model: if you can finish the job alone in 3 hours and your buddy in 6, together you finish in 2.)

22. Downstream speed is $x + 2$, upstream is $x - 2$. Times sum to 5: $\frac{24}{x+2} + \frac{24}{x-2} = 5$. Domain: $x \neq \pm 2$; physically $x > 2$. LCD $= (x + 2)(x - 2)$. Multiply: $24(x - 2) + 24(x + 2) = 5(x^2 - 4)$. Expand: $48x = 5x^2 - 20$, so $5x^2 - 48x - 20 = 0$. Factor: $(5x + 2)(x - 10) = 0$, giving $x = 10$ or $x = -\frac{2}{5}$. Reject the negative root (non-physical). **Check** $x = 10$**:** downstream time $= \frac{24}{12} = 2$ h; upstream time $= \frac{24}{8} = 3$ h; total $= 5$ h ✓. Boat's still-water speed is 10 mph – a clean positive integer, exactly as a well-set word problem should yield.

23. Domain check: the denominator $x - 2$ is zero at $x = 2$, so the original equation is undefined there. Any algebra that produces $x = 2$ has introduced an **extraneous** root. Plug $x = 2$ into the original: both denominators become 0, making the equation meaningless. In fact, subtracting $\frac{2}{x-2}$ from both sides gives $\frac{x-2}{x-2} = 3$, i.e., $1 = 3$ for any $x \neq 2$ – a contradiction. **Conclusion:** the equation has no solution. The lesson: always check candidate solutions in the *original* equation, not just the polynomial version after clearing denominators.

24. Let Ben's speed be x mph; Anna's is $x + 5$. Times: Ben takes $\frac{60}{x}$ hours; Anna takes $\frac{60}{x+5}$. Anna arrives 1 hour earlier: $\frac{60}{x} - \frac{60}{x+5} = 1$. Domain: $x \neq 0, -5$; physically $x > 0$. LCD $= x(x + 5)$. Multiply: $60(x + 5) - 60x = x(x + 5)$, so $300 = x^2 + 5x$, giving $x^2 + 5x - 300 = 0$. Factor: $(x - 15)(x + 20) = 0$, so $x = 15$ or $x = -20$. Reject $x = -20$ (negative speed). **Check** $x = 15$**:** Ben's time $= \frac{60}{15} = 4$ h; Anna's time $= \frac{60}{20} = 3$ h; difference $= 1$ h ✓. So Ben rides at 15 mph and Anna at 20 mph – both clean integer speeds, as a well-set word problem demands.

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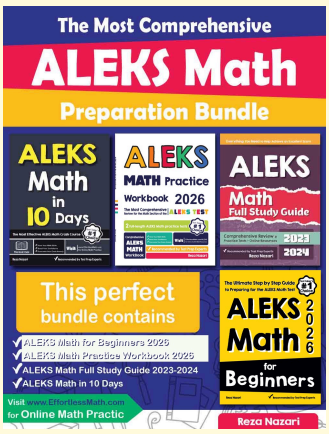

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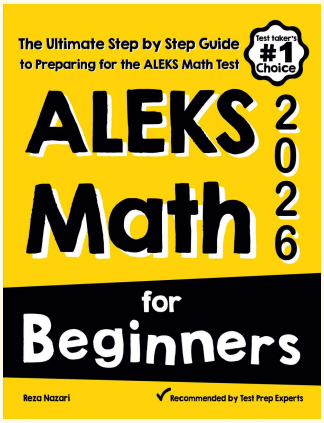

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