Simplifying Rational Expressions

Name: ______________ Date: ______________ Score: ______ / 30

Quick Review

A **rational expression** is a fraction $\frac{P(x)}{Q(x)}$ with $Q(x) \neq 0$. Simplifying one is the same move you make with numeric fractions: factor, cancel common factors, state what the denominator is not allowed to be.

Step 1 – factor everything. Don't even think about canceling until both top and bottom are fully factored. Common patterns: GCF (e.g. $2x^2 - 8 = 2(x-2)(x+2)$), difference of squares ($a^2 - b^2 = (a-b)(a+b)$), and trinomial factoring ($x^2 + 5x + 6 = (x+2)(x+3)$).

Step 2 – cancel *factors*, not terms. You can cross out $(x-3)$ when it appears in both numerator and denominator. You *cannot* cross out the x in $\frac{x+5}{x-3}$ – that x is part of a sum, not a factor. This is the single biggest rational-expression error students make.

Step 3 – state restrictions. Every value of x that makes the *original* denominator zero stays excluded, even if the offending factor cancels. Quick check: $\frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3} = x+3$ – but the answer still requires $x \neq 3$. (At $x = 3$ the original expression is undefined; the simplified version isn't, so they only agree off that one point.)

Sign-flipped factors. $a - b = -(b - a)$. So $\frac{2-x}{x-2} = \frac{-(x-2)}{x-2} = -1$ (for $x \neq 2$). When you see almost-matching factors that disagree in sign, factor out the -1.

Common slips. Canceling across addition (treating $\frac{x+5}{x}$ as 5). Dropping the restriction once the factor cancels. Forgetting that $\frac{0}{x}$ is undefined at $x = 0$, even though it's 0 everywhere else.

PRACTICE

Factor numerator and denominator, cancel common factors, and state every restriction from the original denominator.

1. Simplify $\frac{4x}{8x^2}$. Use the table to check your simplified form against the original at a few inputs. ________

x	1	2	4
$\frac{4x}{8x^2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

2. Simplify $\frac{x^2-9}{x-3}$. The table evaluates the expression away from the excluded value. ________

x	4	5	6
$\frac{x^2-9}{x-3}$	7	8	9

3. Simplify $\frac{x^2-5x+6}{x-2}$. The table gives a few sample values to test against your answer. ________

x	0	1	4
$\frac{x^2-5x+6}{x-2}$	-3	-2	1

4. $\frac{2-x}{x-2}$ ________

5. $\frac{6x^2y}{9xy^3}$ ________

6. $\frac{x^2+x-6}{x^2-4}$ ________

7. $\frac{2x^2-8}{x^2+2x-8}$ ________

8. $\frac{x^2-7x+12}{x^2-9}$ ________

9. $\frac{3x-6}{4-2x}$ ________

Scan Me

10. $\frac{x^2+x-12}{x^2-16}$ ______

11. $\frac{x^2-25}{x+5}$ ______

12. $\frac{x^3-x}{x^2+x}$ ______

13. $\frac{x^2-1}{x^2-2x+1}$ ______

14. $\frac{4x^2-9}{2x+3}$ ______

15. $\frac{5-x}{x^2-25}$ ______

16. $\frac{x^2+6x+9}{x^2-9}$ ______

17. $\frac{2x^2+5x+3}{x+1}$ ______

18. $\frac{x^2-4x}{x^2-16}$ ______

19. $\frac{x^2-x-12}{x^2-2x-8}$ ______

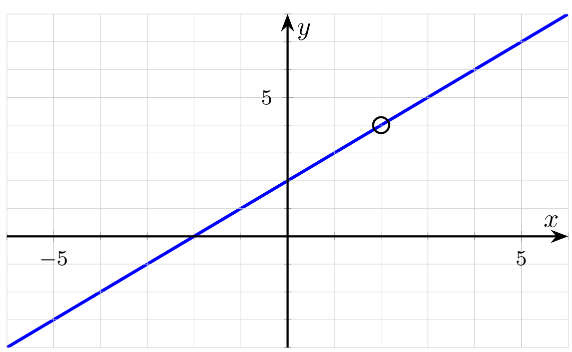
20. $\frac{9-x^2}{x^2-3x}$ ______

♦ Word Problems

21. A rectangle has area x^2+5x+6 square inches and width $x+2$ inches. Find the length, in simplest form, and state the values of x for which the answer makes physical sense. ______

22. A student writes $\frac{x+3}{x}=3$ by "canceling the x." Show what's wrong by plugging in $x=2$ on both sides, and write the correct simplified form (or explain why none is possible). ______

23. The expression $f(x)=\frac{x^2-4}{x-2}$ is graphed below. Simplify $f(x)$ algebraically, state any restriction, and identify the hole shown in the graph. ______



24. A rate problem leads to the expression $\frac{2x^2+7x+3}{x^2-9}$. Simplify it and state every restriction. (The expression came from a real context where $x>3$, so the algebraic restrictions can be checked against the context.) ______

Scan Me

Additional Practice

25. Simplify $\dfrac{x^2-9}{x-3}$. ________

26. Excluded value of $\dfrac{1}{x+4}$. ________

27. Domain of $f(x) = \dfrac{x}{x-5}$. ________

28. Multiply $\dfrac{x}{3} \cdot \dfrac{6}{x}$. ________

29. Divide $\dfrac{x^2}{5} \div \dfrac{x}{10}$. ________

30. Add $\dfrac{3}{x} + \dfrac{5}{x}$. ________

Answer Keys

1. $\boxed{\frac{1}{2x},\ x \neq 0}$
2. $\boxed{x+3,\ x \neq 3}$
3. $\boxed{x-3,\ x \neq 2}$
4. $\boxed{-1,\ x \neq 2}$
5. $\boxed{\frac{2x}{3y^2},\ x \neq 0,\ y \neq 0}$
6. $\boxed{\frac{x+3}{x+2},\ x \neq \pm 2}$
7. $\boxed{\frac{2(x+2)}{x+4},\ x \neq -4,\ 2}$
8. $\boxed{\frac{x-4}{x+3},\ x \neq \pm 3}$
9. $\boxed{-\frac{3}{2},\ x \neq 2}$
10. $\boxed{\frac{x-3}{x-4},\ x \neq \pm 4}$
11. $\boxed{x-5,\ x \neq -5}$
12. $\boxed{x-1,\ x \neq 0,\ -1}$
13. $\boxed{\frac{x+1}{x-1},\ x \neq 1}$
14. $\boxed{2x-3,\ x \neq -\frac{3}{2}}$
15. $\boxed{-\frac{1}{x+5},\ x \neq \pm 5}$
16. $\boxed{\frac{x+3}{x-3},\ x \neq \pm 3}$
17. $\boxed{2x+3,\ x \neq -1}$
18. $\boxed{\frac{x}{x+4},\ x \neq \pm 4}$
19. $\boxed{\frac{x+3}{x+2},\ x \neq -2,\ 4}$
20. $\boxed{-\frac{x+3}{x},\ x \neq 0,\ 3}$
21. $\boxed{x+3 \text{ in},\ x > 0}$
22. $\boxed{\text{Invalid cancel; } f(2) = \frac{5}{2} \neq 3}$
23. $\boxed{f(x) = x+2,\ x \neq 2;\ \text{hole at } (2,4)}$
24. $\boxed{\frac{2x+1}{x-3},\ x \neq \pm 3}$

Additional Practice Answers

25. $\boxed{x+3,\ x \neq 3}$
26. $\boxed{x = -4}$
27. $\boxed{x \neq 5}$
28. $\boxed{2}$
29. $\boxed{2x}$
30. $\boxed{\frac{8}{x}}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. GCF on top and bottom is $4x$. Divide: $\frac{4x}{8x^2} = \frac{1}{2x}$. The original denominator is zero at $x = 0$, so $x \neq 0$. (The table values match $\frac{1}{2x}$ at $x = 1, 2, 4$ – a quick correctness check.)
2. Top is a difference of squares: $(x-3)(x+3)$. Cancel $x-3$. The restriction $x \neq 3$ stays even though the factor cancels – that's the whole point. (Notice the table skips $x = 3$, where the original is undefined.)
3. Factor the trinomial: $x^2 - 5x + 6 = (x-2)(x-3)$. Cancel $x-2$; the simplified form is $x-3$, valid for $x \neq 2$. (The table values agree with $x-3$ at $x = 0, 1, 4$.)
4. Start with the key idea: $2 - x = -(x-2)$. Cancel to get -1. (Sign-flipped factors always collapse to -1.) That gives a quick check on the answer.
5. Coefficients: $\frac{6}{9} = \frac{2}{3}$. Powers: $\frac{x^2}{x} = x$, $\frac{y}{y^3} = \frac{1}{y^2}$. Combine: $\frac{2x}{3y^2}$.
6. Top: $(x+3)(x-2)$. Bottom (difference of squares): $(x-2)(x+2)$. Cancel $x-2$. Both ± 2 remain excluded because they sat in the original denominator.
7. Top: $2(x^2-4) = 2(x-2)(x+2)$. Bottom: $(x-2)(x+4)$. Cancel $x-2$. The factor pulls a 2 along for the ride.
8. Start with the key idea: Top: $(x-3)(x-4)$. Bottom: $(x-3)(x+3)$. Cancel $x-3$. This is the part to check before moving on, because it keeps the answer tied to the original question.
9. Top: $3(x-2)$. Bottom: $-2(x-2)$ (factor out the sign-flipper). Cancel $(x-2)$: $\frac{3}{-2} = -\frac{3}{2}$.
10. Keep the rule visible: Top: $(x+4)(x-3)$. Bottom: $(x-4)(x+4)$. Cancel $x+4$. This is the part to check before moving on, because it keeps the answer tied to the original question.
11. One steady path is: Top: $(x-5)(x+5)$. Cancel $x+5$. This is the part to check before moving on, because it keeps the answer tied to the original question.
12. Top: $x(x^2-1) = x(x-1)(x+1)$. Bottom: $x(x+1)$. Cancel the x and the $(x+1)$. Both $x = 0$ and $x = -1$ stay excluded.
13. Top: $(x-1)(x+1)$. Bottom: $(x-1)^2$. Cancel one copy of $x-1$. (One $x-1$ remains in the denominator – still need $x \neq 1$.)
14. Top is a difference of squares: $(2x-3)(2x+3)$. Cancel $2x+3$. The restriction $2x+3 \neq 0$ becomes $x \neq -\frac{3}{2}$.
15. One steady path is: Bottom: $(x-5)(x+5)$. Top: $-(x-5)$. Cancel $(x-5)$: $\frac{-1}{x+5}$. That gives a quick check on the answer.
16. Start with the key idea: Top: $(x+3)^2$. Bottom: $(x-3)(x+3)$. Cancel one $(x+3)$. This is the part to check before moving on, because it keeps the answer tied to the original question.
17. A careful way to see it: Top: $(2x+3)(x+1)$. Cancel $x+1$. This is the part to check before moving on, because it keeps the answer tied to the original question.
18. Keep the rule visible: Top: $x(x-4)$. Bottom: $(x-4)(x+4)$. Cancel $x-4$. This is the part to check before moving on, because it keeps the answer tied to the original question.
19. Top: $(x-4)(x+3)$. Bottom: $(x-4)(x+2)$. Cancel $x-4$. (Both original denominator factors – $x = -2$ and $x = 4$ – remain excluded.)
20. Top: $9 - x^2 = -(x^2-9) = -(x-3)(x+3)$. Bottom: $x(x-3)$. Cancel $(x-3)$: $\frac{-(x+3)}{x} = -\frac{x+3}{x}$.
21. Length $= \frac{\text{area}}{\text{width}} = \frac{x^2+5x+6}{x+2}$. Factor the top: $(x+2)(x+3)$. Cancel $x+2$: length $= x+3$ in. The restriction from the algebra is $x \neq -2$, but physically the rectangle's dimensions must be positive, so we also need $x > 0$ (which automatically rules out $x = -2$). At $x = 4$, for example, area $= 42$ in^2, width $= 6$ in, length $= 7$ in – consistent with the simplified formula.
22. Plug in $x = 2$: the left side is $\frac{2+3}{2} = \frac{5}{2} = 2.5$, not 3. So the "cancel the x" move is wrong. The reason: x on top is part of a sum $(x+3)$, not a factor. Only *factors* cancel. The expression $\frac{x+3}{x} = 1 + \frac{3}{x}$ is as simplified as it gets without canceling a common factor (and no such factor exists). This trap is one of the most common errors in Algebra 2 – it's worth keeping the numeric check in your back pocket.

Scan Me

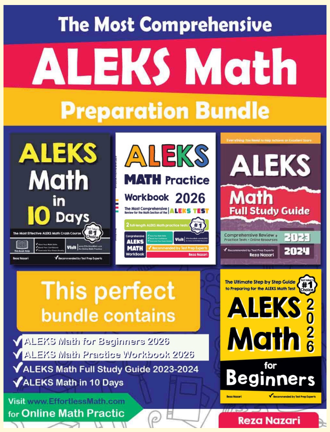
23. Factor the numerator: $x^2-4=(x-2)(x+2)$. So $f(x)=\dfrac{(x-2)(x+2)}{x-2}=x+2$ when $x\neq 2$. At $x=2$ the original expression is $\dfrac{0}{0}$ – undefined – but the simplified version gives $2+2=4$. That mismatch is exactly what produces a *hole* at $(2,4)$ on the graph: a single missing point on an otherwise straight line $y=x+2$. (The open circle on the plot is the visual signature of a canceled factor – a removable discontinuity.)

24. Factor: top $=(2x+1)(x+3)$ (split $7x$ as $6x+x$; group $2x^2+6x=2x(x+3)$ and $x+3$); bottom $=(x-3)(x+3)$. Cancel $x+3$: $\dfrac{2x+1}{x-3}$. Restrictions: $x\neq\pm 3$ (both from the original denominator). The context $x>3$ rules out both algebraic problem values automatically – the simplified rate is safe to use. Sanity check at $x=5$: original $=\dfrac{50+35+3}{25-9}=\dfrac{88}{16}=\dfrac{11}{2}$. Simplified $=\dfrac{11}{2}$. Match.

Scan Me

Keep Building ALEKS Skills

Recommended ALEKS books and bundle



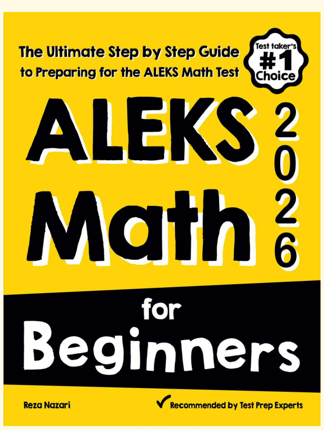
The Most Comprehensive ALEKS Math Preparation Bundle

Prep books, workbooks, full-length tests, detailed explanations, and a formula bonus file in one complete ALEKS study path.

Scan Me

Download Instantly

STUDENT FAVORITE · ALEKS Math for Beginners



ALEKS Math for Beginners 2026

Step-by-step lessons, basic study guides, topic practice, and two full-length ALEKS Math tests for students who need a calm path from review to readiness.

Great companion to these worksheets for self-study, tutoring, and classroom review.

PDF Edition

Scan Me

Amazon

Scan Me

For more ALEKS prep, visit EffortlessMath.com/ALEKS