Adding and Subtracting Rational Expressions

Name: _____	Date: _____	Score: _____ / 34

Quick Review

Adding or subtracting rational expressions is the same logic as adding numeric fractions: you need a **common denominator** first. With polynomials the LCD takes one extra step (factor each denominator), but the rule is identical.

Step 1 – factor every denominator. Once you see the factors, the LCD is the product of each distinct factor at the highest power it appears in any denominator. Quick check: denominators $x-3$ and $x^2-9=(x-3)(x+3)$ share $(x-3)$, so the LCD is $(x-3)(x+3)$.

Step 2 – rewrite each fraction with the LCD. Multiply numerator and denominator of each piece by whatever factor is missing. Keep the new numerators in factored form – you'll need to combine them next.

Step 3 – combine the numerators. Add or subtract across the top. The *biggest* trap on subtraction: the negative sign hits *every term* in the second numerator, not just the first. Use parentheses and distribute carefully.

Step 4 – simplify and state restrictions. Factor the resulting numerator if you can, look for any common factor with the LCD, and cancel. State $x \neq$ values from *every* original denominator (whether or not those factors cancel).

What you should never do. Add numerators when denominators differ. That's the equivalent of saying $\frac{1}{2}+\frac{1}{3}=\frac{2}{5}$ – the universal warning sign of fraction trouble.

The subtraction sign trap. $\frac{a}{d}-\frac{b+c}{d}=\frac{a-b-c}{d}$, not $\frac{a-b+c}{d}$. The minus distributes through the entire second numerator. This catches more students than any other sub-step in the topic.

PRACTICE

Factor denominators, build the LCD, rewrite with it, combine numerators, then simplify and state restrictions.

1. Add $\frac{2}{x}+\frac{3}{x}$. The table evaluates the sum at a few inputs. _____

x	1	2	5
$\frac{2}{x}+\frac{3}{x}$	5	$\frac{5}{2}$	1

2. $\frac{x+5}{x-1}-\frac{x+2}{x-1}$ _____

3. Add $\frac{1}{x}+\frac{2}{x+1}$. The table samples the sum away from the excluded values. _____

x	1	2	3
$\frac{1}{x}+\frac{2}{x+1}$	2	$\frac{7}{6}$	$\frac{5}{6}$

4. Subtract $\frac{3}{x+2}-\frac{2}{x-1}$. The table evaluates the difference at several inputs. _____

x	0	2	3
difference	$\frac{7}{2}$	$-\frac{5}{4}$	$-\frac{2}{5}$

5. What is the LCD of $\frac{1}{x-3}$ and $\frac{2}{x^2-9}$? _____

6. $\frac{1}{x}+\frac{1}{x+1}-\frac{1}{x-1}$ _____

7. $\frac{2}{x-3}+\frac{x}{x+3}$ _____

8. $\frac{3}{x-4}+\frac{2}{x^2-16}$ _____

9. $\frac{x}{x-2}-\frac{5}{x+1}$ _____

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10. $\frac{3}{x+1} - \frac{1}{x}$ _____

11. $\frac{5}{x^2-1} + \frac{3}{x+1}$ _____

12. $\frac{2x}{x-1} + \frac{x+1}{x-1}$ _____

13. $\frac{1}{x+2} + \frac{1}{x-2}$ _____

14. $\frac{x+3}{x-2} - \frac{x-2}{x+3}$ _____

15. $\frac{4}{x} - \frac{1}{2x}$ _____

16. Mark TRUE or FALSE: $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$. _____

17. $\frac{2x}{x+3} - \frac{x-1}{x+3}$ _____

18. $\frac{x}{x^2-4} + \frac{1}{x-2}$ _____

19. $\frac{5}{x-3} + \frac{4}{3-x}$ _____

20. $\frac{x+1}{x} + \frac{1}{x+1}$ _____

♦ Word Problems

21. Two pipes fill a tank. Pipe A fills $\frac{1}{x}$ of the tank per hour, Pipe B fills $\frac{1}{x+1}$ per hour. Working together, what fraction of the tank do they fill in one hour? Give the combined rate as a single simplified rational expression. ____

22. A trip of D miles is driven at speed r mph, then the same distance back at speed $r+10$ mph. Express the total time, in hours, as a single simplified rational expression in r and D. ____

23. A baker uses $\frac{x}{x+2}$ cups of flour for one recipe and $\frac{3}{x+2}$ cups for another. Find the total flour used, in simplest form, and state the physical restriction on x. ____

24. In a parallel-resistor circuit, the total resistance satisfies $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2}$. With $R_1 = x$ ohms and $R_2 = x+4$ ohms, find a single simplified expression for R_{total}. ____

Additional Practice

25. Simplify $\frac{x^2-9}{x-3}$. _____

26. Excluded value of $\frac{1}{x+4}$. _____

27. Domain of $f(x) = \frac{x}{x-5}$. _____

28. Multiply $\frac{x}{3} \cdot \frac{6}{x}$. _____

29. Divide $\frac{x^2}{5} \div \frac{x}{10}$. _____

30. Add $\frac{3}{x} + \frac{5}{x}$. _____

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31. Subtract $\frac{7}{x-1} - \frac{2}{x-1}$. _____

32. Solve $\frac{1}{x} = 4$. _____

33. Solve $\frac{x+2}{x-1} = 3$. _____

34. Vertical asymptote of $y = \frac{4}{x+8}$. _____

Answer Keys

1. $\frac{5}{x},\ x \neq 0$
2. $\frac{3}{x-1},\ x \neq 1$
3. $\frac{3x+1}{x(x+1)},\ x \neq 0,\ -1$
4. $\frac{x-7}{(x+2)(x-1)},\ x \neq -2,\ 1$
5. $(x-3)(x+3)$
6. $\frac{x^2-2x-1}{x(x+1)(x-1)},\ x \neq 0,\ \pm 1$
7. $\frac{x^2-x+6}{(x-3)(x+3)},\ x \neq \pm 3$
8. $\frac{3x+14}{(x-4)(x+4)},\ x \neq \pm 4$
9. $\frac{x^2-4x+10}{(x-2)(x+1)},\ x \neq 2,\ -1$
10. $\frac{2x-1}{x(x+1)},\ x \neq 0,\ -1$
11. $\frac{3x+2}{(x-1)(x+1)},\ x \neq \pm 1$
12. $\frac{3x+1}{x-1},\ x \neq 1$
13. $\frac{2x}{(x-2)(x+2)},\ x \neq \pm 2$
14. $\frac{10x+5}{(x-2)(x+3)},\ x \neq 2,\ -3$
15. $\frac{7}{2x},\ x \neq 0$
16. FALSE
17. $\frac{x+1}{x+3},\ x \neq -3$
18. $\frac{2x+2}{(x-2)(x+2)},\ x \neq \pm 2$
19. $\frac{1}{x-3},\ x \neq 3$
20. $\frac{x^2+3x+1}{x(x+1)},\ x \neq 0,\ -1$
21. $\frac{2x+1}{x(x+1)}$ per hour, $x > 0$
22. $\frac{D(2r+10)}{r(r+10)}$ hours, $r > 0$
23. $\frac{x+3}{x+2}$ cups, $x \geq 0$
24. $R_{\text{total}} = \frac{x(x+4)}{2x+4}$ ohms, $x > 0$

Additional Practice Answers

25. $x+3,\ x \neq 3$
26. $x = -4$
27. $x \neq 5$
28. 2
29. $2x$
30. $\frac{8}{x}$
31. $\frac{5}{x-1}$
32. $x = \frac{1}{4}$
33. $x = \frac{5}{2}$
34. $x = -8$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Same denominator: $\frac{2+3}{x} = \frac{5}{x}$. (The table values match $\frac{5}{x}$ at $x = 1, 2, 5$.) That gives a quick check on the answer.

2. Same denominator: combine numerators. $(x+5)-(x+2) = x+5-x-2 = 3$. Watch the sign: it hits both x and $+2$.

3. LCD $= x(x+1)$. $\frac{x+1}{x(x+1)} + \frac{2x}{x(x+1)} = \frac{3x+1}{x(x+1)}$. (The table matches $\frac{3x+1}{x(x+1)}$ at $x = 1, 2, 3$.)

4. LCD $= (x+2)(x-1)$. Numerators: $3(x-1)-2(x+2) = 3x-3-2x-4 = x-7$. (The table agrees with $\frac{x-7}{(x+2)(x-1)}$ at $x = 0, 2, 3$.)

5. Factor: $x^2-9 = (x-3)(x+3)$. The LCD is $(x-3)(x+3)$ (each factor at its highest power).

6. LCD $= x(x+1)(x-1)$. Numerators: $(x+1)(x-1)+x(x-1)-x(x+1) = (x^2-1)+(x^2-x)-(x^2+x) = x^2-2x-1$.

7. One steady path is: LCD $= (x-3)(x+3)$. Numerators: $2(x+3)+x(x-3) = 2x+6+x^2-3x = x^2-x+6$. That gives a quick check on the answer.

8. Start with the key idea: $x^2-16 = (x-4)(x+4)$. LCD $= (x-4)(x+4)$. $\frac{3(x+4)+2}{(x-4)(x+4)} = \frac{3x+14}{(x-4)(x+4)}$. That gives a quick check on the answer.

9. A careful way to see it: LCD $= (x-2)(x+1)$. Numerators: $x(x+1)-5(x-2) = x^2+x-5x+10 = x^2-4x+10$. That gives a quick check on the answer.

10. Keep the rule visible: LCD $= x(x+1)$. Numerators: $3x-(x+1) = 3x-x-1 = 2x-1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: $x^2-1 = (x-1)(x+1)$. LCD $= (x-1)(x+1)$. Numerators: $5+3(x-1) = 5+3x-3 = 3x+2$. Final: $\frac{3x+2}{(x-1)(x+1)}$. That gives a quick check on the answer.

12. The denominators already match, so just add the tops: $2x+(x+1) = 3x+1$, over the shared $x-1$. The numerator doesn't factor to cancel anything, so keep $x \neq 1$.

13. The LCD is $(x-2)(x+2)$. Rewrite each piece: $\frac{x-2}{(x-2)(x+2)} + \frac{x+2}{(x-2)(x+2)}$. Add the tops: $(x-2)+(x+2) = 2x$, giving $\frac{2x}{(x-2)(x+2)}$. Keep $x \neq \pm 2$.

14. LCD $= (x-2)(x+3)$. Numerators: $(x+3)^2-(x-2)^2 = (x^2+6x+9)-(x^2-4x+4) = 10x+5$.

15. One steady path is: LCD $= 2x$. $\frac{8-1}{2x} = \frac{7}{2x}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Add fractions over a common denominator: $\frac{ad+bc}{bd}$. Adding numerators and denominators separately is one of the most universal fraction errors.

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17. Denominators match, so subtract the tops – but the minus sign hits *both* terms of $(x-1)$: $2x-(x-1)=2x-x+1=x+1$. Result: $\frac{x+1}{x+3}$, with $x \neq -3$.

18. Keep the rule visible: $x^2-4=(x-2)(x+2)$. LCD $=(x-2)(x+2)$. $\frac{x+(x+2)}{(x-2)(x+2)}=\frac{2x+2}{(x-2)(x+2)}$. That gives a quick check on the answer.

19. One steady path is: $3-x=-(x-3)$. So $\frac{4}{3-x}=-\frac{4}{x-3}$. Sum: $\frac{5-4}{x-3}=\frac{1}{x-3}$. (Sign-flip denominators are the same denominator in disguise.) That gives a quick check on the answer.

20. Start with the key idea: LCD $=x(x+1)$. $\frac{(x+1)^2+x}{x(x+1)}=\frac{x^2+2x+1+x}{x(x+1)}=\frac{x^2+3x+1}{x(x+1)}$. That gives a quick check on the answer.

21. Combined rate $=\frac{1}{x}+\frac{1}{x+1}$. LCD $=x(x+1)$. $\frac{(x+1)+x}{x(x+1)}=\frac{2x+1}{x(x+1)}$ per hour. The algebraic restriction is $x \neq 0,-1$; physically the rates must be positive, so $x>0$. Sanity check at $x=2$: $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$. Simplified: $\frac{5}{6}$. Match. (At $x=2$ the two pipes together fill $\frac{5}{6}$ of the tank per hour – so they finish a full tank in $\frac{6}{5}=1.2$ hours.)

22. Time $=$ distance/speed. Out: $\frac{D}{r}$. Back: $\frac{D}{r+10}$. Total: $\frac{D}{r}+\frac{D}{r+10}=\frac{D(r+10)+Dr}{r(r+10)}=\frac{D(2r+10)}{r(r+10)}$. Restriction: $r \neq 0,-10$; physically $r>0$ (and $r+10>0$ automatically). Sanity check with $D=60$, $r=30$: $\frac{60}{30}+\frac{60}{40}=2+1.5=3.5$ h. Simplified: $\frac{60\cdot 70}{30\cdot 40}=\frac{4200}{1200}=3.5$ h. Match.

23. Same denominator: add the numerators. $\frac{x+3}{x+2}$ cups total. Algebraic restriction: $x \neq -2$; physical restriction (cups can't be negative): $x \geq 0$. Sanity check at $x=4$: $\frac{4}{6}+\frac{3}{6}=\frac{7}{6}$ cups. Simplified: $\frac{7}{6}$. Match.

24. Start with the key idea: $\frac{1}{R_{\text{total}}}=\frac{1}{x}+\frac{1}{x+4}=\frac{(x+4)+x}{x(x+4)}=\frac{2x+4}{x(x+4)}$. Take the reciprocal: $R_{\text{total}}=\frac{x(x+4)}{2x+4}$ ohms. Algebraic restrictions: $x \neq 0,-4$; physically $x>0$. Sanity check at $x=4$: $\frac{1}{4}+\frac{1}{8}=\frac{3}{8}$, so $R=\frac{8}{3}\approx 2.67$ ohms. Formula: $\frac{4\cdot 8}{12}=\frac{32}{12}=\frac{8}{3}$. Match. (Parallel resistance is always less than either individual resistor – here $\frac{8}{3}<4$ and $\frac{8}{3}<8$.) That gives a quick check on the answer.

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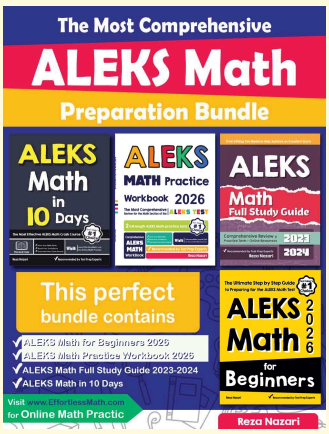

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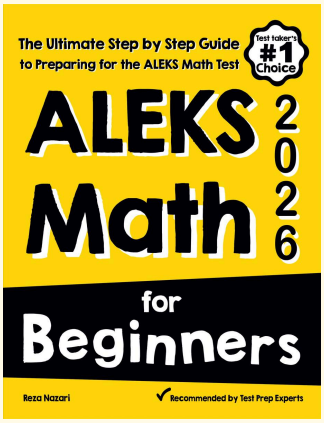

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