Finite Geometric Series

Name: ______________ Date: ______________ Score: ______ / 30

Quick Review

A **finite geometric series** is the sum of the first n terms of a geometric sequence. The formula doesn't ask you to add them up by hand:

Main formula. $S_n = \frac{a_1(1-r^n)}{1-r}$ for any $r \neq 1$. Equivalently (multiply top and bottom by -1): $S_n = \frac{a_1(r^n-1)}{r-1}$. Pick whichever keeps signs friendly – if $r > 1$, the second form has positive numerator and denominator; if $|r| < 1$, the first form is cleaner.

Edge case $r = 1$**.** Every term equals a_1, so $S_n = n \cdot a_1$. The main formula has $1 - r = 0$ in the denominator, undefined – that's why it carries the $r \neq 1$ restriction.

Where the formula comes from. Write $S = a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1}$. Multiply by r to get $rS = a_1r + a_1r^2 + \cdots + a_1r^n$. Subtract: most of the middle terms cancel, leaving $S - rS = a_1 - a_1r^n$. Factor: $S(1-r) = a_1(1-r^n)$, divide.

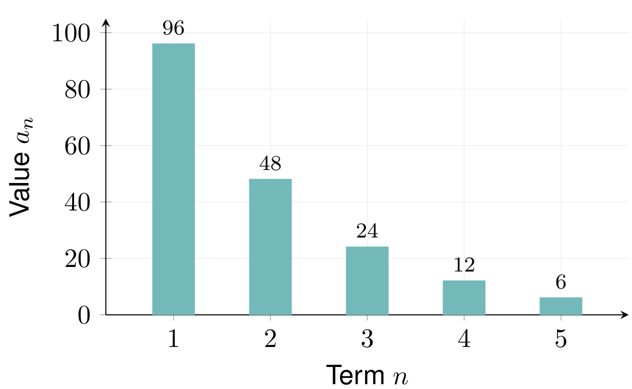
Sigma form. Sums like $\sum_{k=0}^{n-1} a_1r^k$ and $\sum_{k=1}^{n} a_1r^{k-1}$ both give the standard n-term geometric series. The bottom-equals-0 form has n terms running from $k = 0$ to $k = n - 1$.

Common slips. Plugging in r^{n-1} when the formula needs r^n. Counting terms wrong when sigma starts at $k = 0$ – $\sum_{k=0}^{n}$ has $n + 1$ terms, not n. Forgetting that a negative r raised to an even power becomes positive (it changes the sign of $1 - r^n$).

PRACTICE

Compute finite geometric sums. Pick the form of the formula that keeps the arithmetic clean.

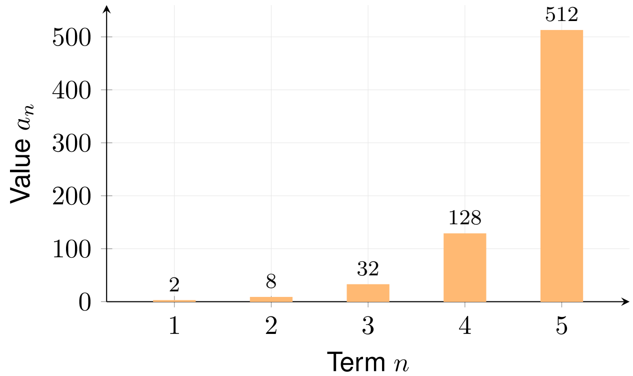
1. Find $1 + 2 + 4 + 8 + 16$. ________
2. Find S_6 for $a_1 = 5$, $r = 2$. ________
3. Find S_4 for $a_1 = 8$, $r = \frac{1}{2}$. ________
4. Compute $\sum_{k=0}^{4} 3^k$. ________
5. Find r if $a_1 = 2$ and $S_4 = 80$ (positive r). ________
6. Find the sum of the first 7 terms with $a_1 = 4$, $r = 3$. ________
7. Find the sum of the first 5 terms with $a_1 = 6$, $r = -2$. ________
8. Use the bar chart to compute S_5 for $a_1 = 96$, $r = \frac{1}{2}$. ________



9. Compute $\sum_{k=1}^{6} 3 \cdot 2^{k-1}$. ________
10. True or False: when $r = 1$, $S_n = \frac{a_1(1-r^n)}{1-r}$ still applies. ________
11. A geometric series has first 4 terms summing to 40 with $r = 3$. Find a_1. ________

Scan Me

12. Find S_5 for $a_1 = 2$, $r = 4$. ______



13. Compute $\sum_{k=1}^{5} 2 \cdot 3^k$. ______

14. A geometric series has $a_1 = 1$, $r = \frac{1}{3}$. Find S_4. ______

15. Find the sum of the first 6 terms with $a_1 = 2$ and $r = -3$. ______

16. Find a_1 if $r = 2$ and $S_5 = 93$. ______

17. A prize starts at \$$3$ in round 1 and doubles each round. Total awarded over 8 rounds? ______

18. Compute $\sum_{k=0}^{5} 5 \cdot 2^k$. ______

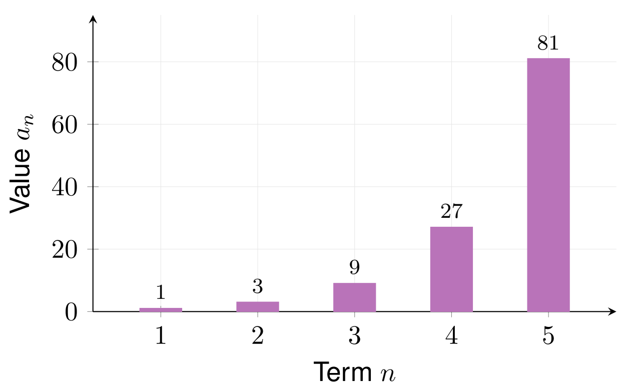
19. True or False: $\sum_{k=1}^{n} a_1 r^{k-1}$ and $\sum_{k=0}^{n-1} a_1 r^k$ give the same sum. ______

20. A geometric series has $a_1 = 4$, $r = \frac{1}{2}$. Find S_6. ______

♦ Word Problems

21. A grandparent gives a child \$$2$ on the first birthday, and doubles the amount every year. What is the total received over the first 10 birthdays? ______

22. A chain message starts when one person forwards it to 3 friends. Each of those friends forwards to 3 more. After 5 rounds (counting the first sender as round 1), how many people total have received the message? ______



23. A patient takes a 200 mg dose once a day. Each day, 25% of the previous day's drug is still in the bloodstream (so 75% has been metabolized). The residual amounts from the first 5 doses (just before the next dose) form a geometric sequence $200(0.25), 200(0.25)^2, \ldots$ What is the total residual from the first 5 doses, just before the sixth dose? ______

Scan Me

24. A savings plan deposits $\$100$ in month 1, and each month after that deposits half as much as the previous month ($\$50$, then $\$25$, then $\$12.50$, and so on). What is the total deposited in the first 8 months? ____

Additional Practice

25. Find the next term: $4, 9, 14, 19, \ldots$. ________

26. Find a_{10} if $a_1 = 3$ and $d = 5$. ________

27. Find the next term: $2, 6, 18, 54, \ldots$. ________

28. Find a_6 if $a_1 = 5$ and $r = 2$. ________

29. Sum $1 + 2 + 3 + \cdots + 20$. ________

30. Find S_5 for $3, 6, 12, 24, 48$. ________

Answer Keys

1. $\boxed{31}$
2. $\boxed{315}$
3. $\boxed{15}$
4. $\boxed{121}$
5. $\boxed{r = 3}$
6. $\boxed{4372}$
7. $\boxed{66}$
8. $\boxed{186}$
9. $\boxed{189}$
10. $\boxed{\text{False}}$
11. $\boxed{a_1 = 1}$
12. $\boxed{682}$
13. $\boxed{726}$
14. $\boxed{\frac{40}{27}}$
15. $\boxed{-364}$
16. $\boxed{a_1 = 3}$
17. $\boxed{\$765}$
18. $\boxed{315}$
19. $\boxed{\text{True}}$
20. $\boxed{\frac{63}{8}}$
21. $\boxed{\$2{,}046}$
22. $\boxed{121 \text{ people}}$
23. $\boxed{\approx 66.6 \text{ mg}}$
24. $\boxed{\$199.22 \text{ (approx.)}}$

Additional Practice Answers

25. $\boxed{24}$
26. $\boxed{48}$
27. $\boxed{162}$
28. $\boxed{160}$
29. $\boxed{210}$
30. $\boxed{93}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Geometric with $a_1 = 1, r = 2, n = 5$. $S_5 = \frac{1(1-2^5)}{1-2} = \frac{1-32}{-1} = 31$. (Or just add directly – five small terms.)

2. Use $S_n = \frac{a_1(1-r^n)}{1-r}$ with $a_1 = 5, r = 2, n = 6$: $S_6 = \frac{5(1-2^6)}{1-2} = \frac{5(1-64)}{-1} = \frac{5(-63)}{-1} = 5 \cdot 63 = 315$. Note the exponent is $n = 6$ here, not $n-1$ – the sum formula uses r^n.

3. Terms: $8, 4, 2, 1$. Sum: $8 + 4 + 2 + 1 = 15$. Formula confirms: $S_4 = \frac{8(1-(1/2)^4)}{1-1/2} = \frac{8(15/16)}{1/2} = \frac{15/2}{1/2} = 15$.

4. Five terms: $1 + 3 + 9 + 27 + 81 = 121$. (Or geometric formula with $a_1 = 1$, $r = 3, n = 5$: $S_5 = \frac{1-3^5}{1-3} = \frac{-242}{-2} = 121$.)

5. Trial-and-error or factor: try $r = 3$: $S_4 = \frac{2(1-81)}{1-3} = \frac{-160}{-2} = 80$ ✓. (Algebraically, $80(1-r) = 2(1-r^4) = 2(1-r)(1+r)(1+r^2)$, so for $r \neq 1$ this reduces to $40 = (1+r)(1+r^2)$; $r = 3$ gives $4 \cdot 10 = 40$.)

6. Keep the rule visible: $S_7 = \frac{4(1-3^7)}{1-3} = \frac{4(1-2187)}{-2} = \frac{4(-2186)}{-2} = 4(1093) = 4372$. That gives a quick check on the answer.

7. Terms: $6, -12, 24, -48, 96$. Sum: $6 - 12 + 24 - 48 + 96 = 66$. Formula: $S_5 = \frac{6(1-(-2)^5)}{1-(-2)} = \frac{6(1+32)}{3} = \frac{6(33)}{3} = 66$. (Negative ratio, odd power: $(-2)^5 = -32$. Watch signs.)

8. From the chart, read off the terms: $96 + 48 + 24 + 12 + 6 = 186$. Formula confirms: $S_5 = \frac{96(1-(1/2)^5)}{1-1/2} = \frac{96(31/32)}{1/2} = 2 \cdot 96 \cdot \frac{31}{32} = 6 \cdot 31 = 186$.

9. Pull out the 3: $3\sum_{k=1}^{6} 2^{k-1} = 3(1+2+4+8+16+32) = 3(63) = 189$. (Or formula with $a_1 = 3, r = 2, n = 6$: $3 \cdot \frac{2^6-1}{2-1} = 3(63) = 189$.)

10. At $r = 1$, both numerator and denominator become 0 – the formula collapses to $0/0$. Use $S_n = n \cdot a_1$ instead (every term equals a_1, so just add n copies).

11. One steady path is: $S_4 = \frac{a_1(1-3^4)}{1-3} = \frac{a_1(-80)}{-2} = 40a_1$. Set $40a_1 = 40 \Rightarrow a_1 = 1$. Sanity: $1 + 3 + 9 + 27 = 40$ ✓. That gives a quick check on the answer.

12. Start with the key idea: $S_5 = \frac{2(1-4^5)}{1-4} = \frac{2(1-1024)}{-3} = \frac{-2046}{-3} = 682$. Direct add: $2 + 8 + 32 + 128 + 512 = 682$ ✓. That gives a quick check on the answer.

13. Pull out: $2\sum_{k=1}^{5} 3^k = 2(3+9+27+81+243) = 2(363) = 726$. (The sum starts at $k = 1$, so the first term is 3, not 1.)

14. Keep the rule visible: $S_4 = \frac{1(1-(1/3)^4)}{1-1/3} = \frac{1-1/81}{2/3} = \frac{80/81}{2/3} = \frac{80}{81} \cdot \frac{3}{2} = \frac{240}{162} = \frac{40}{27}$. That gives a quick check on the answer.

15. One steady path is: $S_6 = \frac{2(1-(-3)^6)}{1-(-3)} = \frac{2(1-729)}{4} = \frac{-1456}{4} = -364$. (Even power: $(-3)^6 = 729$, positive. The negative result comes from $1 - 729 = -728$.) That gives a quick check on the answer.

16. Start with the key idea: $S_5 = \frac{a_1(1-32)}{1-2} = \frac{-31a_1}{-1} = 31a_1$. $31a_1 = 93 \Rightarrow a_1 = 3$. That gives a quick check on the answer.

17. Geometric series with $a_1 = 3, r = 2, n = 8$. $S_8 = \frac{3(1-2^8)}{1-2} = \frac{3(-255)}{-1} = 3(255) = 765$. So \$765 total. (Try the direct add as a check: $3 + 6 + 12 + \cdots + 384 = 765$ ✓.)

18. Six terms ($k = 0, 1, 2, 3, 4, 5$): $5 + 10 + 20 + 40 + 80 + 160 = 315$. (Formula: $a_1 = 5, r = 2, n = 6$. $S_6 = 5 \cdot \frac{2^6-1}{2-1} = 5(63) = 315$.)

19. Both sums have n terms with exponents running from 0 to $n-1$. They're the same series, just indexed differently. (Always count terms by hand if you're unsure: number of terms = upper minus lower plus 1.)

20. Start with the key idea: $S_6 = \frac{4(1-(1/2)^6)}{1-1/2} = \frac{4(1-1/64)}{1/2} = 8\left(\frac{63}{64}\right) = \frac{63}{8}$. That gives a quick check on the answer.

21. Geometric series with $a_1 = 2, r = 2, n = 10$. $S_{10} = \frac{2(1-2^{10})}{1-2} = \frac{2(-1023)}{-1} = 2(1023) = 2046$, so \$2,046 total. (Doubling gets eye-watering fast – birthday 10 alone is $\$2^{10} = 1024$.)

Scan Me

22. Round counts form a geometric sequence: $1, 3, 9, 27, 81$ (the bar chart matches). Total is the finite series with $a_1 = 1$, $r = 3$, $n = 5$: $S_5 = \frac{1-3^5}{1-3} = \frac{-242}{-2} = 121$. So 121 people. (Reality check: the message reaches more people in round 5 alone – 81 – than in all earlier rounds combined; that's how exponential growth works.)

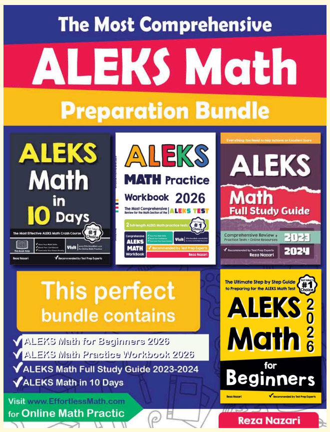
23. Geometric series with $a_1 = 200(0.25) = 50$, $r = 0.25$, $n = 5$. $S_5 = \frac{50(1-(0.25)^5)}{1-0.25} = \frac{50(1-0.000977)}{0.75} = \frac{50(0.999023)}{0.75} \approx 66.6$ mg. (Reality check: positive, less than the original 50 mg first-residual times $5 = 250$ – because each later residual is much smaller.)

24. Geometric series with $a_1 = 100$, $r = 0.5$, $n = 8$. $S_8 = \frac{100(1-(0.5)^8)}{1-0.5} = \frac{100(1-1/256)}{0.5} = 200 \cdot \frac{255}{256} = \frac{255 \cdot 200}{256} = \frac{51000}{256} \approx 199.22$, so about \$$199.22$. (Geometric series with $|r| < 1$ approach a finite limit – here, $\$200$.)

Scan Me

Keep Building ALEKS Skills

Recommended ALEKS books and bundle



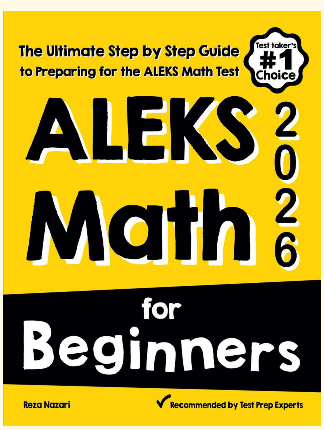
The Most Comprehensive ALEKS Math Preparation Bundle

Prep books, workbooks, full-length tests, detailed explanations, and a formula bonus file in one complete ALEKS study path.

Scan Me

Download Instantly

STUDENT FAVORITE · ALEKS Math for Beginners



ALEKS Math for Beginners 2026

Step-by-step lessons, basic study guides, topic practice, and two full-length ALEKS Math tests for students who need a calm path from review to readiness.

Great companion to these worksheets for self-study, tutoring, and classroom review.

PDF Edition

Scan Me

Amazon

Scan Me

For more ALEKS prep, visit EffortlessMath.com/ALEKS