# Arc Length and Sector Area

| Name: ______________ | Date: ______________ | Score: ______ / 36 |
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## Quick Review

A pie slice cut from a circle is called a **sector**. The curved outer edge of that slice is an **arc**. Both have clean formulas – as long as you measure the central angle in *radians*.

**Arc length.** For a circle of radius $r$ and central angle $\theta$ (radians), the arc length is $s = r\theta$.

**Sector area.** Same setup gives sector area $A = \frac{1}{2}r^2\theta$.

**Why radians?** A radian is defined so that one radian of sweep on a unit circle produces exactly one unit of arc – the formula $s = r\theta$ falls out automatically with no conversion factor. If $\theta$ is given in degrees, *convert it first*: $\theta_{\text{rad}} = \theta_{\text{deg}} \cdot \frac{\pi}{180}$.

**Sanity checks.** A full sweep ($\theta = 2\pi$) gives $s = 2\pi r$ (the circumference) and $A = \frac{1}{2}r^2 \cdot 2\pi = \pi r^2$ (the full area). Half a circle ($\theta = \pi$) gives half each.

**Reverse problems.** Given $s$ and $r$, solve for $\theta = \frac{s}{r}$ (in radians). Given $A$ and $r$, solve for $\theta = \frac{2A}{r^2}$.

**Fraction-of-circle alternative.** A degree-friendly form: the sector is $\frac{\theta_{\text{deg}}}{360}$ of the full circle, so $s = \frac{\theta_{\text{deg}}}{360} \cdot 2\pi r$ and $A = \frac{\theta_{\text{deg}}}{360} \cdot \pi r^2$. Use whichever form is faster for your given units.

**Common slips.** Using degree measure directly in $s = r\theta$ (you'll be off by a factor of $\frac{180}{\pi} \approx 57.3$). Forgetting the $\frac{1}{2}$ in the sector-area formula. Mixing up radius and diameter – $r$ is the radius (half the diameter).

## PRACTICE

*Use $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ with $\theta$ in radians. Convert degree angles before plugging in.*

1. Arc length formula (radians). ________
2. Sector area formula (radians). ________
3. For the sector in the table below, find the arc length $s$. ________

| $r$ | $\theta$ |
| --- | --- |
| $4$ | $\frac{\pi}{3}$ |

4. For the sector in the table below, find the sector area $A$. ________

| $r$ | $\theta$ |
| --- | --- |
| $6$ | $\frac{\pi}{2}$ |

5. Circle of radius $10$ cm, central angle $90°$. Arc length. ________
6. Arc of length $15$ on a circle of radius $5$. Central angle in radians. ________
7. Pizza of radius $9$ inches, slice angle $60°$. Slice area. ________
8. Sector of radius $8$ cm, central angle $45°$. Area. ________
9. Arc of length $6\pi$ ft, central angle $\frac{3\pi}{4}$. Radius. ________

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10. For the sector in the table below, find the arc length $s$. ______

| $r$ | $\theta$ |
|---|---|
| 12 | $\frac{5\pi}{6}$ |

11. Sector of radius $5$, central angle $\frac{\pi}{4}$. Area. ______
12. Arc of length $20$ on a circle of radius $4$. Central angle in radians. ______
13. Sector of area $18\pi$ on a circle of radius $6$. Central angle in radians. ______
14. Circle of radius $7$, central angle $\frac{2\pi}{7}$. Arc length. ______
15. Sector of radius $10$, central angle $\frac{3\pi}{5}$. Area. ______
16. Circle of radius $3$ m, central angle $120°$. Arc length (exact). ______
17. Sector of radius $4$ ft, central angle $\frac{\pi}{6}$. Area. ______
18. True or False: in $s = r\theta$, $\theta$ can be in degrees. ______
19. Circle of radius $15$ in, central angle $\frac{4\pi}{5}$. Arc length. ______
20. A sector has $A = 20\pi$ and $r = 10$. Find the central angle $\theta$ in radians. ______

## ♦ Word Problems

21. A track is circular with radius $50$ meters. A runner covers a $\frac{\pi}{4}$-radian arc of the track. How far did she run? ______
22. A circular pizza of radius $7$ inches is cut into $8$ equal slices. Find the area of one slice exactly. ______
23. A bike wheel of radius $13$ inches makes $5$ full rotations. How many inches has the contact point on the tire traveled? Give the answer in terms of $\pi$ and as a decimal to the nearest inch. ______
24. A circular sector has area $25\pi$ ft$^2$ and central angle $\frac{\pi}{2}$. Find the radius and the arc length. ______

## Additional Practice

25. Find $\sin\theta$ if opposite $= 5$, hypotenuse $= 13$. ______
26. Find $\cos\theta$ if adjacent $= 12$, hypotenuse $= 13$. ______
27. Find $\tan\theta$ if opposite $= 7$, adjacent $= 4$. ______
28. Find $\sin 30°$. ______
29. Find $\cos 60°$. ______
30. Find $\tan 45°$. ______
31. Convert $180°$ to radians. ______
32. Convert $\frac{\pi}{3}$ radians to degrees. ______
33. Find a coterminal angle with $70°$. ______

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**34.** Reference angle for $150^\circ$. ________

**35.** Use $\sin^2\theta + \cos^2\theta$. ________

**36.** If $\sin\theta = \frac{3}{5}$, $\theta$ in QI, find $\cos\theta$. ________

# Answer Keys

1. $\boxed{s = r\theta}$
2. $\boxed{A = \frac{1}{2}r^2\theta}$
3. $\boxed{\frac{4\pi}{3}}$
4. $\boxed{9\pi}$
5. $\boxed{5\pi \text{ cm}}$
6. $\boxed{3}$
7. $\boxed{\frac{27\pi}{2}}$
8. $\boxed{8\pi}$
9. $\boxed{8 \text{ ft}}$
10. $\boxed{10\pi}$
11. $\boxed{\frac{25\pi}{8}}$
12. $\boxed{5}$
13. $\boxed{\pi}$
14. $\boxed{2\pi}$
15. $\boxed{30\pi}$
16. $\boxed{2\pi \text{ m}}$
17. $\boxed{\frac{4\pi}{3}}$
18. $\boxed{\text{False}}$
19. $\boxed{12\pi \text{ in}}$
20. $\boxed{\frac{2\pi}{5}}$
21. $\boxed{\frac{25\pi}{2} \text{ m} \approx 39.3 \text{ m}}$
22. $\boxed{\frac{49\pi}{8} \text{ in}^2}$
23. $\boxed{130\pi \text{ in} \approx 408 \text{ in}}$
24. $\boxed{r = 10 \text{ ft},\ s = 5\pi \text{ ft}}$

### Additional Practice Answers

25. $\boxed{\frac{5}{13}}$
26. $\boxed{\frac{12}{13}}$
27. $\boxed{\frac{7}{4}}$
28. $\boxed{\frac{1}{2}}$
29. $\boxed{\frac{1}{2}}$
30. $\boxed{1}$
31. $\boxed{\pi}$
32. $\boxed{60°}$
33. $\boxed{430°}$
34. $\boxed{30°}$
35. $\boxed{1}$
36. $\boxed{\frac{4}{5}}$

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

**1.** A careful way to see it: With $\theta$ in radians, just multiply radius by angle. This is the part to check before moving on, because it keeps the answer tied to the original question.

**2.** Keep the rule visible: Half of radius-squared times angle, with $\theta$ in radians. This is the part to check before moving on, because it keeps the answer tied to the original question.

**3.** One steady path is: $s = r\theta = 4 \cdot \frac{\pi}{3} = \frac{4\pi}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**4.** Start with the key idea: $A = \frac{1}{2}(36)\left(\frac{\pi}{2}\right) = 9\pi$. (That's a quarter of $36\pi$, the full circle area ✓.) That gives a quick check on the answer.

**5.** A careful way to see it: Convert: $90° = \frac{\pi}{2}$. Then $s = 10 \cdot \frac{\pi}{2} = 5\pi$ cm. This is the part to check before moving on, because it keeps the answer tied to the original question.

**6.** Keep the rule visible: $\theta = \frac{s}{r} = \frac{15}{5} = 3$ rad. This is the part to check before moving on, because it keeps the answer tied to the original question.

**7.** One steady path is: Convert: $60° = \frac{\pi}{3}$. $A = \frac{1}{2}(81)\left(\frac{\pi}{3}\right) = \frac{27\pi}{2}$ in$^2$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**8.** Start with the key idea: Convert: $45° = \frac{\pi}{4}$. $A = \frac{1}{2}(64)\left(\frac{\pi}{4}\right) = 8\pi$ cm$^2$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**9.** A careful way to see it: $r = \frac{s}{\theta} = \frac{6\pi}{3\pi/4} = 6\pi \cdot \frac{4}{3\pi} = 8$ ft. This is the part to check before moving on, because it keeps the answer tied to the original question.

**10.** Keep the rule visible: $s = 12 \cdot \frac{5\pi}{6} = 10\pi$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**11.** One steady path is: $A = \frac{1}{2}(25)\left(\frac{\pi}{4}\right) = \frac{25\pi}{8}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**12.** Start with the key idea: $\theta = \frac{20}{4} = 5$ rad. (More than $2\pi \approx 6.28$, so just under a full sweep.) That gives a quick check on the answer.

**13.** A careful way to see it: $\theta = \frac{2A}{r^2} = \frac{36\pi}{36} = \pi$. (Half the circle.) This is the part to check before moving on, because it keeps the answer tied to the original question.

**14.** Keep the rule visible: $s = 7 \cdot \frac{2\pi}{7} = 2\pi$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**15.** One steady path is: $A = \frac{1}{2}(100)\left(\frac{3\pi}{5}\right) = 30\pi$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**16.** Start with the key idea: $120° = \frac{2\pi}{3}$. $s = 3 \cdot \frac{2\pi}{3} = 2\pi$ m. This is the part to check before moving on, because it keeps the answer tied to the original question.

**17.** A careful way to see it: $A = \frac{1}{2}(16)\left(\frac{\pi}{6}\right) = \frac{16\pi}{12} = \frac{4\pi}{3}$ ft$^2$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**18.** Keep the rule visible: Only radians work directly. For degrees, convert first (multiply by $\frac{\pi}{180}$). That gives a quick check on the answer.

**19.** One steady path is: $s = 15 \cdot \frac{4\pi}{5} = 12\pi$ in. This is the part to check before moving on, because it keeps the answer tied to the original question.

**20.** Start with the key idea: $\theta = \frac{2A}{r^2} = \frac{40\pi}{100} = \frac{2\pi}{5}$. This is the part to check

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before moving on, because it keeps the answer tied to the original question.

**21.** A careful way to see it: $s = r\theta = 50 \cdot \frac{\pi}{4} = \frac{50\pi}{4} = \frac{25\pi}{2} \approx 39.27$ meters. (Reality check: $\frac{\pi}{4}$ rad is $\frac{1}{8}$ of the full circle, so $\frac{1}{8}$ of $100\pi \approx 314$ m gives $\approx 39.3$ m ✓.) That gives a quick check on the answer.

**22.** Each slice has central angle $\frac{2\pi}{8} = \frac{\pi}{4}$. Area: $A = \frac{1}{2}(49)\left(\frac{\pi}{4}\right) = \frac{49\pi}{8} \approx 19.24$ in$^2$. (Sanity: $8$ slices total $\frac{49\pi}{8} \cdot 8 = 49\pi$, the full circle area ✓.)

**23.** Each full rotation sweeps $\theta = 2\pi$ rad, so the arc covered per rotation is $s = r\theta = 13(2\pi) = 26\pi$ in. Five rotations: $5(26\pi) = 130\pi \approx 408.4 \approx 408$ in. (That's about $34$ feet – about half a basketball court.)

**24.** From $A = \frac{1}{2}r^2\theta$: $25\pi = \frac{1}{2}r^2\left(\frac{\pi}{2}\right) = \frac{\pi r^2}{4}$, so $r^2 = 100$ and $r = 10$ ft. Arc length: $s = r\theta = 10 \cdot \frac{\pi}{2} = 5\pi$ ft. (Sanity: a quarter-circle of radius $10$ has area $\frac{1}{4}\pi(100) = 25\pi$ ✓and arc $\frac{1}{4}(2\pi \cdot 10) = 5\pi$ ✓.)

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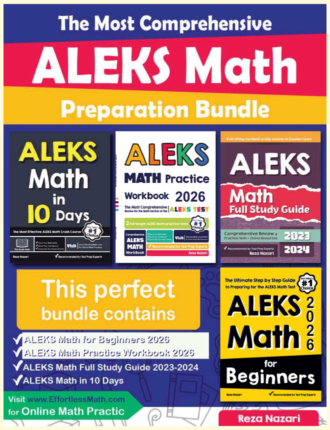


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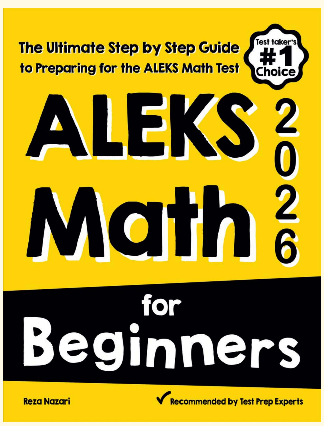


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