

Coterminal Angles and Reference Angles

Name: _____ Date: _____ Score: _____ / 29

Q Quick Review

This section ties the two ideas together: **coterminal angles** share a terminal side, and **reference angles** measure how far that terminal side sits from the nearest x -axis.

Coterminal recipe. To find a positive angle in $[0, 360^\circ)$ coterminal with a given angle, add or subtract 360° as many times as needed. Same for radians, but with 2π .

Reference angle recipe (after reducing to $[0, 360^\circ)$).

Q1: reference = θ .

Q2: reference = $180^\circ - \theta$.

Q3: reference = $\theta - 180^\circ$.

Q4: reference = $360^\circ - \theta$.

On an axis (0, 90, 180, 270): reference is the distance to the nearest x -axis (0 or 90°).

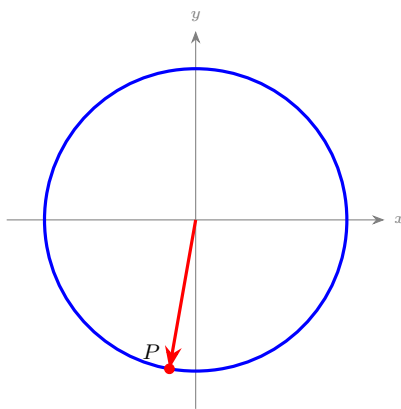
Key fact. Coterminal angles share their reference angle – and their values for all six trig functions. So if you want $\sin(-405^\circ)$, the fastest route is: find a coterminal in $[0, 360^\circ)$, then read off the answer there.

Common slips. Stopping after the coterminal reduction without computing the reference angle (the problem usually wants both). Picking the wrong quadrant rule – always verify which quadrant the reduced angle is in. Reporting a negative reference angle (they're always positive).

PRACTICE

For each angle, give (i) a positive coterminal angle in $[0, 360^\circ)$ (or $[0, 2\pi)$) and (ii) the reference angle.

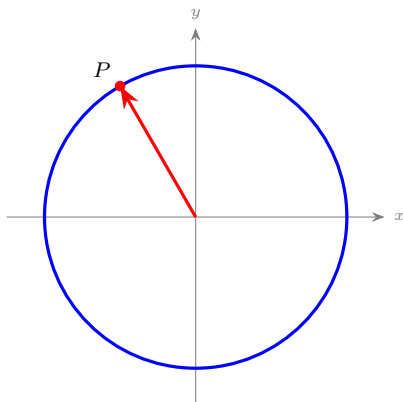
1. The terminal side below is for a -100° angle. Give its positive coterminal angle in $[0, 360^\circ)$. _____



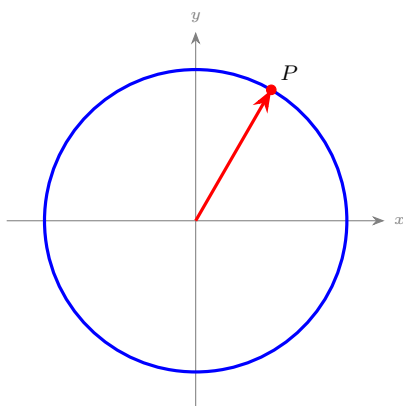
2. 200° : a negative coterminal angle. _____
3. -225° : reference angle. _____



4. A 480° angle reduces to the terminal side drawn below. Find its reference angle. _____



5. The terminal side below is for a -300° angle. Give its positive coterminal angle in $[0, 360^\circ)$ and its reference angle. _____



6. -405° : reference angle. _____

7. -50° : reference angle. _____

8. 810° : positive coterminal in $[0, 360^\circ)$. _____

9. $\frac{7\pi}{4}$: reference angle. _____

10. 725° : reference angle. _____

11. -690° : positive coterminal and reference angle. _____

12. $\frac{11\pi}{4}$: positive coterminal in $[0, 2\pi)$. _____

13. $\frac{3\pi}{4}$: reference angle. _____

14. 420° : reference angle. _____

15. -150° : positive coterminal and reference angle. _____

16. 1000° : positive coterminal in $[0, 360^\circ)$. _____

17. 1000° : reference angle. _____

18. $-\frac{\pi}{3}$: positive coterminal in $[0, 2\pi)$. _____

19. $-\frac{\pi}{3}$: reference angle. _____

20. True or False: coterminal angles share the same reference angle. _____



◆ Word Problems

21. A satellite makes 2.25 counterclockwise rotations from a reference position. In degrees, what is its terminal angle in $[0, 360^\circ)$ and the reference angle of that terminal side? _____
22. An angle measures -585° . Find a positive coterminal angle in $[0, 360^\circ)$ and its reference angle. _____
23. A clock's second hand sweeps from the 12 position through $1,050^\circ$ counterclockwise (a strange clock). What's the equivalent positive angle in $[0, 360^\circ)$, and where does the hand point? _____
24. A wheel sensor logs angle $-\frac{17\pi}{6}$ in radians. Find a positive coterminal angle in $[0, 2\pi)$ and the reference angle. _____

Additional Practice

25. Find $\sin \theta$ if opposite = 5, hypotenuse = 13. _____
26. Find $\cos \theta$ if adjacent = 12, hypotenuse = 13. _____
27. Find $\tan \theta$ if opposite = 7, adjacent = 4. _____
28. Find $\sin 30^\circ$. _____
29. Find $\cos 60^\circ$. _____



Answer Keys

1. 260°
2. -160°
3. 45°
4. 60°
5. 60° ; ref 60°
6. 45°
7. 50°
8. 90°
9. $\frac{\pi}{4}$
10. 5°
11. 30° ; ref 30°
12. $\frac{3\pi}{4}$

Additional Practice Answers

25. $\frac{5}{13}$
26. $\frac{12}{13}$
27. $\frac{7}{4}$

13. $\frac{\pi}{4}$
14. 60°
15. 210° ; ref 30°
16. 280°
17. 80°
18. $\frac{5\pi}{3}$
19. $\frac{\pi}{3}$
20. True
21. 90° ; reference 90°
22. 135° ; reference 45°
23. 330° ; between the 11 and 12
24. $\frac{7\pi}{6}$; reference $\frac{\pi}{6}$

28. $\frac{1}{2}$
29. $\frac{1}{2}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: $-100^\circ + 360^\circ = 260^\circ$ – the same terminal side shown. This is the part to check before moving on, because it keeps the answer tied to the original question.
2. Keep the rule visible: $200^\circ - 360^\circ = -160^\circ$. (Any answer that differs by a multiple of 360° works.) That gives a quick check on the answer.
3. One steady path is: Positive coterminal: $-225^\circ + 360^\circ = 135^\circ$ (Q2). Reference: $180^\circ - 135^\circ = 45^\circ$. That gives a quick check on the answer.
4. Start with the key idea: Reduce: $480^\circ - 360^\circ = 120^\circ$ (Q2). Reference: $180^\circ - 120^\circ = 60^\circ$. This is the part to check before moving on, because it keeps the answer tied to the original question.
5. A careful way to see it: $-300^\circ + 360^\circ = 60^\circ$, which is in Q1, so reference = angle = 60° . That gives a quick check on the answer.
6. Keep the rule visible: $-405^\circ + 720^\circ = 315^\circ$ (Q4). Reference = $360^\circ - 315^\circ = 45^\circ$. This is the part to check before moving on, because it keeps the answer tied to the original question.
7. One steady path is: $-50^\circ + 360^\circ = 310^\circ$ (Q4). Reference = $360^\circ - 310^\circ = 50^\circ$. (Quick rule: the reference angle of $-\theta$ equals the reference angle of $+\theta$.) That gives a quick check on the answer.
8. Start with the key idea: $810^\circ - 720^\circ = 90^\circ$. This is the part to check before moving on, because it keeps the answer tied to the original question.
9. A careful way to see it: Q4. Reference = $2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
10. Keep the rule visible: Reduce: $725^\circ - 720^\circ = 5^\circ$ (Q1). Reference is the angle itself: 5° . That gives a quick check on the answer.
11. One steady path is: $-690^\circ + 720^\circ = 30^\circ$ (Q1). Reference is the angle itself, 30° . That gives a quick check on the answer.
12. Start with the key idea: $\frac{11\pi}{4} - 2\pi = \frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
13. A careful way to see it: Q2. Reference = $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
14. Keep the rule visible: Reduce: $420^\circ - 360^\circ = 60^\circ$ (Q1). Reference = 60° . This is the part to check before moving on, because it keeps the answer tied to the

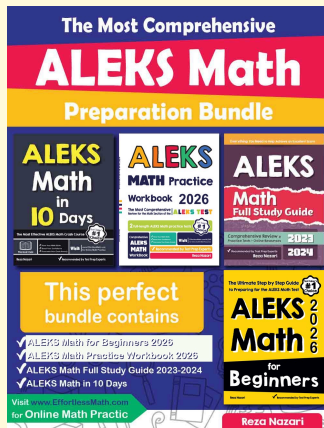
original question.

15. One steady path is: $-150^\circ + 360^\circ = 210^\circ$ (Q3). Reference = $210^\circ - 180^\circ = 30^\circ$. This is the part to check before moving on, because it keeps the answer tied to the original question.
16. Start with the key idea: $1000^\circ - 2(360^\circ) = 1000^\circ - 720^\circ = 280^\circ$. This is the part to check before moving on, because it keeps the answer tied to the original question.
17. A careful way to see it: 280° is in Q4. Reference = $360^\circ - 280^\circ = 80^\circ$. This is the part to check before moving on, because it keeps the answer tied to the original question.
18. Keep the rule visible: $-\frac{\pi}{3} + 2\pi = -\frac{\pi}{3} + \frac{6\pi}{3} = \frac{5\pi}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
19. One steady path is: $\frac{5\pi}{3}$ is in Q4. Reference = $2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
20. They share a terminal side, so they sit in the same quadrant (or on the same axis) and have the same distance to the nearest x -axis – same reference angle.
21. A careful way to see it: 2.25 rotations = $2.25 \cdot 360^\circ = 810^\circ$. Reduce: $810^\circ - 720^\circ = 90^\circ$. The terminal side lies on the positive y -axis, so the reference angle is 90° (the distance to the nearest x -axis). That gives a quick check on the answer.
22. Keep the rule visible: $-585^\circ + 2(360^\circ) = -585^\circ + 720^\circ = 135^\circ$. That sits in Q2, so the reference angle is $180^\circ - 135^\circ = 45^\circ$. That gives a quick check on the answer.
23. One steady path is: $1050^\circ - 2(360^\circ) = 1050^\circ - 720^\circ = 330^\circ$. That's 30° short of a full counterclockwise sweep – so the hand sits one hour mark before the 12, between the 11 and 12. (Each hour mark is 30° .) That gives a quick check on the answer.
24. Add full rotations: $-\frac{17\pi}{6} + 2(2\pi) = -\frac{17\pi}{6} + \frac{24\pi}{6} = \frac{7\pi}{6}$. That's in Q3 (between $\pi = \frac{6\pi}{6}$ and $\frac{3\pi}{2} = \frac{9\pi}{6}$). Reference = $\frac{7\pi}{6} - \pi = \frac{\pi}{6}$.



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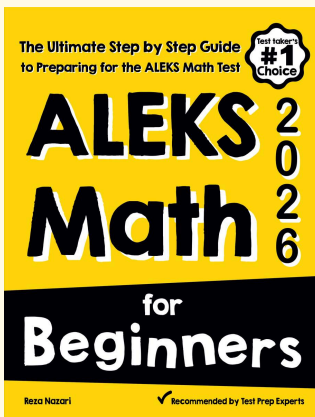
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