Natural Logarithms

Name: ________________ Date: ____________ Score: ______ / 34

Quick Review

The **natural logarithm**, written $\ln(x)$, is just $\log_e(x)$ — a logarithm with base $e \approx 2.71828$. Everything you know about logarithms still applies; the base just happens to be Euler's number, which shows up in continuous-growth models because of its calculus-friendly properties.

The three key values. $\ln(1) = 0$ (since $e^0 = 1$), $\ln(e) = 1$ (since $e^1 = e$), and $\ln(e^n) = n$ for every real n. The natural log peels exponents off powers of e the same way base-2 log peels them off powers of 2.

Inverse with e^x. The natural log undoes the natural exponential: $e^{\ln x} = x$ for $x > 0$, and $\ln(e^x) = x$ for every real x. This is the magic step that solves continuous-growth equations: when the unknown is in the exponent, take $\ln$ of both sides to bring it down.

Why e? Continuous compounding gives $A = Pe^{rt}$, where the rate r is the *instantaneous* growth fraction. Natural logs are the inverse of that machine. Anytime you see e^{kt} in a model, $\ln$ is the tool that solves for t or k.

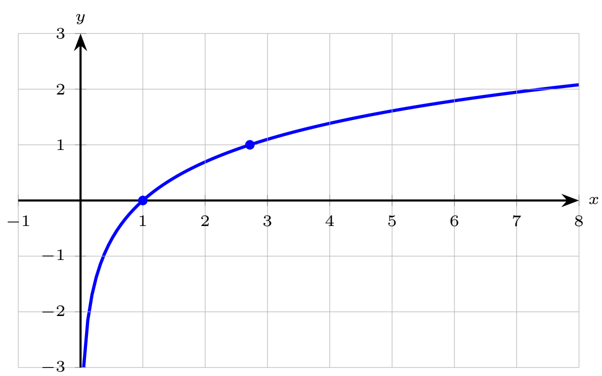
Domain reminder. $\ln(x)$ requires $x > 0$. So $\ln(x-3)$ requires $x > 3$; $\ln(x^2+1)$ is fine for every real x because $x^2 + 1 \geq 1 > 0$.

Common slips. Writing $\ln(M+N)$ as $\ln M + \ln N$ (wrong — same trap as the common log). Using $\log$ (base 10) instead of $\ln$ on the calculator when the problem involves e. Forgetting to back-substitute and check that the log argument stayed positive.

PRACTICE

Evaluate natural logs; isolate exponentials with $\ln$; check the domain after solving.

1. Definition: $\ln(x) =$ ______. ________
2. Evaluate $\ln(1)$. The graph of $y = \ln(x)$ is shown to help you find where the curve crosses the x-axis. ________



3. A short table of values for $g(x) = \ln(x)$ is given below. Use it to estimate $\ln(3)$ to two decimals. ________

x	1	2	3	4	5
$\ln(x)$	0	0.69	1.10	1.39	1.61

4. Evaluate $\ln(e^5)$. ________
5. Simplify $e^{\ln(7)}$. ________
6. Solve $\ln(x) = 4$ for x. ________
7. Simplify $\ln(e^3) - \ln(e^2) + 2\ln(e)$. ________
8. Solve $3e^x - 5 = 16$ to four decimals. ________
9. Mark TRUE or FALSE: $\ln(0) = 0$. ________
10. If $\ln(2x-1) = \ln(11)$, find x. ________
11. Solve $A(t) = 12e^{0.18t} = 30$ for t. Round to two decimals. ________

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12. Solve $\ln(x) + \ln(5) = \ln(15)$.

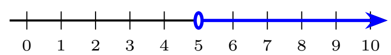
13. Solve $\ln(x - 3) = 2$.

14. Solve $2\ln(x) = \ln(36)$ with $x > 0$.

15. Solve $e^{2x} = 15$ for x to two decimals.

16. Solve $\ln(x) - \ln(2) = \ln(7)$.

17. Find the domain of $f(x) = \ln(x - 5)$. The number line below marks the boundary value of the argument; use it to write the domain as an inequality.



18. Find the domain of $f(x) = \ln(x^2 + 1)$.

19. Solve $e^{x+1} = e^{2x-3}$ for x.

20. Solve $5e^{0.04t} = 20$ for t to two decimals.

♦ Word Problems

21. A radioactive sample decays as $A(t) = 200e^{-0.04t}$ grams, where t is in years. How long until 50 grams remain? Round to two decimals.

22. The number of bacteria in a culture is $N(t) = 500e^{0.12t}$, with t in hours. Find when the population reaches 2000. Round to two decimals.

23. Solve $\ln(x + 3) - \ln(x - 1) = \ln 2$ for x. Check the domain of every log in the original equation.

24. A car's value depreciates continuously, modeled by $V(t) = 25000e^{-0.15t}$ dollars after t years. After how many years does the car's value drop to \$10,000? Round to two decimals.

Additional Practice

25. Evaluate $\log_2 32$.

26. Evaluate $\log_5 125$.

27. Rewrite $\log_3 81 = 4$ exponentially.

28. Solve $\log_4 x = 3$.

29. Domain of $y = \log(x - 7)$.

30. Expand $\log(ab)$.

31. Expand $\log(x^3)$.

32. Condense $\log x + \log 5$.

33. Condense $2\log x$.

34. Solve $\ln x = 0$.

Answer Keys

1. $\boxed{\log_e(x)}$
2. $\boxed{0}$
3. $\boxed{\approx 1.10}$
4. $\boxed{5}$
5. $\boxed{7}$
6. $\boxed{x = e^4}$
7. $\boxed{3}$
8. $\boxed{\ln(7) \approx 1.9459}$
9. FALSE
10. $\boxed{x = 6}$
11. $\boxed{t \approx 5.09}$
12. $\boxed{x = 3}$
13. $\boxed{x = e^2 + 3 \approx 10.39}$
14. $\boxed{x = 6}$
15. $\boxed{x \approx 1.35}$
16. $\boxed{x = 14}$
17. $\boxed{x > 5}$
18. all real numbers
19. $\boxed{x = 4}$
20. $\boxed{t \approx 34.66}$
21. $\boxed{t \approx 34.66 \text{ years}}$
22. $\boxed{t \approx 11.55 \text{ hours}}$
23. $\boxed{x = 5}$
24. $\boxed{t \approx 6.11 \text{ years}}$

Additional Practice Answers

25. $\boxed{5}$
26. $\boxed{3}$
27. $\boxed{3^4 = 81}$
28. $\boxed{x = 64}$
29. $\boxed{x > 7}$
30. $\boxed{\log a + \log b}$
31. $\boxed{3 \log x}$
32. $\boxed{\log(5x)}$
33. $\boxed{\log(x^2)}$
34. $\boxed{x = 1}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. The natural log is just a logarithm whose base is Euler's number $e \approx 2.71828$, so $\ln(x) = \log_e(x)$. Every log rule you know still applies; only the base is fixed.

2. Keep the rule visible: $e^0 = 1$, so $\ln(1) = 0$. (Same identity as every other log: $\log_b(1) = 0$.) The curve crosses the x-axis right at $x = 1$, which is exactly what the marked point $(1, 0)$ shows. That gives a quick check on the answer.

3. Read straight down the $x = 3$ column: $\ln(3) \approx 1.10$. (Check the shape — the values climb but more and more slowly, the signature of a logarithm.)

4. Ask: $e^? = e^5$. The exponents must match, so the answer is 5. Since $\ln = \log_e$, the natural log peels the exponent straight off a power of e: $\ln(e^n) = n$.

5. Here e^x and $\ln$ undo each other in the other order: $\ln(7)$ is the exponent you put on e to get 7, so raising e to it lands right back on 7. The identity is $e^{\ln x} = x$ for $x > 0$.

6. Keep the rule visible: Flip to exponential form: $x = e^4 \approx 54.6$. (Domain check: $e^4 > 0$, so valid.) That gives a quick check on the answer.

7. Use $\ln(e^k) = k$ on each term: $\ln(e^3) = 3$, $\ln(e^2) = 2$, and $\ln(e) = 1$. Substitute and combine: $3 - 2 + 2(1) = 3$.

8. Isolate the exponential first: add 5 to get $3e^x = 21$, then divide by 3 to get $e^x = 7$. Take $\ln$ of both sides so the inverse brings the exponent down: $x = \ln(7) \approx 1.9459$.

9. A careful way to see it: $\ln(0)$ is undefined — no real power of e equals 0. The natural log shoots down to $-\infty$ as the argument approaches 0 from above. That gives a quick check on the answer.

10. Same base, same argument: $2x - 1 = 11$, so $x = 6$. Domain check: $2(6) - 1 = 11 > 0$, valid.

11. Set $12e^{0.18t} = 30$ and divide by 12 to isolate the exponential: $e^{0.18t} = 2.5$. Take $\ln$ so the inverse frees the exponent: $0.18t = \ln(2.5)$. Divide by 0.18: $t = \dfrac{\ln 2.5}{0.18} \approx 5.09$.

12. Product rule (in reverse): $\ln(5x) = \ln(15)$, so $5x = 15$ and $x = 3$. Domain check: $x = 3 > 0$, valid.

13. Exponential form: $x - 3 = e^2$, so $x = e^2 + 3 \approx 7.389 + 3 = 10.39$. Domain: $x - 3 = e^2 > 0$, valid.

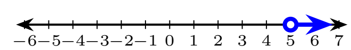
14. Power rule first: $\ln(x^2) = \ln(36)$, so $x^2 = 36$. Since the original $\ln(x)$ requires $x > 0$, only $x = 6$ survives; $x = -6$ is extraneous.

15. The exponential is already isolated, so take $\ln$ of both sides; the inverse pulls the exponent down: $2x = \ln(15)$. Divide by 2: $x = \dfrac{\ln 15}{2} \approx \dfrac{2.708}{2} \approx 1.35$.

16. Quotient property: $\ln(x/2) = \ln(7)$, so $x/2 = 7$ and $x = 14$. Domain check: $x = 14 > 0$, valid.

17. The argument $x - 5$ must be positive: $x - 5 > 0$, so $x > 5$. The open circle at 5 on the number line shows that $x = 5$ itself is excluded (it would make the argument zero).

Answer graph



18. Keep the rule visible: $x^2 + 1 \geq 1 > 0$ for every real x, so the log is defined everywhere. That gives a quick check on the answer.

19. Both sides are powers of the same base e, and e^x is one-to-one, so the exponents must be equal: $x + 1 = 2x - 3$. Solve: subtract x and add 3 to get $x = 4$.

20. Divide by 5 to isolate the exponential: $e^{0.04t} = 4$. Take $\ln$ so the inverse brings the exponent down: $0.04t = \ln(4)$. Divide by 0.04: $t = \dfrac{\ln 4}{0.04} \approx \dfrac{1.3863}{0.04} \approx 34.66$.

21. Set $200e^{-0.04t} = 50$. Divide by 200: $e^{-0.04t} = 0.25$. Take $\ln$ of both sides: $-0.04t = \ln(0.25) \approx -1.3863$. Solve: $t = \dfrac{1.3863}{0.04} \approx 34.66$ years. (Sanity check by plugging back: $200e^{-0.04(34.66)} = 200e^{-1.386} \approx 200(0.25) = 50$. Match.)

22. Set $500e^{0.12t} = 2000$. Divide: $e^{0.12t} = 4$. Take ln: $0.12t = \ln 4 \approx 1.3863$. Solve: $t \approx 11.55$ hours. (Quick check: from 500, two doublings get us to 2000, and one doubling takes $\ln 2/0.12 \approx 5.78$ hours, so two doublings ≈ 11.55 — agrees with the direct calculation.)

23. Quotient property: $\ln\left(\dfrac{x+3}{x-1}\right) = \ln 2$. Same-base logs equal means the arguments are equal: $\dfrac{x+3}{x-1} = 2$, so $x + 3 = 2(x - 1) = 2x - 2$, giving $x = 5$. Domain check: $x + 3 = 8 > 0$ and $x - 1 = 4 > 0$, both valid. (Always check both original arguments stay positive — one of them going non-positive would have killed the solution.)

24. Set $25000e^{-0.15t} = 10000$. Divide by 25000: $e^{-0.15t} = 0.4$. Take ln: $-0.15t = \ln(0.4) \approx -0.9163$. Solve: $t = \dfrac{0.9163}{0.15} \approx 6.11$ years. (Continuous decay is faster than the same nominal rate compounded annually — the $e^{-0.15} \approx 0.861$ per year is a 13.9% effective annual loss.)

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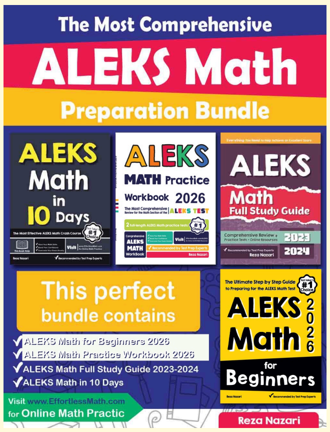

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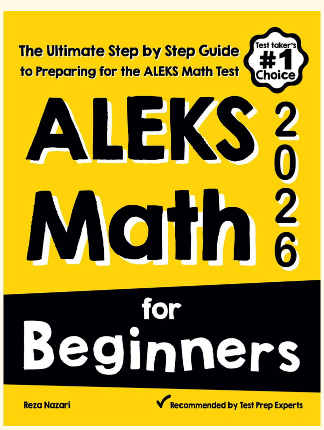

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