Properties of Logarithms

Name: ______________________ Date: _____________ Score: _____ / 35

Quick Review

All four log properties come from one place: logarithms convert *products into sums* (and the other arithmetic operations follow). For valid base b and positive arguments M and N:

Product: $\log_b(MN) = \log_b M + \log_b N$.

Quotient: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$.

Power: $\log_b(M^p) = p\log_b M$. The exponent hops down to the front as a multiplier.

Change of base: $\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$ for any valid base c. With $c = 10$ or $c = e$ you get the two calculator-friendly forms: $\log_b(x) = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$.

Expanding vs condensing. *Expand* means break a single log into a sum/difference. *Condense* means glue a sum/difference back into one log. Either way, you use the same three rules — just in reverse.

The most common slip — by far. $\log_b(M+N) \neq \log_b M + \log_b N$. Logs do NOT distribute over addition. The product property turns multiplication inside the log into addition outside, not the other way around. Same warning for $\log_b(MN) \neq \log_b(M) \cdot \log_b(N)$.

Roots are exponents. $\sqrt{y} = y^{1/2}$, so $\log_b(\sqrt{y}) = \frac{1}{2}\log_b(y)$. Cube roots give $\frac{1}{3}$, and so on.

PRACTICE

Expand, condense, or evaluate using log properties. Keep the domain restrictions in mind.

1. Product property: $\log_b(MN) =$ ______. ________
2. Power property: $\log_b(M^p) =$ ______. ________
3. Expand $\log_2(8x^3)$ (assume $x > 0$). ________
4. Expand $\log\left(\frac{a^2 b}{c}\right)$ for $a, b, c > 0$. ________
5. Condense $2\log x + \log y - 3\log z$ $(x, y, z > 0)$. ________
6. Write $\log_4(50)$ as a natural-log expression. ________
7. Expand $\log\left(\frac{x^3\sqrt{y}}{z^2}\right)$ for $x, y, z > 0$. ________
8. Mark TRUE or FALSE: $\log_b(M+N) = \log_b M + \log_b N$. ________
9. Mark TRUE or FALSE: $\log_b(MN) = \log_b(M) \cdot \log_b(N)$. ________
10. Simplify $\log_3(9x)$ for $x > 0$. ________
11. Condense $3\ln a - \frac{1}{2}\ln b + \ln c$ for $a, b, c > 0$. ________

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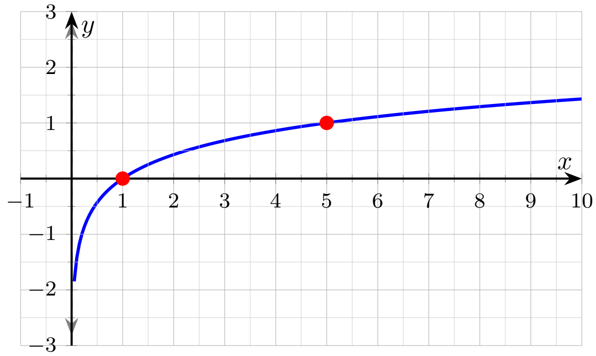
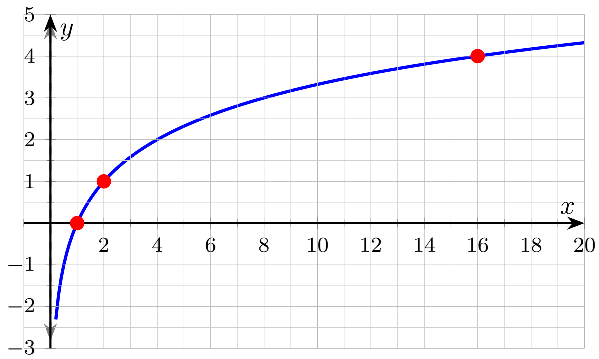
12. Simplify $\log_5(125) + \log_5(25)$ and locate the final value on the curve $y = \log_5(x)$. ______



13. Simplify $\log_2(48) - \log_2(3)$ and read the answer off the curve $y = \log_2(x)$. ______



14. Expand $\ln(x^2y^3)$ for $x, y > 0$. ______

15. Condense $\frac{1}{2}\log x + \log y$ for $x, y > 0$. ______

16. Use change of base to approximate $\log_3(20)$ to two decimals. ______

17. Simplify $\log_6(2) + \log_6(3)$. ______

18. Simplify $\frac{1}{2}\log_4(16)$. ______

19. Expand $\log_5\left(\frac{25}{x^2}\right)$ for $x > 0$. ______

20. Condense $\log_2(x) + \log_2(x + 3)$. ______

♦ Word Problems

21. Expand $\log\left(\frac{100x^3}{\sqrt{y}}\right)$ as far as the properties allow $(x, y > 0)$. Then evaluate the constant piece. ______

22. Suppose $\log_2(a) = 3$ and $\log_2(b) = 5$. Find $\log_2(a^2b)$ and $\log_2\left(\frac{a}{b}\right)$. ______

23. Without a calculator, simplify $\log_2(40) - \log_2(5)$. Then verify by computing each log separately. ______

24. The amount A in an account satisfies $\log_{10}(A) = \log_{10}(P) + t\log_{10}(1 + r)$ with $P = 1000$, $r = 0.05$, $t = 10$. Find A to the nearest dollar. Show how the log properties translate this back to the familiar compound-interest formula. ______

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Additional Practice

25. Evaluate $\log_2 32$. ________

26. Evaluate $\log_5 125$. ________

27. Rewrite $\log_3 81 = 4$ exponentially. ________

28. Solve $\log_4 x = 3$. ________

29. Domain of $y = \log(x - 7)$. ________

30. Expand $\log(ab)$. ________

31. Expand $\log(x^3)$. ________

32. Condense $\log x + \log 5$. ________

33. Condense $2 \log x$. ________

34. Solve $\ln x = 0$. ________

35. Solve $\log_2(x - 1) = 3$. ________

Answer Keys

1. $\boxed{\log_b M + \log_b N}$
2. $\boxed{p \log_b M}$
3. $\boxed{3 + 3\log_2 x}$
4. $\boxed{2\log a + \log b - \log c}$
5. $\boxed{\log\left(\frac{x^2 y}{z^3}\right)}$
6. $\boxed{\frac{\ln 50}{\ln 4}}$
7. $\boxed{3\log x + \frac{1}{2}\log y - 2\log z}$
8. FALSE
9. FALSE
10. $\boxed{2 + \log_3 x}$
11. $\boxed{\ln\left(\frac{a^3 c}{\sqrt{b}}\right)}$
12. $\boxed{5}$
13. $\boxed{4}$
14. $\boxed{2\ln x + 3\ln y}$
15. $\boxed{\log(y\sqrt{x})}$
16. $\boxed{\approx 2.73}$
17. $\boxed{1}$
18. $\boxed{1}$
19. $\boxed{2 - 2\log_5 x}$
20. $\boxed{\log_2(x(x+3))}$
21. $\boxed{2 + 3\log x - \frac{1}{2}\log y}$
22. $\boxed{\log_2(a^2 b) = 11;\ \log_2(a/b) = -2}$
23. $\boxed{3}$
24. $\boxed{A \approx \$1{,}629}$

Additional Practice Answers

25. $\boxed{5}$
26. $\boxed{3}$
27. $\boxed{3^4 = 81}$
28. $\boxed{x = 64}$
29. $\boxed{x > 7}$
30. $\boxed{\log a + \log b}$
31. $\boxed{3\log x}$
32. $\boxed{\log(5x)}$
33. $\boxed{\log(x^2)}$
34. $\boxed{x = 1}$
35. $\boxed{x = 9}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. The product property says a log of a product becomes a *sum* of logs: $\log_b(MN) = \log_b M + \log_b N$. It comes from adding exponents when you multiply powers. Watch the trap — it is a sum, never a product of the two logs.

2. The power property lets the exponent slide to the front as a multiplier: $\log_b(M^p) = p\log_b M$. It comes from M^p meaning p copies of M multiplied, which the product rule turns into p equal logs. Roots count too: $\log_b \sqrt{M} = \frac{1}{2}\log_b M$.

3. Product first: $\log_2(8) + \log_2(x^3)$. Then evaluate $\log_2(8) = 3$ and apply the power rule: $3 + 3\log_2 x$.

4. Quotient handles the division; product handles the multiplication; power handles the a^2. Combine: $\log(a^2) + \log b - \log c = 2\log a + \log b - \log c$.

5. Move each coefficient back into the exponent first: $\log(x^2) + \log y - \log(z^3)$. Sum becomes product, difference becomes quotient: $\log\left(\frac{x^2 y}{z^3}\right)$.

6. Change of base: $\log_b(x) = \frac{\ln x}{\ln b}$. (Be careful: the argument goes on top, the base on the bottom — a flipped fraction is a classic slip.)

7. Quotient first, then product, then power: $\log(x^3) + \log(\sqrt{y}) - \log(z^2) = 3\log x + \frac{1}{2}\log y - 2\log z$. The square root contributes the $\frac{1}{2}$.

8. Logs don't distribute over addition. The product property is about multiplication *inside* the log, not addition. A counterexample is $\log_{10}(10+90) = \log_{10}(100) = 2$, but $\log_{10}(10) + \log_{10}(90) \approx 1 + 1.95 = 2.95$. Different numbers.

9. The log of a product is a *sum* of logs, not a product of them: $\log_b(MN) = \log_b M + \log_b N$. The right side of the claim multiplies two logs, which is a different number entirely. Watch this trap on every problem.

10. Split the product with the product rule: $\log_3(9x) = \log_3(9) + \log_3(x)$. Then evaluate $\log_3(9) = 2$ because $9 = 3^2$. The result is $2 + \log_3 x$.

11. Pull coefficients back into the exponent: $\ln(a^3) - \ln(\sqrt{b}) + \ln c$. Sum becomes product, difference becomes quotient: $\ln\left(\frac{a^3 c}{\sqrt{b}}\right)$.

12. Product: $\log_5(125 \cdot 25) = \log_5(3125) = \log_5(5^5) = 5$. Or compute separately: $3 + 2 = 5$. The figure shows the powers-of-5 anchors $(1,0)$, $(5,1)$, $(25,2)$; the value 5 would land at $x = 5^5 = 3125$ off-window.

13. Quotient: $\log_2\left(\frac{48}{3}\right) = \log_2(16) = 4$. The red dot at $(16,4)$ on the curve confirms.

14. Apply the product rule first to break the product into a sum: $\ln(x^2) + \ln(y^3)$. Then the power rule drops each exponent to the front: $2\ln x + 3\ln y$.

15. One steady path is: $\frac{1}{2}\log x = \log(\sqrt{x})$, then sum becomes product: $\log(\sqrt{x} \cdot y) = \log(y\sqrt{x})$. That gives a quick check on the answer.

16. Start with the key idea: $\log_3(20) = \frac{\ln 20}{\ln 3} \approx \frac{2.996}{1.099} \approx 2.73$. Sanity: $3^2 = 9$, $3^3 = 27$, so the answer sits between 2 and 3, closer to 3. That gives a quick check on the answer.

17. Combine with the product rule: $\log_6(2) + \log_6(3) = \log_6(2 \cdot 3) = \log_6(6)$. Since any base's log of itself is 1, the answer is 1.

18. First evaluate the log: $\log_4(16) = 2$ because $4^2 = 16$. Then multiply by the coefficient: $\frac{1}{2} \cdot 2 = 1$.

19. One steady path is: Quotient: $\log_5(25) - \log_5(x^2) = 2 - 2\log_5 x$. (The 25 becomes 2 because $25 = 5^2$.) That gives a quick check on the answer.

20. Product rule (in reverse): a sum of logs with the same base is the log of a product. (Note the domain restriction: this combination requires $x > 0$ and $x + 3 > 0$, so $x > 0$.)

21. Use the quotient rule first to split off the denominator, then the product rule for the numerator, then the power rule to bring exponents in front: $\log(100) + \log(x^3) - \log(\sqrt{y}) = \log(100) + 3\log x - \frac{1}{2}\log y$. Finally, $\log(100) = 2$ (base 10 assumed), so the answer is $2 + 3\log x - \frac{1}{2}\log y$. (Tip: the square root turns into $\frac{1}{2}$ in front, not $\frac{1}{2}$ inside — a common mix-up.)

22. Use the properties to break each expression into pieces involving $\log_2(a)$ and $\log_2(b)$. For $\log_2(a^2 b)$: $2\log_2(a) + \log_2(b) = 2(3) + 5 = 11$. For $\log_2(a/b)$: $\log_2(a) - \log_2(b) = 3 - 5 = -2$. (Negative answer here is fine because $a < b$, so the ratio is less than 1 and its log is negative.)

23. Quotient property: $\log_2(40) - \log_2(5) = \log_2(40/5) = \log_2(8) = 3$. Verification: $\log_2(40) \approx 5.32$ and $\log_2(5) \approx 2.32$. Their difference is 3.00.

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Match. (The quotient rule turns an ugly pair of decimals into a clean integer — worth reaching for whenever the arguments share a factor.)

24. Run the log properties in reverse to rebuild the formula. Combine the right side: $\log P + t\log(1+r) = \log P + \log((1+r)^t) = \log(P(1+r)^t)$. So $\log A = \log(P(1+r)^t)$ means $A = P(1+r)^t = 1000(1.05)^{10} \approx 1000(1.62889) \approx 1628.89$, or about $\$1{,}629$. (This is what financial calculators do under the hood: take a log to turn the power into a multiplication, then exponentiate at the end.)

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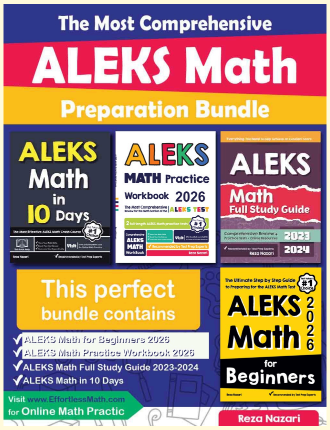

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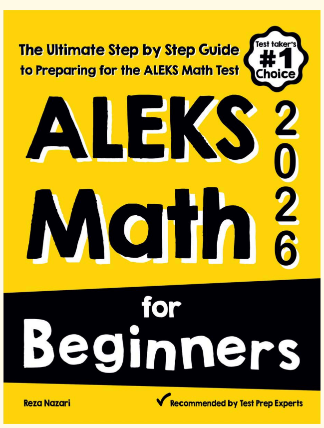

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