Evaluating Logarithms

Name: ______________________	Date: ______________	Score: ______ / 28

Quick Review

A **logarithm** answers one simple question: *what exponent do I need?* The equation $\log_b(x) = y$ is exactly the same statement as $b^y = x$ — written two ways. Every time you see a logarithm, mentally flip it to the exponent form and the problem becomes a power problem you already know how to do.

Three special values to memorize. For any valid base $b > 0$, $b \neq 1$: $\log_b(1) = 0$ (because $b^0 = 1$); $\log_b(b) = 1$ (because $b^1 = b$); and $\log_b(b^n) = n$ (the log peels the exponent off whatever shares its base).

Notation shortcuts. $\log(x)$ with no base means base 10: $\log(100) = 2$. $\ln(x)$ means base e: $\ln(e^3) = 3$.

Domain. The argument must be positive: $\log_b(x)$ requires $x > 0$. A negative input or zero is undefined for real logarithms — no real power of a positive base ever lands there.

Negative or fractional answers. $\log_2(\frac{1}{8}) = -3$ because $2^{-3} = \frac{1}{8}$. $\log_9(3) = \frac{1}{2}$ because $9^{1/2} = 3$. Flip to the exponent form and the sign or fraction falls right out.

Change of base. When the argument isn't a clean power of the base, use $\log_b(x) = \dfrac{\ln x}{\ln b} = \dfrac{\log x}{\log b}$. Quick check: $\log_4(20) = \dfrac{\ln 20}{\ln 4} \approx 2.16$. Sanity check: $4^2 = 16$ and $4^3 = 64$, so the answer should sit a bit above 2.

Common slips. Reading $\log_b(MN)$ as $\log_b(M) \cdot \log_b(N)$ (wrong — it's a sum, see SS 9.2). Forgetting the domain restriction so an extraneous solution sneaks through. Mixing up $\log$ and $\ln$ in a calculator step.**Inverse table.** A logarithm asks for an exponent. Pairing powers and logarithms side by side keeps the base and the answer from trading places.

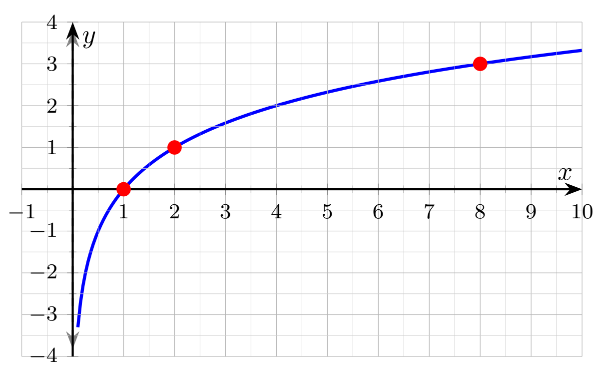
x	1	2	3	4
2^x	2	4	8	16

x	2	4	8	16
$\log_2 x$	1	2	3	4

PRACTICE

Evaluate each logarithm exactly. Use change of base only when no clean power form exists.

1. Evaluate $\log_2(8)$. The graph of $y = \log_2(x)$ is shown to help you locate the input/output pair. ________



2. Evaluate $\log(100)$ (common log, base 10). ________
3. Evaluate $\log_5(5)$. ________
4. Evaluate $\log_7(1)$. ________

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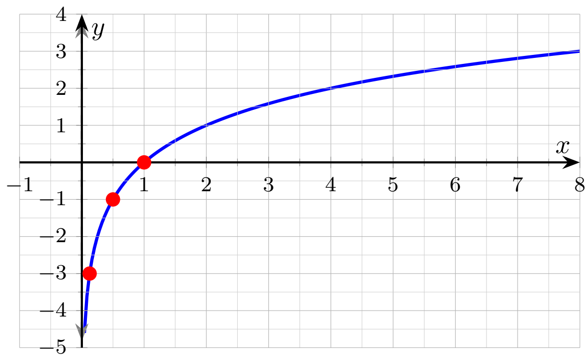
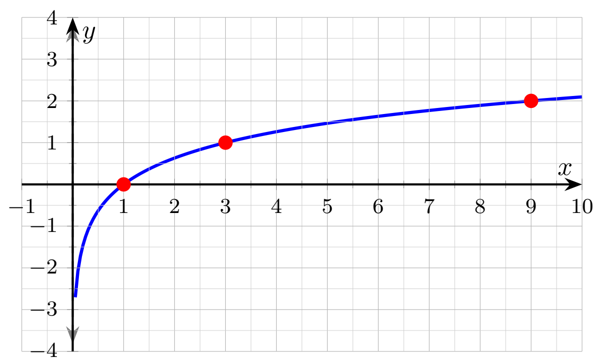
5. Evaluate $\log_2\left(\frac{1}{8}\right)$. The curve $y = \log_2(x)$ is shown; the asymptote at $x = 0$ explains why the log dives toward $-\infty$ as the input shrinks. ________



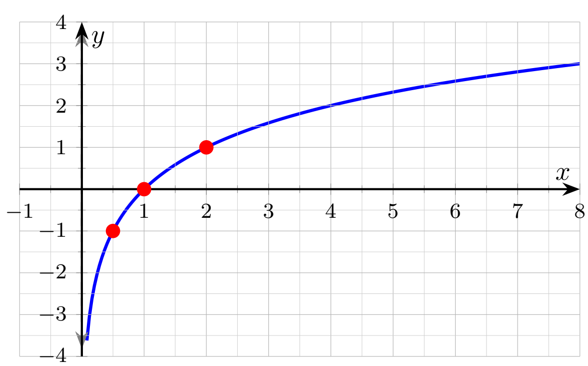
6. Evaluate $\log_3(81)$ and locate the value on the curve $y = \log_3(x)$ shown. ________



7. Evaluate $\log_5\left(\frac{1}{125}\right)$. ________

8. Evaluate $\log_6(\sqrt{6})$. ________

9. Evaluate $\log_2(0.5)$. The curve $y = \log_2(x)$ is shown; the dashed line marks the asymptote at $x = 0$. ________

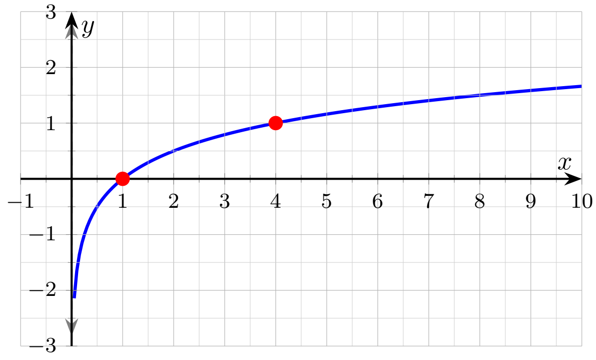


10. Evaluate $\log_{1/3}(27)$. ________

11. Approximate $\log_4(20)$ to two decimal places using change of base. ________

12. Evaluate $\log_{10}(10^7)$. ________

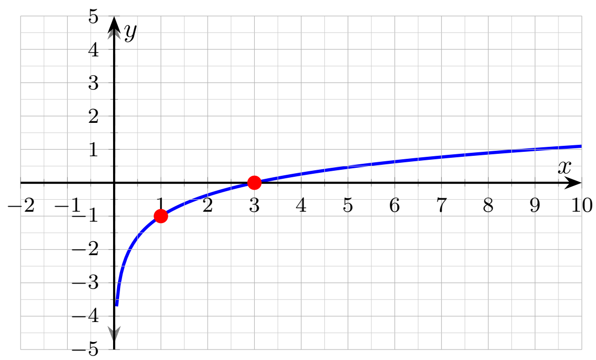
13. Evaluate $\log_4(64)$ and check your answer against the curve $y = \log_4(x)$. ______



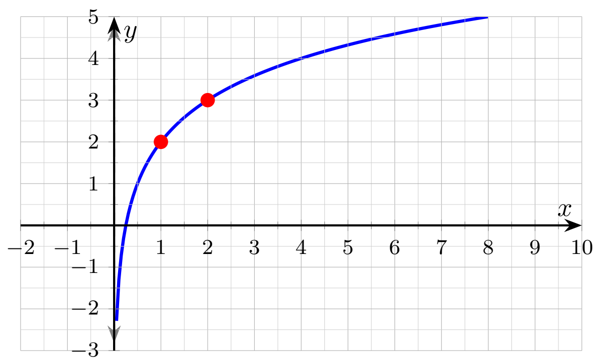
14. Find x if $\log_2(x-3) = 4$. ______

15. Which logarithm identities are TRUE for $b > 0,\ b \neq 1$? $\log_b(1) = 0$; $\log_b(b) = 1$; $\log_b(0)$ defined; $b^{\log_b(x)} = x$ for $x > 0$. ______

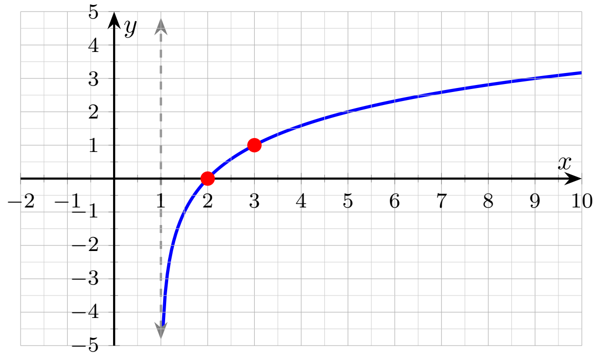
16. Identify the function shown. ______



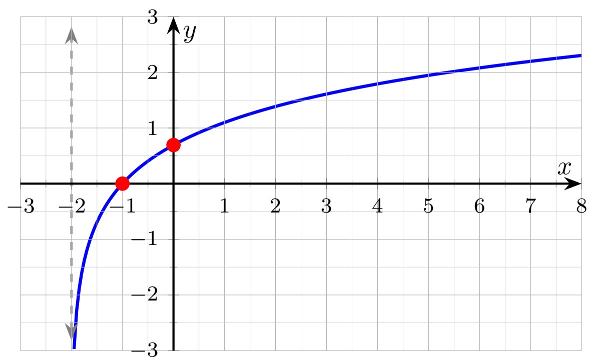
17. Identify the function shown. ______



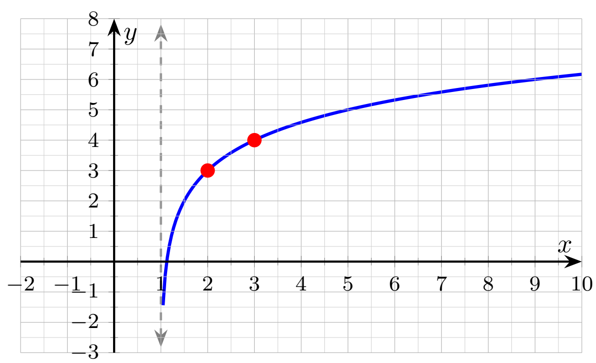
18. Which graph is $y = \log_2(x-1)$? Identify the asymptote and one anchor point. ________



19. For $f(x) = \ln(x+2)$, give the domain. ________



20. For $f(x) = \log_2(x-1) + 3$, identify the asymptote, the value $f(3)$, and the domain. ________



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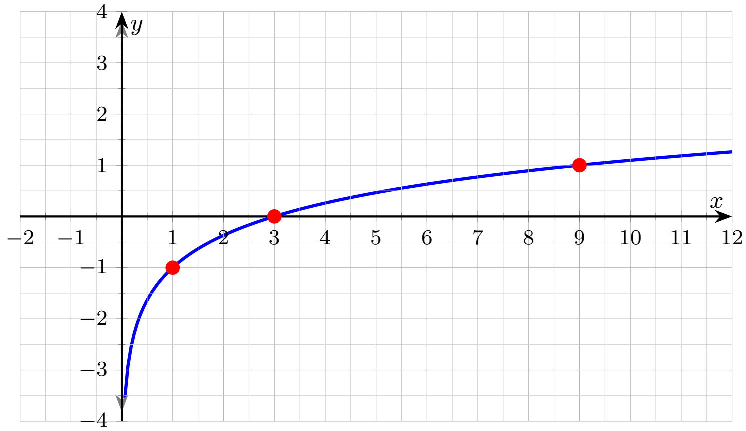
Word Problems

21. A function $f(x) = \log_3(x) - 1$ is graphed below. Use the graph to find $f(9)$, name the vertical asymptote, and state the domain. ________



22. Solve $\log_4(2x + 8) = 3$ for x. Verify the solution satisfies the log's domain. ____

23. You are told $\log_b(81) = 4$ and $\log_b(b^5) = 5$. What is the base b, and what is $\log_b(\sqrt{b})$? ____

24. Approximate $\log_7(50)$ to two decimal places using change of base. Then sanity-check your answer by bracketing it between two integer values of the log. ____

Additional Practice

25. Evaluate $\log_2 32$. ________

26. Evaluate $\log_5 125$. ________

27. Rewrite $\log_3 81 = 4$ exponentially. ________

28. Solve $\log_4 x = 3$. ________

Answer Keys

1. $\boxed{3}$
2. $\boxed{2}$
3. $\boxed{1}$
4. $\boxed{0}$
5. $\boxed{-3}$
6. $\boxed{4}$
7. $\boxed{-3}$
8. $\boxed{\frac{1}{2}}$
9. $\boxed{-1}$
10. $\boxed{-3}$
11. $\boxed{\approx 2.16}$
12. $\boxed{7}$
13. $\boxed{3}$
14. $\boxed{x = 19}$
15. $\boxed{\text{first, second, fourth}}$
16. $\boxed{f(x) = \log_3(x) - 1}$
17. $\boxed{f(x) = \log_2(x) + 2}$
18. $\boxed{\text{asymptote } x = 1;\ (2, 0) \text{ on graph}}$
19. $\boxed{x > -2}$
20. $\boxed{x = 1;\ f(3) = 4;\ x > 1}$
21. $\boxed{f(9) = 1;\ \text{asymptote } x = 0;\ \text{domain } x > 0}$
22. $\boxed{x = 28}$
23. $\boxed{b = 3;\ \log_b(\sqrt{b}) = \frac{1}{2}}$
24. $\boxed{\approx 2.01}$

Additional Practice Answers

25. $\boxed{5}$
26. $\boxed{3}$
27. $\boxed{3^4 = 81}$
28. $\boxed{x = 64}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Ask: $2^? = 8$. Since $8 = 2^3$, the answer is 3. (The log just reads the exponent.) The red dot at $(8, 3)$ on the curve confirms.

2. With no base written, the log is base 10. Ask yourself: $10^? = 100$. Since $10^2 = 100$, the answer is 2. The log just hands you back the exponent.

3. Ask: $5^? = 5$. Any base raised to the first power gives itself, so $5^1 = 5$ and the answer is 1. That is the identity $\log_b(b) = 1$ for every valid base.

4. Ask: $7^? = 1$. Any nonzero base to the zero power is 1, so $7^0 = 1$ and the answer is 0. This is the identity $\log_b(1) = 0$, true for every valid base.

5. A careful way to see it: $\frac{1}{8} = 2^{-3}$, so the log reads -3. (Don't be thrown by the fraction — it just signals a negative exponent.) The red dot at $(0.125, -3)$ on the curve confirms the value. That gives a quick check on the answer.

6. Keep the rule visible: $81 = 3^4$, so the answer is 4. Quick powers-of-3 recall: $3, 9, 27, 81, 243$. The curve in the figure passes through $(1, 0)$, $(3, 1)$, and $(9, 2)$; continuing the same pattern, $(81, 4)$ is the next integer anchor (off the plotted window). That gives a quick check on the answer.

7. Ask: $5^? = \frac{1}{125}$. Since $125 = 5^3$, its reciprocal is $\frac{1}{125} = 5^{-3}$, so the exponent is -3. A reciprocal flips the sign of the exponent — that is what makes the answer negative.

8. Ask: $6^? = \sqrt{6}$. A square root is the same as the $\frac{1}{2}$ power, so $\sqrt{6} = 6^{1/2}$ and the log reads off $\frac{1}{2}$. Roots always turn into fractional exponents.

9. A careful way to see it: $0.5 = \frac{1}{2} = 2^{-1}$. Negative exponent because the input is below 1 (but still positive). The red dot at $(0.5, -1)$ shows where the curve crosses the line $y = -1$. That gives a quick check on the answer.

10. Set $\left(\frac{1}{3}\right)^x = 27$. Since $\frac{1}{3} = 3^{-1}$, $\left(\frac{1}{3}\right)^{-3} = 3^3 = 27$. A fractional base less than 1 needs a negative exponent to climb up to 27.

11. One steady path is: $\log_4(20) = \frac{\ln 20}{\ln 4} \approx \frac{2.996}{1.386} \approx 2.16$. Sanity check: $4^2 = 16$ and $4^3 = 64$, so the answer should sit just above 2. It does. That gives a quick check on the answer.

12. Ask: $10^? = 10^7$. The exponents must match, so the answer is 7. Whenever the argument is the base raised to a power, the log returns that power: $\log_b(b^n) = n$.

13. A careful way to see it: $64 = 4^3$, so the log is 3. (Quick: $4^2 = 16$, $4^3 = 64$.) The visible anchors $(1, 0)$, $(4, 1)$, and $(16, 2)$ on the curve confirm the powers-of-4 pattern; $(64, 3)$ continues it. That gives a quick check on the answer.

14. Flip to exponent form: $x - 3 = 2^4 = 16$, so $x = 19$. Domain check: $x - 3 = 16 > 0$, valid.

15. One steady path is: $\log_b(0)$ is undefined (the input must be positive). The other three are the standard identities — two special values plus the inverse property. That gives a quick check on the answer.

16. Vertical asymptote at $x = 0$, and the curve sits one unit below the parent $\log_3(x)$. At $x = 1$, $y = -1$ (the parent's $y = 0$ shifted down by 1). At $x = 3$, $\log_3(3) - 1 = 1 - 1 = 0$. Both marked dots match.

17. Vertical asymptote at $x = 0$, parent $\log_2(x)$ shifted up by 2. At $x = 1$: $\log_2(1) + 2 = 0 + 2 = 2$. At $x = 2$: $1 + 2 = 3$. The two red dots confirm.

18. The -1 inside the log shifts the parent right by 1. Asymptote: $x = 1$. Anchor: $\log_2(2 - 1) = \log_2(1) = 0$, so $(2, 0)$ is on the curve.

19. The natural log needs its argument positive: $x + 2 > 0$, i.e., $x > -2$. Vertical asymptote at $x = -2$, as the dashed line shows.

20. The -1 inside the log gives the asymptote $x = 1$ and domain $x > 1$. At $x = 3$: $\log_2(2) + 3 = 1 + 3 = 4$. The red dot at $(3, 4)$ matches.

21. At $x = 9$: $\log_3(9) - 1 = 2 - 1 = 1$. The marked dot at $(9, 1)$ confirms. The argument x must be positive, so the domain is $x > 0$ and the vertical asymptote sits at $x = 0$. (The -1 outside the log shifts the whole curve down by 1, but it doesn't move the asymptote — that depends only on what's inside the log.)

22. Flip to exponent form: $2x + 8 = 4^3 = 64$, so $2x = 56$ and $x = 28$. Domain check: the argument $2x + 8 = 2(28) + 8 = 64$, which is positive. Valid. (A negative or zero argument would force us to throw the solution out — this one is clean.)

23. From $\log_b(81) = 4$: $b^4 = 81$, so $b = 3$ (taking the positive real root since logs need a positive base). The second equation is automatic: $\log_b(b^5) = 5$ for any base. For the last part, $\sqrt{b} = b^{1/2}$, so $\log_b(\sqrt{b}) = \frac{1}{2}$ — the log peels off the exponent.

24. Change of base: $\log_7(50) = \frac{\ln 50}{\ln 7} \approx \frac{3.912}{1.946} \approx 2.01$. Bracketing: $7^2 = 49$ and $7^3 = 343$, so $\log_7(50)$ must sit just above 2 (since 50 is just past 49). The decimal answer 2.01 matches the bracket perfectly. (A handy habit: always bracket the answer before trusting the calculator step — it catches base-vs-argument mix-ups every time.)

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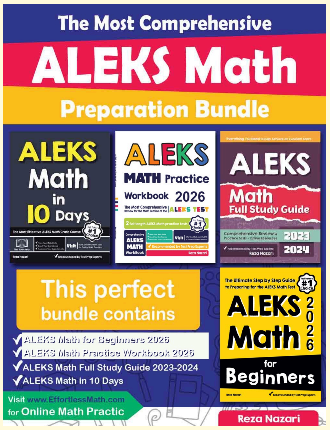

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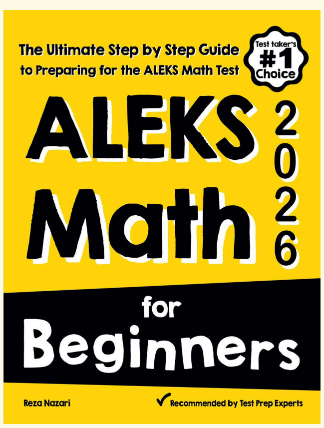

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