Rationalizing Imaginary Denominators

Name: ________	Date: ________	Score: ____ / 33

Quick Review

Standard form for a complex number is $a+bi$, with no i stuck in a denominator. To convert $\frac{1}{i}$ or $\frac{1}{2-3i}$ into that form, you have to *rationalize* the denominator — multiply top and bottom by a carefully chosen factor that clears the i.

Pure i in the denominator: multiply by $\frac{i}{i}$. The denominator becomes $i^2 = -1$, a real number, and the negative sign carries up to the numerator. That's why $\frac{1}{i} = -i$, not $+i$.

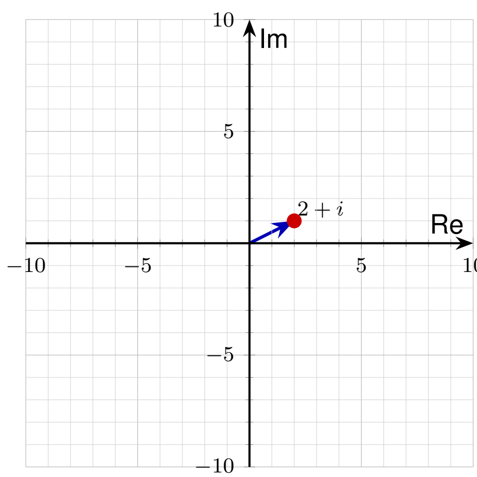
Complex denominator $c+di$: multiply by the *conjugate* $c-di$. The product $(c+di)(c-di) = c^2+d^2$ is a real number, so the new denominator is real. Then split the result into $a+bi$ form. The same idea works on a denominator like $c-di$ — the conjugate is $c+di$. Just flip the sign on the imaginary part; do not touch the real part.

Two traps to flag. First, when you write $\frac{x}{i}\cdot\frac{i}{i} = \frac{xi}{i^2}$, do not stop at $\frac{xi}{1}$; the denominator is -1, not 1. Second, when you FOIL the numerator after multiplying by the conjugate, the $-i^2$ term flips to $+1$ and adds to the real part. Forgetting that flip is the single most common mistake on this whole section.

PRACTICE

Rationalize each denominator. Write the result in standard form $a+bi$.

1. $\frac{1}{i}$ ________
2. $\frac{4}{i}$ ________
3. $\frac{6}{2i}$ ________
4. $\frac{1}{2+i}$. The denominator $2+i$ is plotted below; rationalize using its conjugate. ________



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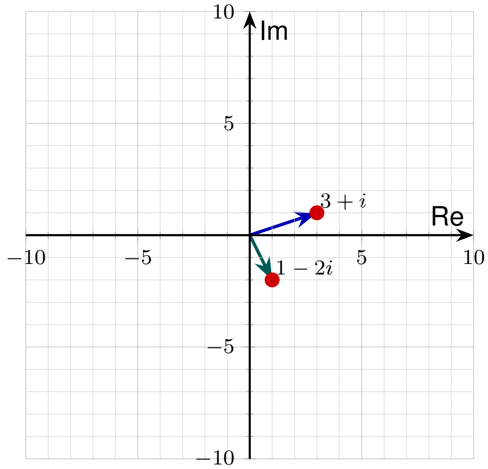
5. $\dfrac{3+i}{1-2i}$. Numerator $3+i$ and denominator $1-2i$ are both plotted below. ________



6. $\dfrac{2}{3i} - \dfrac{1}{i}$ ________

7. $\dfrac{4-i}{3+2i}$ ________

8. $\dfrac{5}{2-3i}$ ________

9. $\dfrac{2-3i}{4+i}$ ________

10. $\dfrac{3+2i}{1-i}$ ________

11. $\dfrac{1}{3i}$ ________

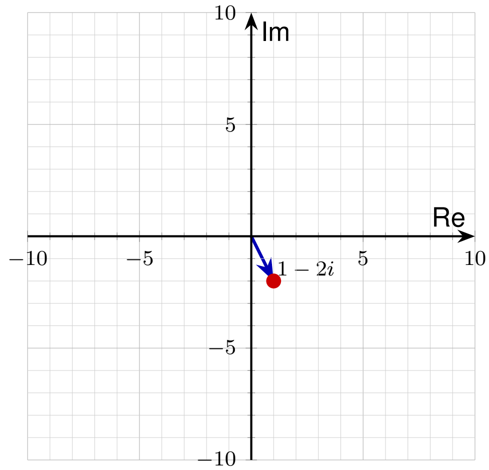
12. $\dfrac{2i}{1+i}$ ________

13. $\dfrac{5+i}{i}$ ________

14. $\dfrac{6-2i}{2i}$ ________

15. $\dfrac{1}{(1+i)(1-i)}$ ________

16. $\frac{-3}{1-2i}$. The denominator $1-2i$ is plotted below; clear the i with its conjugate. ______



17. $\frac{2+i}{2-i}$ ______

18. $\frac{4}{i^3}$ ______

19. $\frac{1+i}{1-i}+\frac{1-i}{1+i}$ ______

20. $\frac{1}{4+5i}$ ______

♦ Word Problems

21. In an AC circuit, admittance Y is the reciprocal of impedance: $Y=\frac{1}{Z}$. If the impedance is $Z=3+4i$ ohms, find the admittance in standard form. ______

22. Maria simplifies $\frac{2}{1-i}$ and writes $1+i$ as her answer. She wants to check her work without re-doing the calculation. What product should equal 2 as the verification? ______

23. In a quantum-mechanics calculation, an amplitude comes out as $\frac{1+2i}{3-i}$. Express it in standard form $a+bi$ so the real and imaginary parts are easy to read off. ______

24. A signal processor outputs $\frac{5}{i}$ in raw form and $\frac{10}{2+i}$ from a second channel. Express each in standard form and find the sum of the two simplified values. ______

Additional Practice

25. Add $(3+2i)+(5-i)$. ______

26. Subtract $(4-i)-(1+6i)$. ______

27. Multiply $(2+3i)(1-i)$. ______

28. Simplify i^{17}. ______

29. Simplify i^{22}. ______

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30. Find the conjugate of $6-5i$. ________

31. Find $|3+4i|$. ________

32. Write $\sqrt{-36}$ in simplest form. ________

33. Divide $\frac{8+6i}{2}$. ________

Answer Keys

1. $-i$
2. $-4i$
3. $-3i$
4. $\frac{2}{5}-\frac{1}{5}i$
5. $\frac{1}{5}+\frac{7}{5}i$
6. $\frac{1}{3}i$
7. $\frac{10}{13}-\frac{11}{13}i$
8. $\frac{10}{13}+\frac{15}{13}i$
9. $\frac{5}{17}-\frac{14}{17}i$
10. $\frac{1}{2}+\frac{5}{2}i$
11. $-\frac{1}{3}i$
12. $1+i$
13. $1-5i$
14. $-1-3i$
15. $\frac{1}{2}$
16. $-\frac{3}{5}-\frac{6}{5}i$
17. $\frac{3}{5}+\frac{4}{5}i$
18. $4i$
19. 0
20. $\frac{4}{41}-\frac{5}{41}i$
21. $Y=\frac{3}{25}-\frac{4}{25}i$ siemens
22. $(1-i)(1+i)=2$
23. $\frac{1}{10}+\frac{7}{10}i$
24. $4-7i$

Additional Practice Answers

25. $8+i$
26. $3-7i$
27. $5+i$
28. i
29. -1
30. $6+5i$
31. 5
32. $6i$
33. $4+3i$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. To clear a lone i, multiply top and bottom by i: $\frac{1}{i}\cdot\frac{i}{i}=\frac{i}{i^2}$. Now $i^2=-1$, so this is $\frac{i}{-1}=-i$. The sign flip comes entirely from $i^2=-1$ — watch out for stopping at $+i$.

2. Multiply top and bottom by i: $\frac{4}{i}\cdot\frac{i}{i}=\frac{4i}{i^2}=\frac{4i}{-1}=-4i$. The coefficient 4 rides along with the i; the minus comes from $i^2=-1$.

3. Reduce the numeric part first: $\frac{6}{2i}=\frac{3}{i}$. Then multiply by $\frac{i}{i}$: $\frac{3i}{i^2}=\frac{3i}{-1}=-3i$.

4. Conjugate of $2+i$ is $2-i$. Bottom: $(2+i)(2-i)=4+1=5$. Top: $1\cdot(2-i)=2-i$. So $\frac{2-i}{5}=\frac{2}{5}-\frac{1}{5}i$.

5. Conjugate of $1-2i$ is $1+2i$. Bottom: $1+4=5$. Top: $(3+i)(1+2i)=3+6i+i+2i^2=3+7i-2=1+7i$. Answer: $\frac{1+7i}{5}=\frac{1}{5}+\frac{7}{5}i$.

6. Rationalize each piece. $\frac{2}{3i}\cdot\frac{i}{i}=\frac{2i}{-3}=-\frac{2}{3}i$. $\frac{1}{i}=-i$. Subtract: $-\frac{2}{3}i-(-i)=-\frac{2}{3}i+\frac{3}{3}i=\frac{1}{3}i$.

7. Conjugate of $3+2i$ is $3-2i$. Bottom: $9+4=13$. Top: $(4-i)(3-2i)=12-8i-3i+2i^2=12-11i-2=10-11i$. Answer: $\frac{10-11i}{13}=\frac{10}{13}-\frac{11}{13}i$.

8. Conjugate of $2-3i$ is $2+3i$. Bottom: $4+9=13$. Top: $5(2+3i)=10+15i$. So $\frac{10+15i}{13}=\frac{10}{13}+\frac{15}{13}i$.

9. Conjugate of $4+i$ is $4-i$. Bottom: $16+1=17$. Top: $(2-3i)(4-i)=8-2i-12i+3i^2=8-14i-3=5-14i$. Answer: $\frac{5-14i}{17}=\frac{5}{17}-\frac{14}{17}i$.

10. Conjugate of $1-i$ is $1+i$. Bottom: $1+1=2$. Top: $(3+2i)(1+i)=3+3i+2i+2i^2=3+5i-2=1+5i$. Answer: $\frac{1+5i}{2}=\frac{1}{2}+\frac{5}{2}i$.

11. Multiply top and bottom by i: $\frac{1}{3i}\cdot\frac{i}{i}=\frac{i}{3i^2}$. Since $i^2=-1$ the bottom is -3, giving $\frac{i}{-3}=-\frac{1}{3}i$.

12. Conjugate $1-i$. Bottom: 2. Top: $2i(1-i)=2i-2i^2=2i+2=2+2i$. So $\frac{2+2i}{2}=1+i$.

13. Multiply by $\frac{i}{i}$: $\frac{(5+i)i}{i^2}=\frac{5i+i^2}{-1}=\frac{5i-1}{-1}=1-5i$. (You can also split: $\frac{5}{i}+\frac{i}{i}=-5i+1$.)

14. Keep the rule visible: Multiply by $\frac{i}{i}$: $\frac{(6-2i)i}{2i^2}=\frac{6i-2i^2}{-2}=\frac{6i+2}{-2}=\frac{2+6i}{-2}=-1-3i$. That gives a quick check on the answer.

15. The denominator is already a conjugate pair: $(1+i)(1-i)=1+1=2$. So the expression is $\frac{1}{2}$ — no rationalization needed because the denominator was already real.

16. Conjugate $1+2i$. Bottom: $1+4=5$. Top: $-3(1+2i)=-3-6i$. So $\frac{-3-6i}{5}=-\frac{3}{5}-\frac{6}{5}i$.

17. Conjugate $2+i$. Bottom: $4+1=5$. Top: $(2+i)(2+i)=(2+i)^2=4+4i+i^2=3+4i$. So $\frac{3+4i}{5}=\frac{3}{5}+\frac{4}{5}i$.

18. First simplify the power: $i^3=-i$, so the expression is $\frac{4}{-i}$. Multiply top and bottom by i: $\frac{4i}{-i^2}=\frac{4i}{1}=4i$ (since $-i^2=+1$). Knowing the i-cycle turns this

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into one clean step.

19. Each fraction rationalizes separately. $\frac{1+i}{1-i}\cdot\frac{1+i}{1+i}=\frac{(1+i)^2}{2}=\frac{2i}{2}=i.$ By symmetry the second is $\frac{(1-i)^2}{2}=\frac{-2i}{2}=-i$. Sum: $i+(-i)=0$.

20. Conjugate $4-5i$. Bottom: $16+25=41$. Top: $4-5i$. Answer: $\frac{4-5i}{41}=\frac{4}{41}-\frac{5}{41}i.$

21. A careful way to see it: $Y=\frac{1}{3+4i}\cdot\frac{3-4i}{3-4i}=\frac{3-4i}{9+16}=\frac{3-4i}{25}=\frac{3}{25}-\frac{4}{25}i$ siemens. (The unit for admittance is the siemens, reciprocal of ohms.) That gives a quick check on the answer.

22. If $\frac{2}{1-i}=1+i$, then $2=(1-i)(1+i)$. Check: $(1-i)(1+i)=1-i^2=1-(-1)=2$ ✓. (Re-multiplying the denominator by the boxed answer is the quickest sanity check for any rationalization.)

23. Multiply by the conjugate $3+i$. Bottom: $(3-i)(3+i)=9+1=10$. Top: $(1+2i)(3+i)=3+i+6i+2i^2=3+7i-2=1+7i$. Standard form: $\frac{1+7i}{10}=\frac{1}{10}+\frac{7}{10}i.$

24. First: $\frac{5}{i}\cdot\frac{i}{i}=\frac{5i}{-1}=-5i$. Second: $\frac{10}{2+i}\cdot\frac{2-i}{2-i}=\frac{10(2-i)}{5}=2(2-i)=4-2i$. Sum: $(-5i)+(4-2i)=4-7i$.

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