Radical Equations

Name: ____________ Date: ____________ Score: ______ / 24

Quick Review

A **radical equation** has the variable inside a root. There's one game plan, two warnings, and one habit that catches every wrong move: *always check.*

The game plan. Isolate the radical on one side. Raise both sides to the matching power (square for $\sqrt{\ }$, cube for $\sqrt[3]{\ }$, etc.) to clear the radical. Solve whatever remains. Then check every candidate in the *original* equation.

Why the check matters — extraneous roots. Squaring is not reversible. The step "square both sides" creates the *same* new equation from $\sqrt{A} = B$ *and* from $\sqrt{A} = -B$, so it can introduce solutions that don't satisfy the original. The classic case: $\sqrt{x+6} = x$ squares to $x + 6 = x^2$, giving $x = 3$ or $x = -2$. Plug back: $\sqrt{9} = 3$ (yes), but $\sqrt{4} = 2 \neq -2$ (no). Throw out $x = -2$. The *check is the algebra step*, not the verification — it's how you finish the problem.

Cube roots don't introduce extraneous roots. Cubing is one-to-one over the real numbers. $\sqrt[3]{A} = B$ becomes $A = B^3$ with no sign trickery. (Still worth a quick check for arithmetic errors, but no extraneous-root surprises.)

Same-base equations. If both sides are roots of the same index, set the radicands equal: $\sqrt{x+4} = \sqrt{2x-5} \Rightarrow x + 4 = 2x - 5 \Rightarrow x = 9$. (Check: $\sqrt{13} = \sqrt{13}$, valid.)

Reading the graph. The graph of $y = \sqrt{ax+b}$ and $y = mx + c$ together makes the geometry visible. Real solutions are intersection points; extraneous roots from squaring show up as intersections of $y = mx + c$ with the *reflection* of the radical curve below the x-axis — points where the parabola $y^2 = ax + b$ crosses but $y = \sqrt{ax+b}$ does not.

Common slips. Skipping the check. Squaring without isolating the radical first (which produces a mess of cross-terms). Cubing both sides and then "checking" for extraneous roots out of habit (cubing doesn't create them, but checking never hurts).

PRACTICE

Isolate, raise to the matching power, solve, and CHECK every candidate in the original equation.

1. Solve $\sqrt{x} = 5$. ________

2. Solve $\sqrt{x} + 3 = 7$. ________

3. Solve $\sqrt{2x-1} = 5$. ________

4. Solve $\sqrt{x+6} = x$. Identify any extraneous roots. ________

5. Solve $\sqrt[3]{x-1} = 2$. ________

6. Solve $\sqrt{x+4} = \sqrt{2x-5}$. ________

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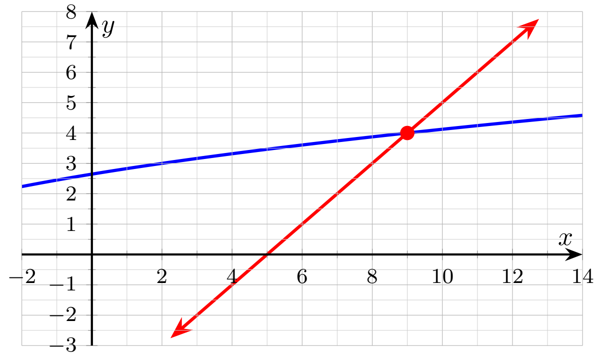
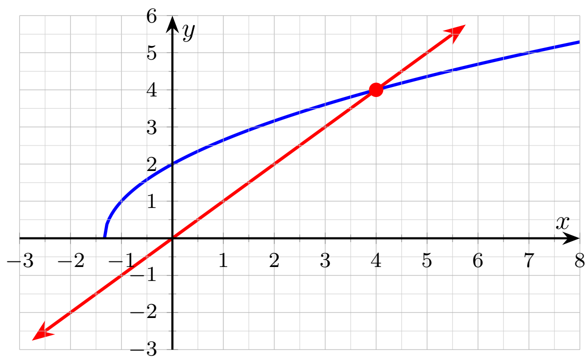
7. Solve $\sqrt{x+7} = x - 5$. Identify any extraneous roots. ________



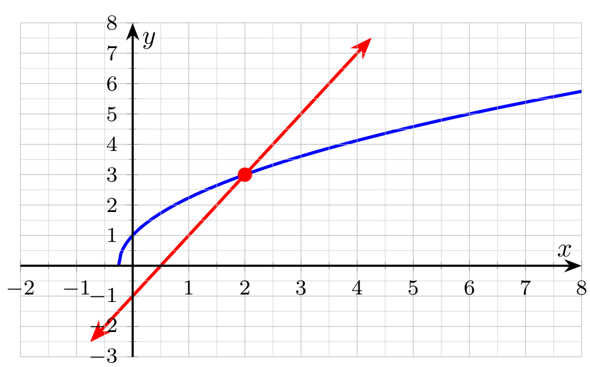
8. Solve $\sqrt{3x+4} = x$. Identify any extraneous roots. ________



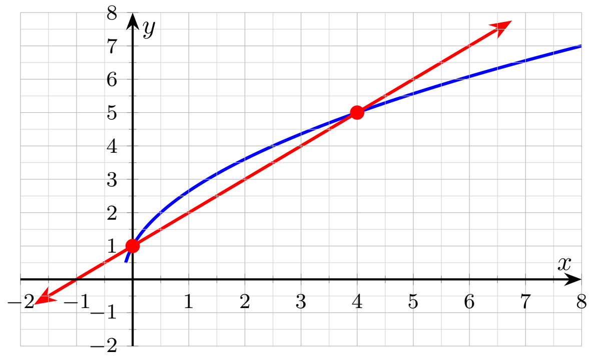
9. Solve $\sqrt{2x+7} = x + 1$. Identify any extraneous roots. ________

10. Mark TRUE or FALSE: Squaring both sides always preserves the solution set without introducing extraneous roots. ________

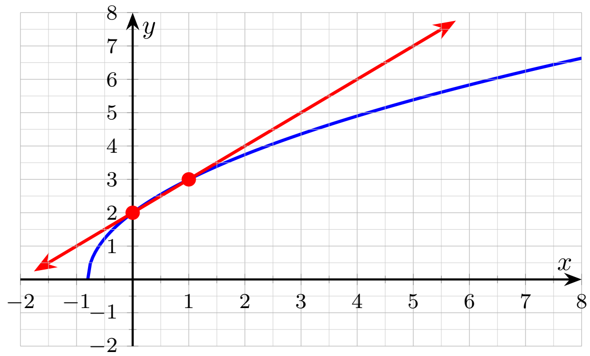
11. Solve $\sqrt{4x+1} = 2x - 1$. Identify any extraneous roots. ________



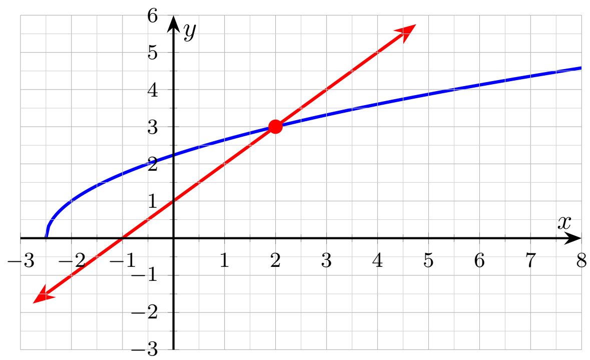
12. Solve $\sqrt{6x+1} = x + 1$. Identify any extraneous roots. ________



13. Solve $\sqrt{5x+4} = x + 2$. Identify any extraneous roots. ________



14. Solve $\sqrt{2x+5} = x + 1$. Identify any extraneous roots. ________



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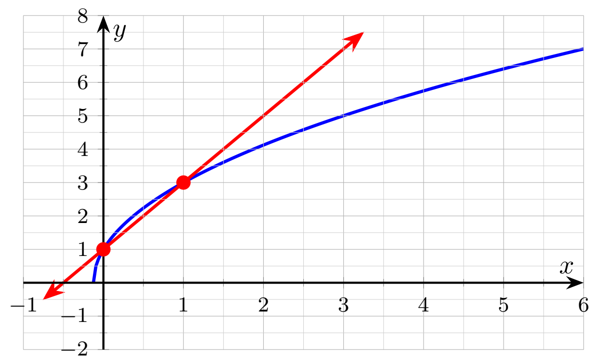
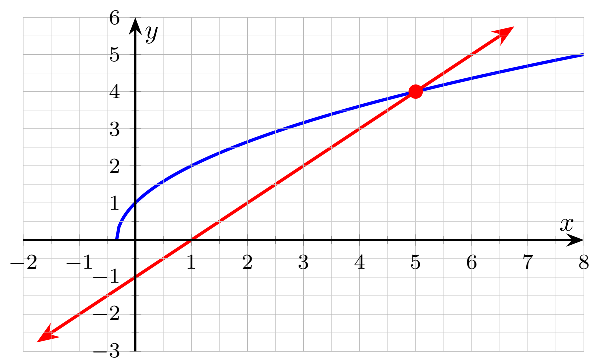
15. Solve $\sqrt{8x+1} = 2x+1$. Identify any extraneous roots. ________



16. Solve $\sqrt{3x+1} = x-1$. Identify any extraneous roots. ________



17. Solve $\sqrt[3]{2x+3} = 3$. ________

18. Solve $2\sqrt{x-1} = 8$. ________

19. Solve $\sqrt{x+1} = \sqrt{3x-7}$. ________

20. Solve $\sqrt{x} + 2 = x$. Identify any extraneous roots. ________

♦ Word Problems

21. On a flat road, the speed (mph) at which a car will begin to skid in a circular turn of radius r (feet) is $v = \sqrt{2.5r}$. Find the radius of a turn for which the skid speed is 30 mph. Show the check that verifies the candidate radius. ____

22. Solve $\sqrt{2x+3} + x = 3$ and explain which (if any) candidate solutions are extraneous. ____

23. The time (seconds) for a pendulum of length L (feet) to complete one swing is $T = \frac{\pi}{4}\sqrt{L}$. Find the exact length that produces a 2-second swing. Verify by substituting back. ____

24. Solve $\sqrt{x+11} = \sqrt{x-1} + 2$. Carry out the check step explicitly — this is a two-radical equation where extraneous roots are especially common. ____

Answer Keys

1. $\boxed{x = 25}$
2. $\boxed{x = 16}$
3. $\boxed{x = 13}$
4. $\boxed{x = 3}$
5. $\boxed{x = 9}$
6. $\boxed{x = 9}$
7. $\boxed{x = 9}$
8. $\boxed{x = 4}$
9. $\boxed{x = \sqrt{6}}$
10. FALSE
11. $\boxed{x = 2}$
12. $\boxed{x = 0 \text{ and } x = 4}$
13. $\boxed{x = 0 \text{ and } x = 1}$
14. $\boxed{x = 2}$
15. $\boxed{x = 0 \text{ and } x = 1}$
16. $\boxed{x = 5}$
17. $\boxed{x = 12}$
18. $\boxed{x = 17}$
19. $\boxed{x = 4}$
20. $\boxed{x = 4}$
21. $\boxed{r = 360 \text{ ft}}$
22. $\boxed{x = 4 - \sqrt{10}}$
23. $\boxed{L = \frac{64}{\pi^2} \text{ ft} \approx 6.48 \text{ ft}}$
24. $\boxed{x = 5}$

Step-by-Step Explanations

1. The radical is already isolated, so square both sides to clear it: $(\sqrt{x})^2 = 5^2$, giving $x = 25$. **Check:** $\sqrt{25} = 5$ ✓, so it's valid. Don't write ± 5 — the radical symbol returns only the principal (non-negative) root.

2. Isolate the radical first by subtracting 3: $\sqrt{x} = 4$. Then square both sides: $x = 16$. **Check** in the original: $\sqrt{16} + 3 = 4 + 3 = 7$ ✓. Isolating before squaring keeps the algebra clean.

3. The radical is isolated, so square both sides: $2x - 1 = 25$. Solve: $2x = 26$, so $x = 13$. **Check:** $\sqrt{2(13) - 1} = \sqrt{25} = 5$ ✓— it satisfies the original, so it's valid.

4. Square: $x + 6 = x^2$, so $x^2 - x - 6 = 0$ and $(x - 3)(x + 2) = 0$, giving $x = 3$ or $x = -2$. **Check** $x = 3$: $\sqrt{9} = 3$ ✓. **Check** $x = -2$: $\sqrt{4} = 2 \neq -2$ (extraneous). Only $x = 3$ survives.

5. Cube: $x - 1 = 8$, so $x = 9$. **Check:** $\sqrt[3]{8} = 2$ ✓. (Cube roots don't introduce extraneous roots, but verifying the arithmetic still pays.)

6. Keep the rule visible: Same index ⇒ set radicands equal: $x + 4 = 2x - 5$, so $x = 9$. **Check:** $\sqrt{13} = \sqrt{13}$ ✓. That gives a quick check on the answer.

7. Square: $x + 7 = x^2 - 10x + 25$, so $x^2 - 11x + 18 = 0$, giving $(x - 9)(x - 2) = 0$ and $x = 9$ or $x = 2$. **Check** $x = 9$: $\sqrt{16} = 4$ and $9 - 5 = 4$ ✓. **Check** $x = 2$: $\sqrt{9} = 3$ but $2 - 5 = -3$ ✗. The line sits below the x-axis at $x = 2$, where the radical curve cannot reach. Only $x = 9$ is valid.

8. Square: $3x + 4 = x^2$, so $x^2 - 3x - 4 = 0$ and $(x - 4)(x + 1) = 0$, giving $x = 4$ or $x = -1$. **Check** $x = 4$: $\sqrt{16} = 4$ ✓. **Check** $x = -1$: $\sqrt{1} = 1 \neq -1$ ✗. The line passes through $(-1, -1)$, below the radical's reach. Only $x = 4$ survives.

9. Square: $2x + 7 = x^2 + 2x + 1$, so $x^2 = 6$ and $x = \pm\sqrt{6}$. The original requires $x + 1 \geq 0$, i.e., $x \geq -1$. **Check** $x = \sqrt{6} \approx 2.45$: LHS $= \sqrt{2(2.45) + 7} = \sqrt{11.9} \approx 3.45$; RHS $= 2.45 + 1 = 3.45$ ✓. **Check** $x = -\sqrt{6} \approx -2.45$: fails the $x \geq -1$ constraint — extraneous. Only $x = \sqrt{6}$ is valid.

10. Squaring is not reversible — it can create extraneous roots that satisfy the squared equation but not the original. That's why the check is mandatory.

11. Square: $4x + 1 = (2x - 1)^2 = 4x^2 - 4x + 1$, so $8x = 4x^2$, giving $4x(x - 2) = 0$ and $x = 0$ or $x = 2$. **Check** $x = 0$: $\sqrt{1} = 1$ but $2(0) - 1 = -1$ ✗. **Check** $x = 2$: $\sqrt{9} = 3$ and $2(2) - 1 = 3$ ✓. Only $x = 2$ survives. The graph shows the line $y = 2x - 1$ dipping below the x-axis near $x = 0$, where the radical can't follow.

12. Square: $6x + 1 = x^2 + 2x + 1$, so $x^2 - 4x = 0$ and $x(x - 4) = 0$, giving $x = 0$ or $x = 4$. **Check** $x = 0$: $\sqrt{1} = 1$ and $0 + 1 = 1$ ✓. **Check** $x = 4$: $\sqrt{25} = 5$ and $4 + 1 = 5$ ✓. Both check out — the line $y = x + 1$ meets the radical curve at two valid intersections. (Not every radical equation has an extraneous root; sometimes the algebra is honest.)

13. Square: $5x + 4 = x^2 + 4x + 4$, so $x^2 - x = 0$ and $x(x - 1) = 0$, giving $x = 0$ or $x = 1$. **Check** $x = 0$: $\sqrt{4} = 2$ and $0 + 2 = 2$ ✓. **Check** $x = 1$: $\sqrt{9} = 3$ and $1 + 2 = 3$ ✓. Both valid.

14. Square: $2x + 5 = x^2 + 2x + 1$, so $x^2 = 4$ and $x = \pm 2$. **Check** $x = 2$: $\sqrt{9} = 3$ and $2 + 1 = 3$ ✓. **Check** $x = -2$: $\sqrt{1} = 1$ but $-2 + 1 = -1$ ✗. Only $x = 2$ valid.

15. Square: $8x + 1 = (2x + 1)^2 = 4x^2 + 4x + 1$, so $4x = 4x^2$, giving $4x(x - 1) = 0$ and $x = 0$ or $x = 1$. **Check** $x = 0$: $\sqrt{1} = 1$ and $2(0) + 1 = 1$ ✓. **Check** $x = 1$: $\sqrt{9} = 3$ and $2(1) + 1 = 3$ ✓. Both valid.

16. Square: $3x + 1 = x^2 - 2x + 1$, so $x^2 - 5x = 0$ and $x(x - 5) = 0$, giving $x = 0$ or $x = 5$. **Check** $x = 5$: $\sqrt{16} = 4$ and $5 - 1 = 4$ ✓. **Check** $x = 0$: $\sqrt{1} = 1$ but $0 - 1 = -1$ ✗. Only $x = 5$ survives.

17. Cube both sides to clear the cube root: $2x + 3 = 3^3 = 27$. Solve: $2x = 24$, so $x = 12$. **Check:** $\sqrt[3]{2(12) + 3} = \sqrt[3]{27} = 3$ ✓. Cubing is one-to-one, so it never introduces extraneous roots.

18. Isolate the radical by dividing both sides by 2: $\sqrt{x - 1} = 4$. Square: $x - 1 = 16$, so $x = 17$. **Check:** $2\sqrt{17 - 1} = 2\sqrt{16} = 2(4) = 8$ ✓. Clear the coefficient before squaring or you'll square a mess.

19. Both sides are square roots of the same index, so square once to set the radicands equal: $x + 1 = 3x - 7$. Solve: $2x = 8$, so $x = 4$. **Check:** both sides give $\sqrt{5}$ ✓. (Still confirm the radicands stay non-negative at the answer — here $5 \geq 0$, so we're fine.)

20. Isolate: $\sqrt{x} = x - 2$. Square: $x = (x - 2)^2 = x^2 - 4x + 4$, so $x^2 - 5x + 4 = 0$ and $(x - 1)(x - 4) = 0$, giving $x = 1$ or $x = 4$. **Check** $x = 1$: $\sqrt{1} + 2 = 3 \neq 1$ ✗. **Check** $x = 4$: $\sqrt{4} + 2 = 4$ ✓. Only $x = 4$ valid.

21. Set $\sqrt{2.5r} = 30$. Square: $2.5r = 900$, so $r = 360$ ft. **Check:** $\sqrt{2.5(360)} = \sqrt{900} = 30$ ✓. Valid. (A 30-mph corner needs a 360-ft radius — a moderately long curve, which matches highway-design tables.)

22. Isolate the radical first: $\sqrt{2x + 3} = 3 - x$. The radical is non-negative, so the right side must be too: $3 - x \geq 0$, giving $x \leq 3$. Domain of the radicand: $2x + 3 \geq 0$, so $x \geq -\frac{3}{2}$. **Square both sides:** $2x + 3 = (3 - x)^2 = 9 - 6x + x^2$, which rearranges to $x^2 - 8x + 6 = 0$. Quadratic formula: $x = \frac{8 \pm \sqrt{64 - 24}}{2} = \frac{8 \pm \sqrt{40}}{2} = 4 \pm \sqrt{10}$. **Check** $x = 4 - \sqrt{10} \approx 0.84$: satisfies $-\frac{3}{2} \leq x \leq 3$. Plug back: $\sqrt{2(0.84) + 3} \approx \sqrt{4.68} \approx 2.16$, and $3 - 0.84 = 2.16$. Match ✓. **Check** $x = 4 + \sqrt{10} \approx 7.16$: fails $x \leq 3$. Extraneous — squaring created this root. Only $x = 4 - \sqrt{10}$ is valid.

23. Set $\frac{\pi}{4}\sqrt{L} = 2$, so $\sqrt{L} = \frac{8}{\pi}$. Square: $L = \frac{64}{\pi^2} \approx 6.48$ ft. **Check:** $T = \frac{\pi}{4}\sqrt{\frac{64}{\pi^2}} = \frac{\pi}{4} \cdot \frac{8}{\pi} = 2$ ✓. (The constant $\pi/4$ encodes the physics: longer pendulums swing more slowly, but the relationship is square-root, not linear. Quadrupling the length only doubles the period — consistent with the length ratio $64/\pi^2$ producing a clean $T = 2$ s.)

24. Square both sides: $x + 11 = (x - 1) + 4\sqrt{x - 1} + 4$, so $x + 11 = x + 3 + 4\sqrt{x - 1}$. Subtract: $8 = 4\sqrt{x - 1}$, giving $\sqrt{x - 1} = 2$ and $x - 1 = 4$, so $x = 5$. **Check:** $\sqrt{16} = 4$ on the left; $\sqrt{4} + 2 = 2 + 2 = 4$ on the right. Match. ✓. (When two radicals appear, squaring once leaves one radical; isolating and squaring a second time finishes the job. Each squaring step is a chance for extraneous roots, so the final check is non-negotiable.)

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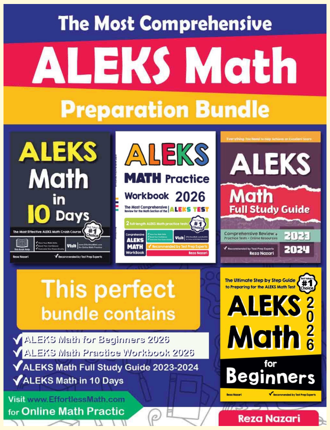

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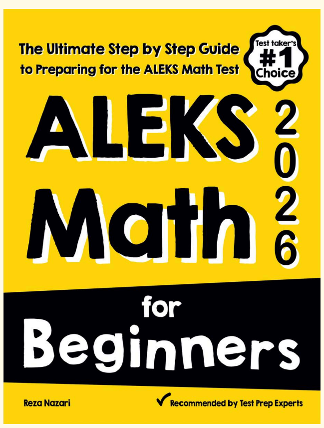

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