Domain and Range of Radical Functions

Name: ______________	Date: ______________	Score: ______ / 33

Quick Review

A **radical function** packs a variable inside a root: $f(x) = \sqrt{\text{(expression in } x)}$ or a cube root, fourth root, etc. The big question is always the same: *for which x does this formula spit out a real number?*

Square roots (and every even-index root). The radicand must be ≥ 0. So $f(x) = \sqrt{x-5}$ requires $x - 5 \geq 0$, meaning $x \geq 5$. The output is always non-negative: $\sqrt{x} \geq 0$ wherever it's defined.

Cube roots (and every odd-index root). The radicand can be anything. $f(x) = \sqrt[3]{x+7}$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$. Cubes keep signs — the function isn't bounded above or below. This is the big distinction that shows up on tests.

Shifts and stretches reshape the range. The parent $f(x) = \sqrt{x}$ has range $[0, \infty)$. A vertical shift $f(x) = \sqrt{x} + c$ shifts the floor: range $[c, \infty)$. A reflection $f(x) = -\sqrt{x}$ flips the range upside-down: $(-\infty, 0]$. A horizontal shift $\sqrt{x-h}$ moves the domain endpoint to h but leaves the range starting at 0.

Watch the sign inside. For $f(x) = \sqrt{4-x}$, set $4 - x \geq 0$, which flips the inequality when you isolate x: $x \leq 4$. The leading negative reverses the direction — a reliable trap on standardized tests.

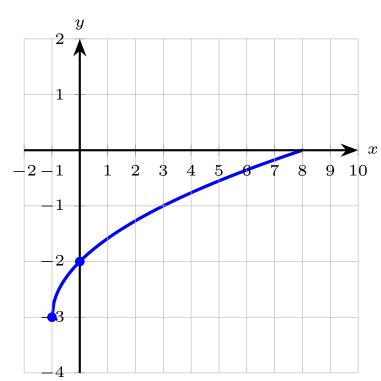
Range of a transformed square root. Trace the parent's output through the transformations. $f(x) = -2\sqrt{x+5}+1$ starts from $\sqrt{x+5} \geq 0$, then $-2\sqrt{x+5} \leq 0$, then $-2\sqrt{x+5} + 1 \leq 1$. Range: $(-\infty, 1]$.

Common slips. Treating cube roots like square roots (they don't need a non-negative radicand). Forgetting to flip the inequality when the radicand has a negative coefficient on x. Saying the range of $\sqrt{x}$ is "all reals".

PRACTICE

Find domain and range for each radical function. Track the parent's transformations.

1. Find the domain of $f(x) = \sqrt{x-5}$. ________
2. Find the range of $f(x) = \sqrt{x}$. ________
3. Find the domain of $g(x) = \sqrt{2x+6}$. ________
4. Find the domain of $h(x) = \sqrt{4-x}$. ________
5. Find the domain of $f(x) = \sqrt[3]{x+7}$. ________
6. The graph shows $f(x) = \sqrt{x+1} - 3$, the parent curve shifted. Two points are marked to anchor it. Use the graph to find $f(8)$. ________



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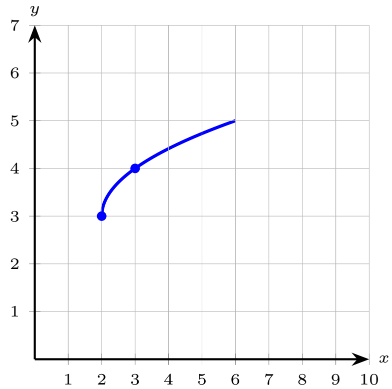
7. The graph shows $f(x) = \sqrt{x-2} + 3$. Use it to find $f(6)$, the height of the curve at $x = 6$. ______



8. Mark TRUE or FALSE: The range of $f(x) = \sqrt{x}$ is all real numbers. ______

9. Find domain and range of $f(x) = -2\sqrt{x+5} + 1$. ______

10. Find the domain of $g(x) = \sqrt{9-2x} + 4$. ______

11. Find the range of $f(x) = \sqrt[3]{x} - 1$. ______

12. The table lists values of $f(x) = \sqrt{x-1}$. Using the pattern, find $f(17)$. ______

x	1	2	5	10
$f(x)$	0	1	2	3

13. Find the range of $g(x) = -\sqrt{x-1}$. ______

14. Find domain and range of $f(x) = \sqrt[4]{x-3}$. ______

15. Find the domain of $f(x) = \sqrt{x^2 - 9}$. ______

16. Mark TRUE or FALSE: The cube-root function requires a non-negative radicand. ______

17. Find the range of $f(x) = 3\sqrt{x} + 5$. ______

18. Find the domain of $f(x) = \sqrt{-x+8}$. ______

19. Find domain and range of $f(x) = \sqrt{x-4} + 2$. ______

20. Find the domain of $f(x) = \sqrt[3]{2x-1} + 5$. ______

Word Problems

21. The free-fall time (seconds) from a height h (meters) is $t(h) = \sqrt{h/4.9}$. For which heights h is the model defined, and what's the range of times it produces? ____

22. A speed-from-skid-mark formula is $v(s) = 2\sqrt{5s}$ (mph), where s is the skid-mark length in feet. Find the domain and range that make physical sense. For what speed range does the formula work on skids up to 80 ft? ____

23. For the function $f(x) = \sqrt{16 - x^2}$, find the domain and range. ____

24. A current-vs-temperature model is $I(T) = \sqrt[3]{T-12} + 5$, where T is in °C. Find the domain and range. What current does the model give at $T = 20$°C? ____

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Additional Practice

25. Simplify $\sqrt{72}$. ________

26. Simplify $\sqrt{45}$. ________

27. Simplify $\sqrt[3]{64}$. ________

28. Solve $\sqrt{x+5} = 9$. ________

29. Solve $\sqrt{x} - 3 = 4$. ________

30. Domain of $y = \sqrt{x-6}$. ________

31. Add $3\sqrt{5} + 2\sqrt{5}$. ________

32. Multiply $\sqrt{3} \cdot \sqrt{12}$. ________

33. Rationalize $\frac{4}{\sqrt{2}}$. ________

Answer Keys

1. $x \geq 5$
2. $y \geq 0$
3. $x \geq -3$
4. $x \leq 4$
5. all real numbers
6. 0
7. 5
8. FALSE
9. domain: $x \geq -5$; range: $y \leq 1$
10. $x \leq \frac{9}{2}$
11. all real numbers
12. 4
13. $y \leq 0$
14. domain: $x \geq 3$; range: $y \geq 0$
15. $x \leq -3$ or $x \geq 3$
16. FALSE
17. $y \geq 5$
18. $x \leq 8$
19. domain: $x \geq 4$; range: $y \geq 2$
20. all real numbers
21. domain: $h \geq 0$; range: $t \geq 0$
22. domain: $0 \leq s \leq 80$; speed up to 40 mph
23. domain: $-4 \leq x \leq 4$; range: $0 \leq y \leq 4$
24. domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$; $I(20) = 7$

Additional Practice Answers

25. $6\sqrt{2}$
26. $3\sqrt{5}$
27. 4
28. $x = 76$
29. $x = 49$
30. $x \geq 6$
31. $5\sqrt{5}$
32. 6
33. $2\sqrt{2}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A square root needs a non-negative radicand to stay real, so set $x - 5 \geq 0$ and solve: $x \geq 5$. Any smaller x would put a negative under the root. In interval form that's $[5, \infty)$.

2. The principal square root never returns a negative value, so its outputs start at 0 (reached at $x = 0$) and climb without bound. The range is $[0, \infty)$ — saying "all reals" is the classic range mistake here.

3. Set the radicand non-negative: $2x + 6 \geq 0$. Subtract 6 to get $2x \geq -6$, then divide by the positive 2 (no flip) for $x \geq -3$. So the domain is $[-3, \infty)$.

4. Start with the key idea: $4 - x \geq 0$, so $-x \geq -4$. Dividing by -1 flips the inequality: $x \leq 4$. The flip is the move most students miss. That gives a quick check on the answer.

5. Cube roots accept every real input. The domain is $(-\infty, \infty)$. (This is the key cube-root vs. square-root distinction.)

6. At $x = 8$: $f(8) = \sqrt{8+1} - 3 = \sqrt{9} - 3 = 3 - 3 = 0$, so the curve crosses the x-axis there. The marked points $(-1, -3)$ (the starting corner) and $(0, -2)$ trace the -3 vertical shift of the parent $\sqrt{x}$.

7. At $x = 6$: $f(6) = \sqrt{6-2} + 3 = \sqrt{4} + 3 = 2 + 3 = 5$. The marked corner $(2, 3)$ is where the curve begins (its domain edge), and $(3, 4)$ shows the first unit of rise off that corner.

8. Start with the key idea: Range is $[0, \infty)$ only. The principal square root is never negative. That gives a quick check on the answer.

9. Domain: $x + 5 \geq 0$, so $x \geq -5$. Range: $\sqrt{x+5} \geq 0 \Rightarrow -2\sqrt{x+5} \leq 0 \Rightarrow -2\sqrt{x+5} + 1 \leq 1$. The negative coefficient flips the bound from *floor* to *ceiling*.

10. Only the radicand matters, so set $9 - 2x \geq 0$, giving $-2x \geq -9$. Dividing by the negative -2 flips the inequality: $x \leq \frac{9}{2}$. The $+4$ outside the root never affects the domain. Watch that sign flip — it's the step most students miss.

11. Cube root sweeps every real value, so $\sqrt[3]{x} - 1$ also covers $(-\infty, \infty)$. (No domain or range bound with odd-index radicals.)

12. Each output is $\sqrt{x-1}$: $f(10) = \sqrt{9} = 3$. For $x = 17$, $f(17) = \sqrt{17-1} = \sqrt{16} = 4$. (The table deliberately omits $x = 17$ — the inputs $1, 2, 5, 10, 17$ are exactly one more than the perfect squares $0, 1, 4, 9, 16$.)

13. Start from the parent: $\sqrt{x-1} \geq 0$ always. The leading minus sign reflects every output across the x-axis, so $-\sqrt{x-1} \leq 0$. The largest value is 0 (when $x = 1$), and outputs go down from there, giving range $(-\infty, 0]$.

14. A fourth root has an even index, so it behaves like a square root: the radicand must be ≥ 0, giving $x - 3 \geq 0$, i.e. $x \geq 3$. And an even root is never negative, so the outputs start at 0 and rise: range $y \geq 0$.

15. Need $x^2 - 9 \geq 0$, so $x^2 \geq 9$. Taking square roots gives $|x| \geq 3$, which splits into $x \leq -3$ or $x \geq 3$. A squared variable inside the root carves the domain into two separate rays — don't write just $x \geq 3$ and lose the negative branch.

16. Start with the key idea: Cube roots accept any real number. Even-index roots are the picky ones. That gives a quick check on the answer.

17. Build up from the parent: $\sqrt{x} \geq 0$, so multiplying by 3 keeps it ≥ 0, then adding 5 shifts the floor up: $3\sqrt{x} + 5 \geq 5$. The smallest output is 5 (at $x = 0$), so the range is $[5, \infty)$.

18. Set the radicand non-negative: $-x + 8 \geq 0$, so $-x \geq -8$. Dividing (or multiplying) by -1 flips the inequality: $x \leq 8$. The negative coefficient on x is what reverses the direction — a reliable test trap.

19. Domain comes from the radicand: $x - 4 \geq 0$, so $x \geq 4$. For the range, start at $\sqrt{x-4} \geq 0$ and add 2, raising the floor to $\sqrt{x-4} + 2 \geq 2$. The lowest output 2 happens right at the domain edge $x = 4$.

20. Cube roots are defined for every real radicand; the $+5$ outside doesn't restrict anything. Domain: $(-\infty, \infty)$.

21. A radicand must be non-negative: $h/4.9 \geq 0$, which gives $h \geq 0$ (heights can't be negative anyway in this context). The square root output is always ≥ 0, so $t \geq 0$. (Both bounds are physical: you can't have a negative height or a negative fall time.)

22. The radicand $5s \geq 0$ requires $s \geq 0$, and the physical cap is $s \leq 80$ ft. At $s = 80$: $v = 2\sqrt{400} = 2 \cdot 20 = 40$ mph. So speeds run from 0 to 40 mph over this skid-length range. (Forensic investigators use variants of this formula; the radical comes from kinetic-energy / work-energy reasoning.)

23. Domain: $16 - x^2 \geq 0$, so $x^2 \leq 16$ and $-4 \leq x \leq 4$. Range: $\sqrt{16 - x^2}$ is smallest when $16 - x^2 = 0$ (giving 0) and largest when $16 - x^2 = 16$ (at $x = 0$, giving 4). So $0 \leq y \leq 4$. (Geometrically this is the upper half of a circle of radius 4.)

24. Cube roots accept any real radicand, so the domain is all reals; the range is also all reals because $\sqrt[3]{\cdot}$ is unbounded both ways. At $T = 20$: $I = \sqrt[3]{20 - 12} + 5 = \sqrt[3]{8} + 5 = 2 + 5 = 7$. (Cube-root models show up whenever a signal is expected to handle both positive and negative shifts symmetrically — no domain headache.)

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