

# Simplifying Radicals with Fractions

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: \_\_\_\_\_ / 34

## Q Quick Review

Two new tools join the radical kit when fractions appear: the **quotient rule** and **rationalizing** a denominator.

**Quotient rule.** For  $a \geq 0$  and  $b > 0$ ,  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ . So  $\sqrt{\frac{9}{16}} = \frac{3}{4}$  and  $\sqrt{\frac{50}{9}} = \frac{5\sqrt{2}}{3}$ . (Same rule going backward:  $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ , which often collapses an ugly quotient into a perfect square.)

**Why rationalize?** A simplified-radical answer never leaves a radical in the denominator. It's a tidiness convention — it makes answers easier to compare and decimals easier to compute by hand. To clear a single  $\sqrt{c}$  from a denominator, multiply numerator and denominator by  $\sqrt{c}$ :

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

**Binomial denominators — use the conjugate.** The *conjugate* of  $a + \sqrt{c}$  is  $a - \sqrt{c}$  (and vice versa). Multiplying by the conjugate triggers the difference-of-squares pattern  $(a + b)(a - b) = a^2 - b^2$ , which kills the radical:  $\frac{1}{\sqrt{5} - 2} \cdot \frac{\sqrt{5} + 2}{\sqrt{5} + 2} = \frac{\sqrt{5} + 2}{5 - 4} = \sqrt{5} + 2$ .

**Order matters in the sign step.** When the denominator after the conjugate step is negative (e.g.  $4 - 7 = -3$  in  $\frac{3}{2 + \sqrt{7}} \cdot \frac{2 - \sqrt{7}}{2 - \sqrt{7}}$ ), the minus sign flips both terms in the numerator. Don't lose it.

**Common slips.** Leaving  $\sqrt{\quad}$  in the denominator. Forgetting that the conjugate only flips the sign on the radical term, not the rational one.

Writing  $\sqrt{\frac{a}{b}} = \sqrt{a} - \sqrt{b}$  (wrong — radicals don't split over addition or subtraction).

## PRACTICE

Simplify and rationalize. The final form should never have a radical in the denominator.

1. Simplify  $\sqrt{\frac{9}{16}}$ . \_\_\_\_\_

2. Rationalize  $\frac{1}{\sqrt{2}}$ . \_\_\_\_\_

3. Simplify  $\sqrt{\frac{50}{9}}$ . \_\_\_\_\_

4. Rationalize  $\frac{6}{\sqrt{3}}$ . \_\_\_\_\_

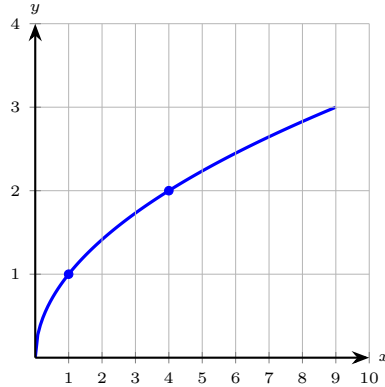
5. Rationalize  $\frac{1}{\sqrt{5} - 2}$ . \_\_\_\_\_

6. Simplify  $\frac{\sqrt{12x^3}}{\sqrt{3x}}$  for  $x > 0$ . \_\_\_\_\_



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7. First simplify  $\frac{\sqrt{45}}{\sqrt{5}}$  to a single number  $N$ . Then use the graph of  $f(x) = \sqrt{x}$  to read off  $f(N)$ . \_\_\_\_\_



8. Mark TRUE or FALSE: A radical fraction is fully simplified even if a radical remains in the denominator. \_\_\_\_\_

9. Rationalize  $\frac{3}{2 + \sqrt{7}}$ . \_\_\_\_\_

10. Simplify  $\frac{\sqrt{80r^5}}{\sqrt{5r}}$  for  $r > 0$ . \_\_\_\_\_

11. Simplify  $\sqrt{\frac{8}{25}}$ . \_\_\_\_\_

12. Rationalize  $\frac{4}{\sqrt{6}}$ . \_\_\_\_\_

13. Rationalize  $\frac{2}{\sqrt{3} + 1}$ . \_\_\_\_\_

14. Simplify  $\sqrt{\frac{49a^2}{25}}$  for  $a \geq 0$ . \_\_\_\_\_

15. Mark TRUE or FALSE:  $\sqrt{\frac{a}{b}} = \sqrt{a} - \sqrt{b}$  for  $a, b \geq 0$ . \_\_\_\_\_

16. Rationalize  $\frac{5}{\sqrt{10}}$ . \_\_\_\_\_

17. The table gives values of  $g(x) = \sqrt{x}$  at several perfect squares. Using the pattern, find  $g(64)$ . \_\_\_\_\_

$x$	4	9	25	36
$g(x)$	2	3	5	6

18. Rationalize  $\frac{\sqrt{2}}{\sqrt{3}}$ . \_\_\_\_\_

19. Rationalize  $\frac{\sqrt{2}}{\sqrt{5} - \sqrt{3}}$ . \_\_\_\_\_

20. Simplify  $\frac{\sqrt{27}}{\sqrt{75}}$ . \_\_\_\_\_



## ◆ Word Problems

21. A right triangle has legs of length 1 and  $\sqrt{3}$  units. Find the exact ratio (hypotenuse)  $\div$  (shorter leg), in simplified radical form with no radical in the denominator. \_\_\_\_\_
22. A track shaped like a rectangle has area  $\sqrt{72}$  square units and width  $\sqrt{8}$  units. Find the exact length in simplified form, with no radical in the denominator. \_\_\_\_\_
23. Free-fall from a tall building takes time  $t = \sqrt{\frac{2h}{g}}$  seconds, with  $h$  in meters and  $g = 10 \text{ m/s}^2$ . For  $h = 45 \text{ m}$ , give  $t$  exactly in simplified radical form, then approximate to two decimals. \_\_\_\_\_
24. The golden-ratio expression  $\frac{1 + \sqrt{5}}{2}$  appears in many places. Find its reciprocal in the form  $a + b\sqrt{5}$  with no radical in the denominator. \_\_\_\_\_

## Additional Practice

25. Simplify  $\sqrt{72}$ . \_\_\_\_\_
26. Simplify  $\sqrt{45}$ . \_\_\_\_\_
27. Simplify  $\sqrt[3]{64}$ . \_\_\_\_\_
28. Solve  $\sqrt{x + 5} = 9$ . \_\_\_\_\_
29. Solve  $\sqrt{x} - 3 = 4$ . \_\_\_\_\_
30. Domain of  $y = \sqrt{x - 6}$ . \_\_\_\_\_
31. Add  $3\sqrt{5} + 2\sqrt{5}$ . \_\_\_\_\_
32. Multiply  $\sqrt{3} \cdot \sqrt{12}$ . \_\_\_\_\_
33. Rationalize  $\frac{4}{\sqrt{2}}$ . \_\_\_\_\_
34. Write  $x^{3/2}$  using radicals. \_\_\_\_\_



Answer Keys

1. $\frac{3}{4}$	13. $\frac{\sqrt{3}-1}{2}$
2. $\frac{\sqrt{2}}{2}$	14. $\frac{7a}{5}$
3. $\frac{5\sqrt{2}}{3}$	15. FALSE
4. $2\sqrt{3}$	16. $\frac{\sqrt{10}}{2}$
5. $\sqrt{5}+2$	17. 8
6. $2x$	18. $\frac{\sqrt{6}}{3}$
7. $f(9) = 3$	19. $\frac{\sqrt{10} + \sqrt{6}}{2}$
8. FALSE	20. $\frac{3}{5}$
9. $\sqrt{7}-2$	21. 2
10. $4r^2$	22. 3
11. $\frac{2\sqrt{2}}{5}$	23. 3 s (exact), $\approx 3.00$ s
12. $\frac{2\sqrt{6}}{3}$	24. $-\frac{1}{2} + \frac{1}{2}\sqrt{5}$

**Additional Practice Answers**

25. $6\sqrt{2}$	30. $x \geq 6$
26. $3\sqrt{5}$	31. $5\sqrt{5}$
27. 4	32. 6
28. $x = 76$	33. $2\sqrt{2}$
29. $x = 49$	34. $\sqrt{x^3}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- Quotient rule:  $\frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$ . Both numerator and denominator are perfect squares, so nothing's left under the radical.
- Multiply top and bottom by  $\sqrt{2}$ :  $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ . The denominator becomes  $(\sqrt{2})^2 = 2$ .
- Split:  $\frac{\sqrt{50}}{3}$ . Then  $\sqrt{50} = 5\sqrt{2}$ , so the answer is  $\frac{5\sqrt{2}}{3}$ . The denominator is already a perfect square — no further rationalizing needed.
- Start with the key idea:  $\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$ . Reducing  $\frac{6}{3}$  at the end gives a clean integer coefficient. That gives a quick check on the answer.
- Conjugate of  $\sqrt{5}-2$  is  $\sqrt{5}+2$ . Multiply:  $\frac{1}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$ . The difference-of-squares pattern wipes the radical from below.
- Use the quotient rule backward to merge the two radicals into one:  $\frac{\sqrt{12x^3}}{\sqrt{3x}} = \sqrt{\frac{12x^3}{3x}} = \sqrt{4x^2} = 2|x|$ . Since  $x > 0$  is given,  $|x| = x$ , so the answer is  $2x$ . Combining first turns a messy quotient into a clean perfect square.
- Combine first:  $\frac{\sqrt{45}}{\sqrt{5}} = \sqrt{\frac{45}{5}} = \sqrt{9} = 3$ , so  $N = 9$ . Now trace the curve at  $x = 9$ : the height is 3, i.e.  $f(9) = \sqrt{9} = 3$ . (Combining the radicals first turns an ugly quotient into a clean integer.)
- Standard simplified form clears every radical out of the denominator. (Calculators don't care, but the convention makes hand-comparison easy.)
- Conjugate =  $2 - \sqrt{7}$ . Multiply:  $\frac{3(2 - \sqrt{7})}{4 - 7} = \frac{3(2 - \sqrt{7})}{-3} = -(2 - \sqrt{7}) = \sqrt{7} - 2$ . The negative denominator flips both numerator terms — the sign step is

where this problem hides.

- Merge under one radical with the quotient rule:  $\frac{\sqrt{80r^5}}{\sqrt{5r}} = \sqrt{\frac{80r^5}{5r}} = \sqrt{16r^4}$ . Both 16 and  $r^4$  are perfect squares, so this is  $4r^2$ . The  $r > 0$  tag keeps the denominator nonzero and lets the root come out without bars.
- Split with the quotient rule:  $\sqrt{\frac{8}{25}} = \frac{\sqrt{8}}{\sqrt{25}} = \frac{\sqrt{8}}{5}$ . The denominator  $\sqrt{25} = 5$  is already rational, so just simplify the top:  $\sqrt{8} = 2\sqrt{2}$ , giving  $\frac{2\sqrt{2}}{5}$ . No conjugate needed here.
- Start with the key idea:  $\frac{4}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$ . Always reduce the fraction at the end. That gives a quick check on the answer.
- The conjugate of  $\sqrt{3}+1$  is  $\sqrt{3}-1$ . Multiply top and bottom by it; the denominator becomes the difference of squares  $(\sqrt{3})^2 - 1^2 = 3 - 1 = 2$ :  $\frac{2(\sqrt{3}-1)}{2} = \sqrt{3}-1$ . The radical vanishes from below, exactly what the conjugate is for.
- Apply the quotient rule, then root each perfect square:  $\sqrt{\frac{49a^2}{25}} = \frac{\sqrt{49a^2}}{\sqrt{25}} = \frac{7|a|}{5}$ . Since  $a \geq 0$  is given,  $|a| = a$ , so the answer is  $\frac{7a}{5}$  — already radical-free below.
- Radicals don't turn quotients into differences. The quotient rule says  $\sqrt{a/b} = \sqrt{a}/\sqrt{b}$  — a *quotient* of radicals, not a difference.
- Multiply top and bottom by  $\sqrt{10}$  to clear the root:  $\frac{5}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{5\sqrt{10}}{10}$ . Then reduce  $\frac{5}{10}$  to  $\frac{1}{2}$ , leaving  $\frac{\sqrt{10}}{2}$ . Always reduce the fraction after rationalizing.



17. Every entry is a perfect square mapped to its root:  $g(36) = 6$  since  $6^2 = 36$ . As  $64 = 8^2$ , the pattern continues to  $g(64) = 8$ . (The table omits  $x = 64$  on purpose — extend the pattern rather than reading it off.)

18. Multiply top and bottom by  $\sqrt{3}$  to clear the denominator:  $\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{2} \cdot \sqrt{3}}{3} = \frac{\sqrt{6}}{3}$ . The two top radicals merge by the product rule into  $\sqrt{6}$ , and the bottom becomes the rational 3.

19. The conjugate of  $\sqrt{5} - \sqrt{3}$  is  $\sqrt{5} + \sqrt{3}$ . Multiply top and bottom by it; the denominator becomes the difference of squares  $(\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$ . The numerator distributes:  $\sqrt{2} \cdot \sqrt{5} + \sqrt{2} \cdot \sqrt{3} = \sqrt{10} + \sqrt{6}$ , giving  $\frac{\sqrt{10} + \sqrt{6}}{2}$ .

20. Combine under one radical first:  $\frac{\sqrt{27}}{\sqrt{75}} = \sqrt{\frac{27}{75}}$ . Reduce the fraction  $\frac{27}{75} = \frac{9}{25}$ , which exposes a perfect square on top and bottom, so  $\sqrt{\frac{9}{25}} = \frac{3}{5}$ . Reducing before rooting saves you from simplifying two ugly radicals.

21. By the Pythagorean theorem, hypotenuse =  $\sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$ . The ratio is  $\frac{2}{1} = 2$ , already radical-free. (This is the 30-60-90 triangle in disguise — worth recognizing.)

22. Length =  $\frac{\sqrt{72}}{\sqrt{8}} = \sqrt{\frac{72}{8}} = \sqrt{9} = 3$ . Combine the radicals before simplifying — the quotient under the radical reduces to a perfect square. (Try it the long way:  $\sqrt{72} = 6\sqrt{2}$ ,  $\sqrt{8} = 2\sqrt{2}$ , ratio =  $6\sqrt{2}/(2\sqrt{2}) = 3$ . Same answer, more steps.)

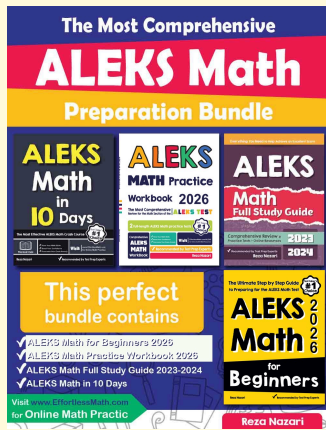
23. Plug in:  $t = \sqrt{\frac{2(45)}{10}} = \sqrt{9} = 3$  s exactly. Decimal: 3.00 s. (The numbers were chosen to give a clean integer — when the radicand reduces this smoothly, double-check the problem expects an integer answer.)

24. Reciprocal =  $\frac{2}{1 + \sqrt{5}}$ . Multiply by the conjugate  $1 - \sqrt{5}$ :  $\frac{2(1 - \sqrt{5})}{(1)^2 - (\sqrt{5})^2} = \frac{2(1 - \sqrt{5})}{1 - 5} = \frac{2(1 - \sqrt{5})}{-4} = \frac{\sqrt{5} - 1}{2} = -\frac{1}{2} + \frac{1}{2}\sqrt{5}$ . (Cute fact: this is exactly  $\varphi - 1$ , where  $\varphi = (1 + \sqrt{5})/2$  is the golden ratio. Its reciprocal differs from  $\varphi$  by exactly 1.)



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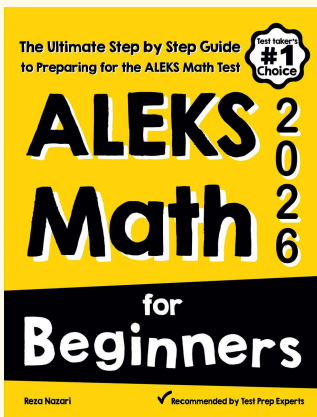
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