

# Solving a Quadratic Equation

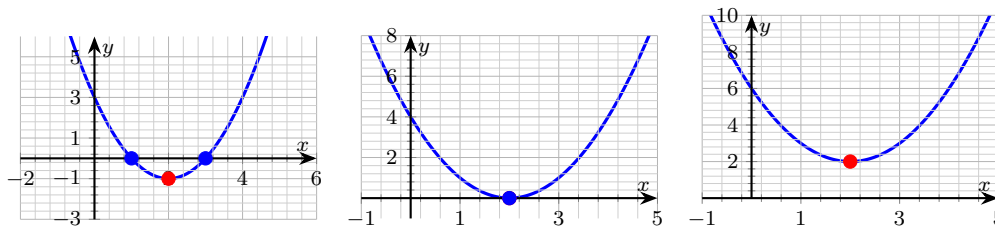
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## Q Quick Review

A **quadratic equation** is anything that can be rearranged into the form  $ax^2 + bx + c = 0$  with  $a \neq 0$ . Four reliable tools solve them, and the smart move is to size up the equation before reaching for one. **Factoring** is fastest when the trinomial splits cleanly — look for two numbers that multiply to  $ac$  and add to  $b$ . **Square roots** work whenever you can isolate a perfect square: from  $(x - 3)^2 = 16$  jump straight to  $x - 3 = \pm 4$ . **Completing the square** is the all-purpose workhorse and is how the next tool was derived in the first place. The **quadratic formula**  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  never fails.

The **discriminant**  $\Delta = b^2 - 4ac$  tells you how many real roots before you commit: positive  $\Rightarrow$  two real, zero  $\Rightarrow$  one repeated, negative  $\Rightarrow$  two complex conjugates (the parabola never touches the  $x$ -axis). Two traps to flag: the sign on  $b$  inside the formula —  $-b$  with  $b = -4$  is  $+4$ , not  $-4$  — and the order of operations under the radical (square  $b$  first, then subtract  $4ac$ ). Always plug your roots back into the original equation. It's the quickest way to catch a sign slip.

Three pictures, one for each discriminant sign:

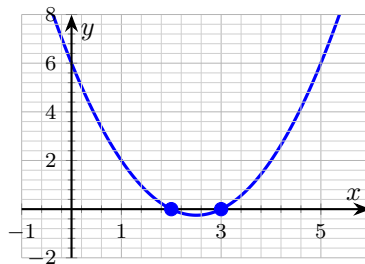


Two real roots ( $\Delta > 0$ ), one double root ( $\Delta = 0$ ), no real roots ( $\Delta < 0$ ).

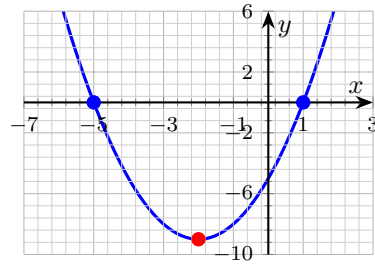
## PRACTICE

Solve each quadratic. State both roots (real or complex).

- Solve  $x^2 - 5x + 6 = 0$ . Confirm with the graph.

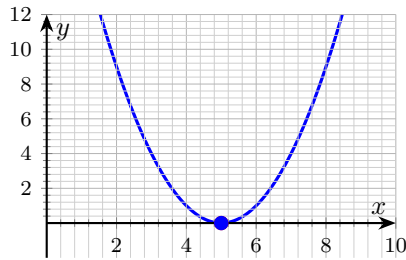


2. Solve  $x^2 + 4x - 5 = 0$ . Use the graph as a sanity check. \_\_\_\_\_

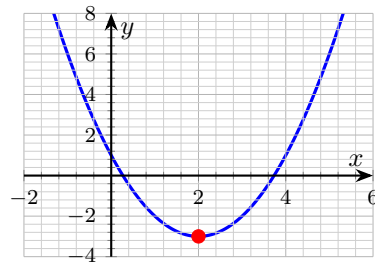


3.  $2x^2 - 5x - 3 = 0$  \_\_\_\_\_

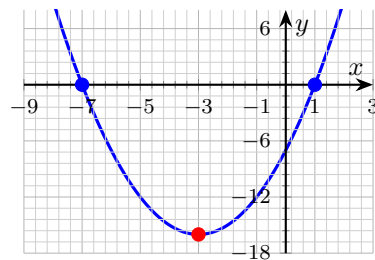
4. Solve  $x^2 - 10x + 25 = 0$ . (Graph is tangent.) \_\_\_\_\_



5. Solve  $x^2 - 4x + 1 = 0$  (irrational roots). \_\_\_\_\_



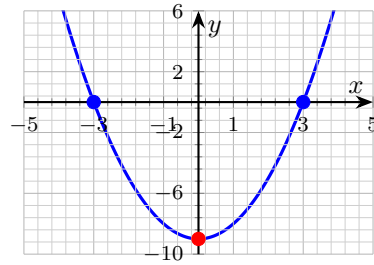
6. Solve  $x^2 + 6x - 7 = 0$  and check against the graph. \_\_\_\_\_



7.  $3x^2 + 2x - 7 = 0$  \_\_\_\_\_

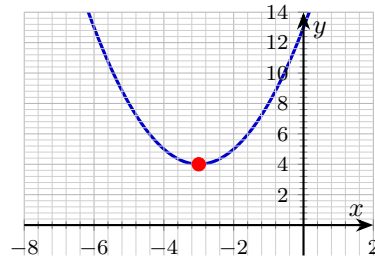


8. Solve  $x^2 - 9 = 0$ . Graph confirms the roots. \_\_\_\_\_

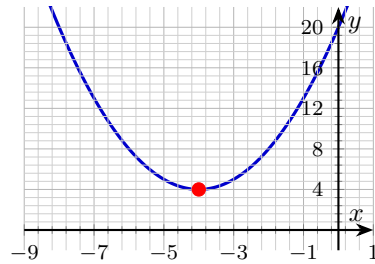


9.  $3x^2 + 2x + 5 = 0$  \_\_\_\_\_

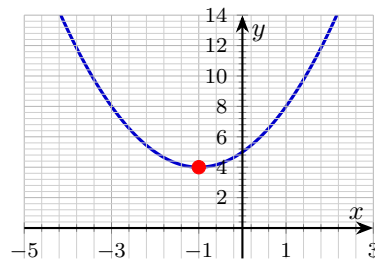
10. Solve  $x^2 + 6x + 13 = 0$ . (The graph confirms complex roots.) \_\_\_\_\_



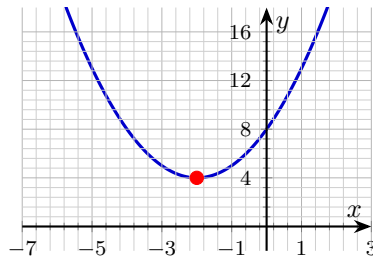
11. Solve  $x^2 + 8x + 20 = 0$ . Graph shown. \_\_\_\_\_



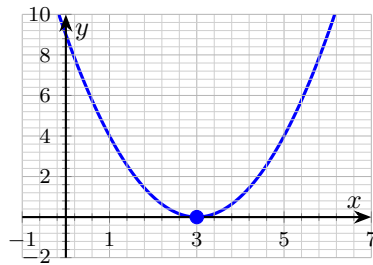
12. Solve  $x^2 + 2x + 5 = 0$ . \_\_\_\_\_



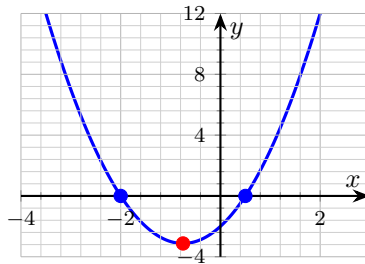
13. Solve  $x^2 + 4x + 8 = 0$ . (Notice the vertex's height.) \_\_\_\_\_



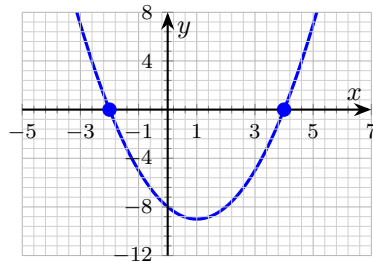
14. Solve  $x^2 - 6x + 9 = 0$ . (Graph is tangent to the  $x$ -axis.) \_\_\_\_\_



15. Solve  $2x^2 + 3x - 2 = 0$  and use the graph as a check. \_\_\_\_\_



16. Solve  $x^2 - 2x - 8 = 0$ . (Graph shows the two real roots.) \_\_\_\_\_

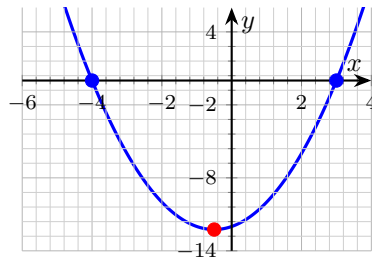


17. How many real roots:  $2x^2 + 5x + 4 = 0$ ? \_\_\_\_\_

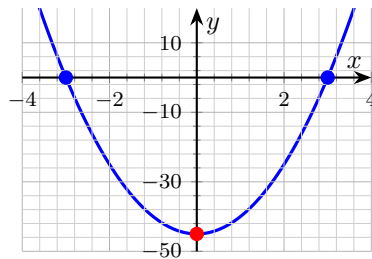
18. How many real roots:  $4x^2 - 12x + 9 = 0$ ? \_\_\_\_\_



19. Solve  $x^2 + x - 12 = 0$ . Confirm the roots from the graph. \_\_\_\_\_



20. Solve  $5x^2 = 45$ . Check with the graph of  $y = 5x^2 - 45$ . \_\_\_\_\_



◆ Word Problems

- 21. A small launched object has height  $h(t) = -16t^2 + 48t + 64$  feet, where  $t$  is seconds after launch. At what positive time does it hit the ground? \_\_\_\_\_
- 22. A rectangular swimming pool has length 2 meters more than twice its width. The area is  $60 \text{ m}^2$ . Find the width. \_\_\_\_\_
- 23. A rectangular garden has length 6 feet more than twice its width. Its area is  $140 \text{ ft}^2$ . Find the width. \_\_\_\_\_
- 24. The product of two consecutive positive integers is 132. Find the integers. \_\_\_\_\_

Additional Practice

- 25. Solve  $x^2 - 5x + 6 = 0$ . \_\_\_\_\_
- 26. Solve  $x^2 = 49$ . \_\_\_\_\_
- 27. Find the vertex of  $y = (x - 3)^2 - 4$ . \_\_\_\_\_
- 28. Find the axis of symmetry of  $y = x^2 + 6x + 1$ . \_\_\_\_\_



## Answer Keys

<p>1. <math>x = 2, 3</math></p> <p>2. <math>x = 1, -5</math></p> <p>3. <math>x = 3, -\frac{1}{2}</math></p> <p>4. <math>x = 5</math> (double root)</p> <p>5. <math>x = 2 \pm \sqrt{3}</math></p> <p>6. <math>x = 1, -7</math></p> <p>7. <math>x = \frac{-1 \pm \sqrt{22}}{3}</math></p> <p>8. <math>x = \pm 3</math></p> <p>9. <math>x = \frac{-1 \pm i\sqrt{14}}{3}</math></p> <p>10. <math>x = -3 \pm 2i</math></p> <p>11. <math>x = -4 \pm 2i</math></p> <p>12. <math>x = -1 \pm 2i</math></p> <p><b>Additional Practice Answers</b></p> <p>25. <math>x = 2, 3</math></p> <p>26. <math>x = -7, 7</math></p>	<p>13. <math>x = -2 \pm 2i</math></p> <p>14. <math>x = 3</math> (double root)</p> <p>15. <math>x = \frac{1}{2}, -2</math></p> <p>16. <math>x = 4, -2</math></p> <p>17. 0 (two complex)</p> <p>18. 1 (double root)</p> <p>19. <math>x = 3, -4</math></p> <p>20. <math>x = \pm 3</math></p> <p>21. <math>t = 4</math> seconds</p> <p>22. 5 meters</p> <p>23. 7 feet</p> <p>24. 11 and 12</p> <p>27. <math>(3, -4)</math></p> <p>28. <math>x = -3</math></p>
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**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

### Step-by-Step Explanations

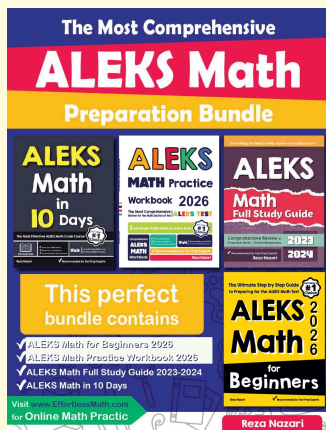
1. Two numbers that multiply to 6 and add to -5: -2 and -3. Factor:  $(x - 2)(x - 3) = 0$ , so  $x = 2$  or  $x = 3$ . The graph crosses the  $x$ -axis at exactly those values.
2. Two numbers multiplying to -5 and adding to 4: 5 and -1. Factor:  $(x - 1)(x + 5) = 0$ . The signs flip from factor to root — that's the part students miss.
3. Try factoring:  $(2x + 1)(x - 3) = 0$ . So  $2x + 1 = 0$  gives  $x = -\frac{1}{2}$ , and  $x - 3 = 0$  gives  $x = 3$ . Quadratic formula confirms:  $x = \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5 \pm 7}{4}$ , matching 3 and  $-\frac{1}{2}$ .
4. Perfect square:  $(x - 5)^2 = 0$ , so  $x = 5$  shows up twice. Discriminant  $100 - 100 = 0$  — the signature of a repeated root.
5. Doesn't factor over the integers, so reach for the formula:  $x = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$ . The  $\pm 2\sqrt{3}$  over 2 reduces cleanly — watch that step or you'll leave a 2 behind.
6. Two numbers multiplying to -7 and adding to 6: 7 and -1. Factor:  $(x + 7)(x - 1) = 0$ , so  $x = -7$  or  $x = 1$ .
7. Doesn't factor. Use the formula with  $a = 3, b = 2, c = -7$ :  $x = \frac{-2 \pm \sqrt{4 + 84}}{6} = \frac{-2 \pm \sqrt{88}}{6} = \frac{-2 \pm 2\sqrt{22}}{6} = \frac{-1 \pm \sqrt{22}}{3}$ .
8. Difference of squares:  $(x - 3)(x + 3) = 0$ , so  $x = \pm 3$ . (Equivalently,  $x^2 = 9$  gives  $x = \pm\sqrt{9} = \pm 3$ .)
9. Discriminant first:  $4 - 60 = -56 < 0$ . Two complex roots.  $x = \frac{-2 \pm \sqrt{-56}}{6} = \frac{-2 \pm 2i\sqrt{14}}{6} = \frac{-1 \pm i\sqrt{14}}{3}$ .
10. Keep the rule visible:  $\Delta = 36 - 52 = -16$ , so complex conjugates.  $x = \frac{-6 \pm \sqrt{-16}}{2} = \frac{-6 \pm 4i}{2} = -3 \pm 2i$ . The parabola sits entirely above the  $x$ -axis. That gives a quick check on the answer.
11. One steady path is:  $\Delta = 64 - 80 = -16$ , complex roots.  $x = \frac{-8 \pm 4i}{2} = -4 \pm 2i$ . The graph never crosses the  $x$ -axis — exactly what complex roots look like. That gives a quick check on the answer.

12. Start with the key idea:  $\Delta = 4 - 20 = -16$ .  $x = \frac{-2 \pm 4i}{2} = -1 \pm 2i$ . The parabola hovers above the  $x$ -axis. That gives a quick check on the answer.
13. A careful way to see it:  $\Delta = 16 - 32 = -16$ , complex pair.  $x = \frac{-4 \pm 4i}{2} = -2 \pm 2i$ . The vertex  $(-2, 4)$  sits above the  $x$ -axis, so no real crossings. That gives a quick check on the answer.
14. Perfect square:  $(x - 3)^2 = 0$ .  $\Delta = 36 - 36 = 0$ , so the parabola is tangent to the  $x$ -axis at  $x = 3$  — one repeated root.
15. Factor:  $(2x - 1)(x + 2) = 0$ . So  $x = \frac{1}{2}$  or  $x = -2$ . Quadratic formula confirms:  $x = \frac{-3 \pm \sqrt{9 + 16}}{4} = \frac{-3 \pm 5}{4}$ .
16. Factor:  $(x - 4)(x + 2) = 0$ . The parabola crosses the  $x$ -axis at the two roots, exactly where the factors vanish.
17. Discriminant:  $25 - 32 = -7 < 0$ . Negative  $\Rightarrow$  no real roots. The parabola never reaches the  $x$ -axis.
18. Discriminant:  $144 - 144 = 0$ . A zero discriminant means one repeated real root (the vertex sits on the  $x$ -axis).
19. Two numbers multiplying to -12 and adding to 1: 4 and -3. Factor:  $(x - 3)(x + 4) = 0$ , giving  $x = 3$  or  $x = -4$ .
20. Divide both sides by 5:  $x^2 = 9$ . Square roots:  $x = \pm 3$ . (Don't divide 45 by 5 to get 9 and then forget the  $\pm$  — both roots count.)
21. Ground means  $h = 0$ . Set  $-16t^2 + 48t + 64 = 0$ , divide by -16 to clean up:  $t^2 - 3t - 4 = 0$ . Factor:  $(t - 4)(t + 1) = 0$ , so  $t = 4$  or  $t = -1$ . Time can't be negative here, so the answer is  $t = 4$  seconds.
22. Let  $w$  be the width; length is  $2w + 2$ . Area:  $w(2w + 2) = 60$ , so  $2w^2 + 2w - 60 = 0$ , then  $w^2 + w - 30 = 0$ . Factor:  $(w + 6)(w - 5) = 0$ , giving  $w = 5$  or  $w = -6$ . Width can't be negative, so  $w = 5$  meters. (Length is  $2(5) + 2 = 12$ , and  $5 \times 12 = 60$  checks out.)
23. Width  $w$ , length  $2w + 6$ . Area:  $w(2w + 6) = 140$ , so  $2w^2 + 6w - 140 = 0$ , then  $w^2 + 3w - 70 = 0$ . Factor:  $(w + 10)(w - 7) = 0$ , so  $w = 7$  or  $w = -10$ . Toss the negative; width is 7 feet. Check:  $7 \times (2(7) + 6) = 7 \times 20 = 140$ .
24. Let the integers be  $n$  and  $n + 1$ . Then  $n(n + 1) = 132$ , so  $n^2 + n - 132 = 0$ . Factor:  $(n - 11)(n + 12) = 0$ , giving  $n = 11$  or  $n = -12$ . Positive integers only, so  $n = 11$  and the pair is 11, 12. (Quick check:  $11 \times 12 = 132$ .)



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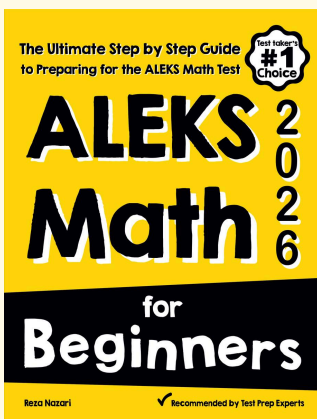
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