# Composition of Functions

| Name: ____________________ | Date: ____________ | Score: ______ / 29 |
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## Quick Review

**Composition** chains two functions: $(f \circ g)(x) = f(g(x))$ means "apply $g$ first, then feed the result into $f$." Read it inside-out. The little circle $\circ$ is not multiplication — don't confuse $(f \circ g)$ with $(f \cdot g)$.

**Order matters.** Composition is *not* commutative. Usually $(f \circ g)(x) \neq (g \circ f)(x)$. Example with $f(x) = 2x + 1$ and $g(x) = x^2$: $(f \circ g)(x) = 2x^2 + 1$, but $(g \circ f)(x) = (2x+1)^2 = 4x^2 + 4x + 1$. Different polynomials.

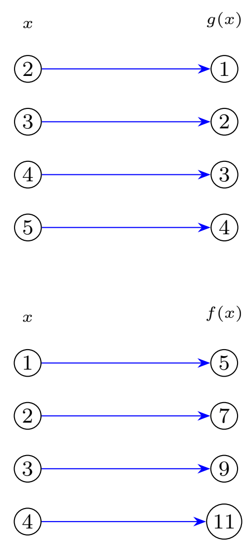
**To evaluate at a number**, work inside-out. For $(f \circ g)(4)$ with $f(x) = 2x + 3$, $g(x) = x - 1$: first $g(4) = 3$, then $f(3) = 9$. So $(f \circ g)(4) = 9$. **To build the formula**, substitute the inner expression wherever the outer function asks for $x$.

**Domain.** $(f \circ g)$ is defined wherever $x$ is in the domain of $g$ *and* $g(x)$ is in the domain of $f$. The **identity** function $I(x) = x$ is the do-nothing function: $f \circ I = I \circ f = f$. Composition is also **associative**: $(f \circ g) \circ h = f \circ (g \circ h)$. Two functions are inverses of each other when $f \circ g$ and $g \circ f$ both equal the identity — that's what an inverse *does* for a living.

## PRACTICE

*Compose the functions as indicated. Watch the order.*

**1.** The mapping diagrams show $g$ (left) and $f$ (right). Find $(f \circ g)(4)$. ________




**2.** $(f \circ g)(x)$ means: ________

**3.** $f(x) = x + 5,\ g(x) = 3x;\ (f \circ g)(x)$ ________

**4.** $f(x) = x^2,\ g(x) = x + 2;\ (f \circ g)(x)$ ________

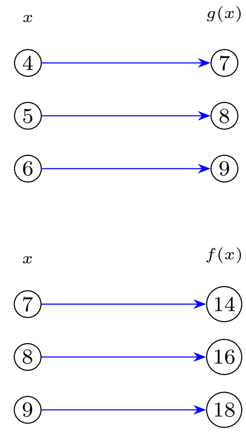
**5.** $f(x) = 2x + 1,\ g(x) = x^2;\ (g \circ f)(x)$ ________

**6.** $f(x) = 3x - 6;\ g(x)$ so that $(f \circ g)(x) = x$ ________

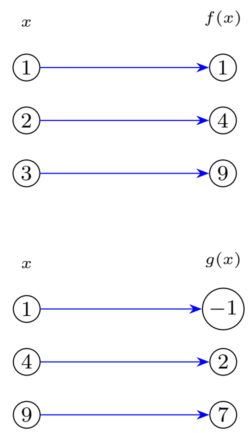
**7.** $f(x) = x^2 + 1,\ g(x) = x - 3;\ (f \circ g)(x)$ ________

**8.** $f(x) = \sqrt{x - 1},\ g(x) = 2x + 5;\ (f \circ g)(x)$ ________

**9.** The mapping diagrams show $g$ (left) and $f$ (right). Find $(f \circ g)(5)$. ________




**10.** The mapping diagrams show $f$ (left) and $g$ (right). Find $(g \circ f)(3)$. ________




**11.** $f(x) = 3x + 2,\ g(x) = x;\ (f \circ g)(x)$ ________

**12.** $f(x) = \dfrac{1}{x},\ g(x) = x + 1;\ (f \circ g)(x)$ ________

**13.** $f(x) = 2x - 1,\ g(x) = 3x + 4;\ (g \circ f)(x)$ ________

**14.** $f(x) = x^2,\ g(x) = x + 1$. Is $(f \circ g)(x)$ the same as $(g \circ f)(x)$? ________

**15.** $S(w) = 4w + 9$ shipping cost; $p(x) = x + 2$ packaging weight in pounds for an $x$-pound item. Cost in terms of $x$: ________

**16.** $f(x) = x^3,\ g(x) = x - 2;\ (f \circ g)(3)$ ________

**17.** $f(x) = 2x + 5,\ g(x) = \dfrac{x - 5}{2}$. Show $(f \circ g)(x) = x$. ________

**18.** $f(x) = x + 4,\ g(x) = 3x;\ (g \circ f)(x)$ ________

**19.** $f(x) = \sqrt{x},\ g(x) = x - 1$. Find $(f \circ g)(x)$ and the domain. ________

**20.** $f(x) = 2x + 1,\ g(x) = \dfrac{x - 1}{2}$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$. ________

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## Word Problems

**21.** A shipping company charges $S(w) = 4w + 9$ dollars to ship a package weighing $w$ pounds. A packing machine increases an item's weight by $2$ pounds before shipping, so $p(x) = x + 2$. Write the total shipping cost as a function of the original item weight $x$, then find the cost for a $5$-pound item. ____

**22.** A circle's radius after $t$ seconds is $r(t) = 2t + 1$ centimeters. Its area in terms of radius is $A(r) = \pi r^2$. Write the area as a function of time, $A(t)$, then find $A(3)$. ____

**23.** Let $f(x) = 2x - 3$ and $g(x) = \dfrac{x+3}{2}$. Show that $f$ and $g$ are inverses by computing both compositions. ____

**24.** A clothing store applies a $20\%$ discount and then charges $8\%$ sales tax. Let $d(p) = 0.80p$ be the discount step (output: discounted price) and $t(p) = 1.08p$ be the tax step. Write the total-cost function $C(p) = t(d(p))$ for an item with sticker price $p$, then find the cost of a $\$50$ item. ____

## Additional Practice

**25.** If $f(x) = 2x - 5$, find $f(4)$. ________

**26.** If $g(x) = x^2 + 1$, find $g(-3)$. ________

**27.** For $f(x) = 3x + 2$, solve $f(x) = 14$. ________

**28.** Find $(f + g)(x)$ if $f = x + 1$, $g = 2x - 5$. ________

**29.** Find $(fg)(x)$ if $f = x - 2$, $g = x + 3$. ________

# Answer Keys

1. $\boxed{9}$
2. $\boxed{f(g(x))}$
3. $\boxed{3x+5}$
4. $\boxed{(x+2)^2}$
5. $\boxed{(2x+1)^2}$
6. $\boxed{g(x)=\dfrac{x+6}{3}}$
7. $\boxed{x^2-6x+10}$
8. $\boxed{\sqrt{2x+4}}$
9. $\boxed{16}$
10. $\boxed{7}$
11. $\boxed{3x+2}$
12. $\boxed{\dfrac{1}{x+1}}$
13. $\boxed{6x+1}$
14. $\boxed{\text{no}}$
15. $\boxed{4x+17}$
16. $\boxed{1}$
17. $\boxed{x}$
18. $\boxed{3x+12}$
19. $\boxed{\sqrt{x-1},\ x\geq 1}$
20. $\boxed{\text{both equal } x}$
21. $\boxed{(S\circ p)(x)=4x+17;\ (S\circ p)(5)=\$37}$
22. $\boxed{A(t)=\pi(2t+1)^2,\ A(3)=49\pi\text{ cm}^2}$
23. $\boxed{f\circ g=g\circ f=I(x)=x}$
24. $\boxed{C(p)=0.864p,\ C(50)=\$43.20}$

### Additional Practice Answers

25. $\boxed{3}$
26. $\boxed{10}$
27. $\boxed{x=4}$
28. $\boxed{3x-4}$
29. $\boxed{x^2+x-6}$

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

**1.** Work inside-out. Trace $g$ first: the arrow from $4$ lands on $3$, so $g(4)=3$. Feed that into $f$: the arrow from $3$ lands on $9$, so $f(3)=9$. Thus $(f\circ g)(4)=9$.
**2.** Apply $g$ first, feed result into $f$. The little circle is composition, not multiplication.
**3.** One steady path is: $f(g(x))=f(3x)=3x+5$. Plug the entire $3x$ into where $x$ appeared in $f$. That gives a quick check on the answer.
**4.** Start with the key idea: $f(g(x))=f(x+2)=(x+2)^2$. Equivalently $x^2+4x+4$. This is the part to check before moving on, because it keeps the answer tied to the original question.
**5.** A careful way to see it: $g(f(x))=g(2x+1)=(2x+1)^2=4x^2+4x+1$. Different from $(f\circ g)(x)=2x^2+1$. That gives a quick check on the answer.
**6.** Keep the rule visible: $g$ must undo $f$. Solve $y=3x-6$ for $x$: $x=\dfrac{y+6}{3}$, so $g(x)=\dfrac{x+6}{3}$. Check: $f(g(x))=3\cdot\dfrac{x+6}{3}-6=x+6-6=x$. (That's the inverse.) That gives a quick check on the answer.
**7.** One steady path is: $f(x-3)=(x-3)^2+1=x^2-6x+9+1=x^2-6x+10$. This is the part to check before moving on, because it keeps the answer tied to the original question.
**8.** Start with the key idea: $f(2x+5)=\sqrt{(2x+5)-1}=\sqrt{2x+4}$. Domain: $2x+4\geq 0$, so $x\geq -2$. This is the part to check before moving on, because it keeps the answer tied to the original question.
**9.** Inside-out: $g$ sends $5$ to $8$, so $g(5)=8$. Then $f$ sends $8$ to $16$. So $(f\circ g)(5)=16$.
**10.** Apply $f$ first: $f(3)=9$. Then apply $g$ to that result: $g(9)=7$. So $(g\circ f)(3)=7$. Order matters — the inner function runs first.
**11.** One steady path is: $g$ is the identity, so $f\circ g=f$. The identity is the do-nothing function under composition. That gives a quick check on the answer.
**12.** Start with the key idea: $f(x+1)=\dfrac{1}{x+1}$, defined for $x\neq -1$. This is the part to check before moving on, because it keeps the answer tied to the original question.
**13.** A careful way to see it: $g(f(x))=g(2x-1)=3(2x-1)+4=6x-3+4=6x+1$. This is the part to check before moving on, because it keeps the answer tied to the original question.
**14.** Keep the rule visible: $(f\circ g)(x)=(x+1)^2=x^2+2x+1$; $(g\circ f)(x)=x^2+1$. Different polynomials — composition is not commutative. That gives a quick check on the answer.
**15.** One steady path is: $S(p(x))=4(x+2)+9=4x+8+9=4x+17$. This is the part to check before moving on, because it keeps the answer tied to the original question.
**16.** Start with the key idea: $g(3)=1$, $f(1)=1$. This is the part to check before moving on, because it keeps the answer tied to the original question.
**17.** A careful way to see it: $f(g(x))=2\cdot\dfrac{x-5}{2}+5=(x-5)+5=x$. (Hence $g=f^{-1}$.) This is the part to check before moving on, because it keeps the answer tied to the original question.
**18.** Keep the rule visible: $g(f(x))=g(x+4)=3(x+4)=3x+12$. (Compare $(f\circ g)(x)=3x+4$.) That gives a quick check on the answer.
**19.** One steady path is: $f(g(x))=\sqrt{x-1}$. The radicand must be $\geq 0$, so the domain is $x\geq 1$. That gives a quick check on the answer.
**20.** Start with the key idea: $f(g(x))=2\cdot\dfrac{x-1}{2}+1=(x-1)+1=x$. $g(f(x))=\dfrac{(2x+1)-1}{2}=\dfrac{2x}{2}=x$. So $f$ and $g$ are inverses. That gives a quick check on the answer.
**21.** The shipping function uses the packed weight, so compose: $S(p(x))=4(x+2)+9=4x+8+9=4x+17$. For a 5-pound item: $4(5)+17=37$ dollars. Sanity check: packed weight is $7$ pounds, and $4(7)+9=37$. Same number both ways.
**22.** Compose: $A(r(t))=\pi(2t+1)^2$. At $t=3$: $r(3)=7$, so $A=\pi(49)=49\pi$ cm$^2$. Composing lets us skip the intermediate radius and write area directly as a function of time — useful when modeling a growing ripple.
**23.** Compute $f(g(x))=2\cdot\dfrac{x+3}{2}-3=(x+3)-3=x$. Compute $g(f(x))=\dfrac{(2x-3)+3}{2}=\dfrac{2x}{2}=x$. Both compositions return the input untouched, which is the identity function. That's exactly the definition of inverse: each function undoes the other.
**24.** Discount first, then tax: $t(d(p))=1.08(0.80p)=0.864p$. So you pay $86.4\%$ of the sticker price. For $p=50$: $C(50)=43.20$. (The two steps are simple scalings, which is why they commute — discount-then-tax and tax-then-discount give the same dollar total.)

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