

# Permutations and Combinations

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 37

## Q Quick Review

**Factorial**  $n! = n \cdot (n - 1) \cdot (n - 2) \cdots 2 \cdot 1$ . By convention,  $0! = 1$  – a tidy choice that keeps the formulas clean.

**Permutation**  $P(n, r) = \frac{n!}{(n - r)!}$ . Use it when *order matters*: arranging books on a shelf, ordering a podium of medalists, building a password from distinct digits.

**Combination**  $C(n, r) = \binom{n}{r} = \frac{n!}{r!(n - r)!}$ . Use it when *order doesn't matter*: picking a committee, choosing a team, selecting a hand of cards. The extra  $r!$  in the denominator divides out the redundant orderings of the same chosen set.

**The pivot question:** *Does swapping two of my selections change the answer?* If yes, it's a permutation. If no, a combination.

**Symmetry:**  $\binom{n}{r} = \binom{n}{n - r}$ . Picking which  $r$  to include is the same as picking which  $n - r$  to leave out.

**Arrangements with repeats.** For a word like BOOKKEEPER with letters  $O$  twice,  $K$  twice,  $E$  three times, the number of distinct arrangements is  $\frac{10!}{2! \cdot 2! \cdot 3!} = 151,200$ . Divide by the factorial of each repeat group to kill duplicate orderings.

**Common slips.** Using  $n^r$  when no repetition is allowed (that formula allows repeats). Using  $P(n, r)$  for a committee (committees don't care about order). Confusing  $n!/r!$  with the permutation formula – the correct denominator is  $(n - r)!$ .

## PRACTICE

Decide permutation vs. combination, then compute.

1.  $5!$  \_\_\_\_\_
2. Using the factorial table below, evaluate  $7!$ . \_\_\_\_\_

$n$	1	2	3	4	5	6	7
$n!$	1	2	6	24	120	720	?

3.  $P(6, 2)$  \_\_\_\_\_
4.  $\binom{8}{3}$  \_\_\_\_\_
5.  $\binom{12}{5}$  \_\_\_\_\_
6.  $P(10, 4)$  \_\_\_\_\_
7.  $\binom{10}{4}$  \_\_\_\_\_
8.  $0!$  \_\_\_\_\_
9. Using the letter tally below, count the distinct arrangements of all letters in BANANA. \_\_\_\_\_

Letter	B	A	N
Count	1	3	2

10. Simplify  $\binom{n}{n-r}$  using symmetry. \_\_\_\_\_
11.  $\binom{9}{2}$  \_\_\_\_\_
12.  $P(7, 3)$  \_\_\_\_\_
13. How many 3-digit codes from  $\{0, \dots, 9\}$  with no repeat \_\_\_\_\_



14. Using the letter tally below, count the distinct arrangements of MISSISSIPPI. \_\_\_\_\_

Letter	M	I	S	P
Count	1	4	4	2

15.  $\binom{52}{5}$  \_\_\_\_\_

16. Lock with 4 digits, repeats allowed: total combos \_\_\_\_\_

17.  $\binom{15}{4}$  \_\_\_\_\_

18. Arrangements of all letters in LEVEL \_\_\_\_\_

19.  $P(9, 3)$  \_\_\_\_\_

20.  $\binom{20}{2}$  \_\_\_\_\_

### ◆ Word Problems

21. A bookshelf has 7 different books. In how many different orders can you arrange all 7 books in a row? \_\_\_\_\_

22. A school is forming a 5-member committee from 12 candidates. The roles are interchangeable – everyone on the committee has the same job. How many different committees are possible? \_\_\_\_\_

23. A password consists of 4 distinct digits arranged in order, chosen from 0 through 9. How many such passwords are possible? \_\_\_\_\_

24. Find the number of distinct arrangements of all the letters in the word BOOKKEEPER. \_\_\_\_\_

### Additional Practice

25. Probability of rolling an even number on a fair die. \_\_\_\_\_

26. Probability of drawing a heart from a standard deck. \_\_\_\_\_

27. Complement of  $P(A) = 0.37$ . \_\_\_\_\_

28. If events are independent,  $P(A) = 0.4$ ,  $P(B) = 0.5$ , find  $P(A \cap B)$ . \_\_\_\_\_

29. Find  $P(A \cup B)$  if  $P(A) = 0.6$ ,  $P(B) = 0.3$ ,  $P(A \cap B) = 0.1$ . \_\_\_\_\_

30. Choose 3 from 8. \_\_\_\_\_

31. Arrange 4 distinct books. \_\_\_\_\_

32. Find  ${}^7P_2$ . \_\_\_\_\_

33. Find  ${}^7C_2$ . \_\_\_\_\_

34. Probability of two heads in two coin flips. \_\_\_\_\_

35. Expected wins in 80 trials with  $p = 0.25$ . \_\_\_\_\_

36. Is drawing without replacement independent? \_\_\_\_\_

37. If 50 of 200 students play chess, find relative frequency. \_\_\_\_\_



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## Answer Keys

1. 120  
 2. 5040  
 3. 30  
 4. 56  
 5. 792  
 6. 5040  
 7. 210  
 8. 1  
 9. 60  
 10.  $\binom{n}{r}$   
 11. 36  
 12. 210

## Additional Practice Answers

25.  $\frac{1}{2}$   
 26.  $\frac{1}{4}$   
 27. 0.63  
 28. 0.20  
 29. 0.8  
 30. 56
13. 720  
 14. 34,650  
 15. 2,598,960  
 16. 10,000  
 17. 1365  
 18. 30  
 19. 504  
 20. 190  
 21. 5040  
 22. 792  
 23. 5040  
 24. 151,200
31. 24  
 32. 42  
 33. 21  
 34.  $\frac{1}{4}$   
 35. 20  
 36. no  
 37. 0.25

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

1. A factorial multiplies every whole number down to 1:  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ . It counts the orderings of 5 distinct items.

2. Each entry is  $n$  times the previous:  $7! = 7 \cdot 6! = 7 \cdot 720 = 5040$ . A useful number to recognize.

3. Permutation:  $P(n, r) = \frac{n!}{(n-r)!}$ , so  $P(6, 2) = \frac{6!}{4!} = 6 \cdot 5 = 30$ . Order matters, so the first slot has 6 choices and the second has 5.

4. Combination (order doesn't matter):  $\binom{8}{3} = \frac{8!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = \frac{336}{6} = 56$ . The 3! in the denominator removes the duplicate orderings a permutation would count.

5. A careful way to see it:  $\frac{12!}{5! \cdot 7!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{120} = 792$ . Useful for committees of 5 from 12 people. That gives a quick check on the answer.

6. Permutation  $P(10, 4) = \frac{10!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$ . Each slot uses one fewer digit since repeats aren't allowed and order matters – exactly 4-digit codes with distinct digits.

7. One steady path is:  $\binom{10}{4} = \frac{P(10, 4)}{4!} = \frac{5040}{24} = 210$ . Dividing the permutation by 4! collapses the 24 orderings of each chosen group into one selection. That gives a quick check on the answer.

8. Start with the key idea: By definition. Keeps formulas like  $\binom{n}{0} = \frac{n!}{0! \cdot n!} = 1$  working out cleanly. That gives a quick check on the answer.

9. A careful way to see it:  $\frac{6!}{3! \cdot 2!} = \frac{720}{12} = 60$ . Three As and two Ns repeat, so divide by 3! and 2!. That gives a quick check on the answer.

10. Symmetry: choosing  $r$  to include is the same as choosing  $n - r$  to exclude. Both counts match.

11. One steady path is:  $\binom{9}{2} = \frac{9 \cdot 8}{2 \cdot 1} = \frac{72}{2} = 36$ . A handshake involves an unordered pair, so combinations fit – 9 people give 36 handshakes. That gives a quick check on the answer.

12. Start with the key idea:  $P(7, 3) = \frac{7!}{4!} = 7 \cdot 6 \cdot 5 = 210$ . Three positions filled in order, each from the dwindling pool 7, 6, 5. That gives a quick check on the answer.

13. A careful way to see it:  $10 \cdot 9 \cdot 8 = P(10, 3) = 720$ . Order matters; repeats banned. This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible:  $\frac{11!}{4! \cdot 4! \cdot 2!} = \frac{39916800}{1152} = 34,650$ . Four Ss, four Is, two Ps. That gives a quick check on the answer.

15. A hand ignores order, so use a combination:  $\binom{52}{5} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!} = 2,598,960$ . That's every possible 5-card poker hand.

16. Start with the key idea:  $10^4 = 10,000$ . Each slot independent with 10 choices, hence the exponent. That gives a quick check on the answer.

17. A careful way to see it:  $\frac{15 \cdot 14 \cdot 13 \cdot 12}{24} = \frac{32760}{24} = 1365$ . This is the part to check before moving on, because it keeps the answer tied to the original question.

18. LEVEL has 5 letters with two Ls and two Es repeating. Divide 5! by the factorial of each repeat group:  $\frac{5!}{2! \cdot 2!} = \frac{120}{4} = 30$  distinct arrangements.

19. One steady path is:  $P(9, 3) = \frac{9!}{6!} = 9 \cdot 8 \cdot 7 = 504$ . Order matters, so multiply the three descending choices. That gives a quick check on the answer.

20. Start with the key idea:  $\binom{20}{2} = \frac{20 \cdot 19}{2 \cdot 1} = \frac{380}{2} = 190$ . Unordered pairs, so it's a combination – 190 handshakes among 20 people. That gives a quick check on the answer.

21. Order matters when arranging a row, so this is a permutation. With all 7 books in 7 positions:  $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$  distinct arrangements. (A useful sanity check: with 3 books you'd have 6 orders, with 4 you'd have 24. The factorial grows fast.)

22. Committee membership doesn't depend on order: {Alice, Bob, Carol, Dan, Eve} is the same committee no matter how you list the names. So this is a combination.  $\binom{12}{5} = \frac{12!}{5! \cdot 7!} = 792$  different committees. (If the roles were



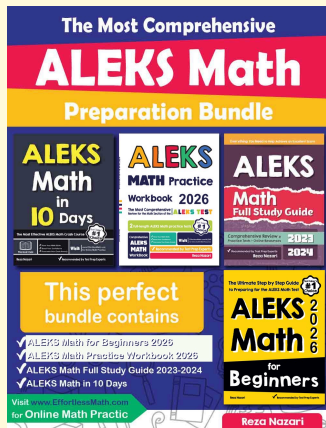
distinct – president, vice-president, etc. – you'd use  $P(12, 5) = 95,040$  instead.)  
**23.** Order matters (a different order is a different password), and digits must be distinct. So this is a permutation of 4 items from 10:  $P(10, 4) = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$  passwords. (If digits could repeat, you'd have  $10^4 = 10,000$  – repetition allows more options. If order didn't matter,  $\binom{10}{4} = 210$ . Three different counts, three different setups.)

**24.** BOOKKEEPER has 10 letters with these repeats: B(1), O(2), K(2), E(3), P(1), R(1). For arrangements with repeats, divide  $n!$  by the factorial of each repeat group's size:  $\frac{10!}{2! \cdot 2! \cdot 3!} = \frac{3628800}{24} = 151,200$ . (Without correcting for repeats you'd get  $10! = 3,628,800$  – but those count BOOKKEEPER and BOOKKEEPER with the two O's swapped as different. Dividing kills that overcount.)



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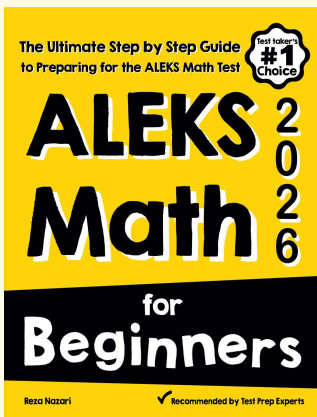
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