

# Solving Quadratics by Completing the Square

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: \_\_\_\_\_ / 24

## Q Quick Review

**Completing the square** converts a quadratic into vertex form so you can solve by isolating a perfect square. Steps for  $x^2 + bx + c = 0$ : **(1)** move  $c$  to the other side; **(2)** add  $(\frac{b}{2})^2$  to both sides (this completes the square on the left); **(3)** factor the left as  $(x + \frac{b}{2})^2$ ; **(4)** take square roots of both sides (don't forget the  $\pm$ ); **(5)** solve for  $x$ . If the leading coefficient isn't 1, divide first or factor it out. Completing the square always works (unlike factoring), and it's how the quadratic formula is derived.

## PRACTICE

Solve each by completing the square.

1.  $x^2 + 4x + 3 = 0$  \_\_\_\_\_

2.  $x^2 - 2x - 3 = 0$  \_\_\_\_\_

3.  $x^2 + 6x = 0$  \_\_\_\_\_

4.  $x^2 - 8x + 12 = 0$  \_\_\_\_\_

5.  $x^2 + 2x - 15 = 0$  \_\_\_\_\_

6.  $x^2 - 10x + 9 = 0$  \_\_\_\_\_

7.  $x^2 + 4x - 5 = 0$  \_\_\_\_\_

8.  $x^2 - 6x + 8 = 0$  \_\_\_\_\_

9.  $x^2 + 2x = 24$  \_\_\_\_\_

10.  $x^2 - 4x = 5$  \_\_\_\_\_

11.  $x^2 + 10x + 21 = 0$  \_\_\_\_\_

12.  $x^2 - 12x + 27 = 0$  \_\_\_\_\_

13.  $x^2 + 8x + 7 = 0$  \_\_\_\_\_

14.  $x^2 - 14x + 45 = 0$  \_\_\_\_\_

15.  $x^2 + 6x + 2 = 0$  \_\_\_\_\_

16.  $x^2 - 4x + 1 = 0$  \_\_\_\_\_

17.  $x^2 + 2x + 2 = 0$  \_\_\_\_\_

18.  $2x^2 + 8x - 10 = 0$  \_\_\_\_\_

19.  $x^2 - 6x + 5 = 0$  \_\_\_\_\_

20.  $x^2 + 2x - 8 = 0$  \_\_\_\_\_

## ◆ Word Problems

21. A landscape designer's area model leads to  $x^2 + 8x + 5 = 0$ . Rewrite the equation in completed-square form so the shifted square is clear.

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22. A rectangular banner design leads to the equation  $x^2 - 10x = -9$ . Solve the equation by completing the square.

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23. A garden has area  $80 \text{ m}^2$  and length 4 more than width. Find the dimensions using completing the square.

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24. A profit model is  $y = x^2 - 6x + 4$ . Convert the model to vertex form and identify the turning point from that

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## Answer Keys

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| <ol style="list-style-type: none"> <li>1. <math>x = -1, -3</math></li> <li>2. <math>x = 3, -1</math></li> <li>3. <math>x = 0, -6</math></li> <li>4. <math>x = 2, 6</math></li> <li>5. <math>x = 3, -5</math></li> <li>6. <math>x = 1, 9</math></li> <li>7. <math>x = 1, -5</math></li> <li>8. <math>x = 2, 4</math></li> <li>9. <math>x = 4, -6</math></li> <li>10. <math>x = 5, -1</math></li> <li>11. <math>x = -3, -7</math></li> <li>12. <math>x = 3, 9</math></li> </ol> | <ol style="list-style-type: none"> <li>13. <math>x = -1, -7</math></li> <li>14. <math>x = 5, 9</math></li> <li>15. <math>x = -3 \pm \sqrt{7}</math></li> <li>16. <math>x = 2 \pm \sqrt{3}</math></li> <li>17. no real solution</li> <li>18. <math>x = 1, -5</math></li> <li>19. <math>x = 1, 5</math></li> <li>20. <math>x = 2, -4</math></li> <li>21. <math>(x + 4)^2 = 11</math></li> <li>22. <math>x = 1, 9</math></li> <li>23. <math>w \approx 7.17</math> m, <math>\ell \approx 11.17</math> m</li> <li>24. <math>y = (x - 3)^2 - 5</math></li> </ol> |
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### Step-by-Step Tutor Notes

1. Keep the order of operations in view, then simplify without skipping the sign check.  $x^2 + 4x = -3$ . Add 4:  $(x + 2)^2 = 1$ .  $x + 2 = \pm 1$ . After simplifying, the answer is  $x = -1, -3$ .
2. Keep the order of operations in view, then simplify without skipping the sign check.  $x^2 - 2x = 3$ . Add 1:  $(x - 1)^2 = 4$ .  $x - 1 = \pm 2$ . After simplifying, the answer is  $x = 3, -1$ .
3. Read the table by matching the correct row and column first, then use the count or total that fits the question.  $x^2 + 6x + 9 = 9 \Rightarrow (x + 3)^2 = 9 \Rightarrow x + 3 = \pm 3$ . This gives  $x = 0, -6$ .
4. Use the labels on the display first; they tell you which count or total belongs in the answer.  $x^2 - 8x = -12 \Rightarrow (x - 4)^2 = 4 \Rightarrow x - 4 = \pm 2$ . This gives  $x = 2, 6$ .
5. For a table question, slow down and locate the exact row, column, or cell before calculating.  $(x + 1)^2 = 16 \Rightarrow x + 1 = \pm 4$ . This gives  $x = 3, -5$ .
6. Read the table by matching the correct row and column first, then use the count or total that fits the question.  $(x - 5)^2 = 16 \Rightarrow x - 5 = \pm 4$ . This gives  $x = 1, 9$ .
7. Use the labels on the display first; they tell you which count or total belongs in the answer.  $(x + 2)^2 = 9 \Rightarrow x + 2 = \pm 3$ . This gives  $x = 1, -5$ .
8. For a table question, slow down and locate the exact row, column, or cell before calculating.  $(x - 3)^2 = 1 \Rightarrow x - 3 = \pm 1$ . This gives  $x = 2, 4$ .
9. For a table question, slow down and locate the exact row, column, or cell before calculating.  $(x + 1)^2 = 25 \Rightarrow x + 1 = \pm 5$ . This gives  $x = 4, -6$ .
10. Read the table by matching the correct row and column first, then use the count or total that fits the question.  $(x - 2)^2 = 9 \Rightarrow x - 2 = \pm 3$ . This gives  $x = 5, -1$ .
11. Read the table by matching the correct row and column first, then use the count or total that fits the question.  $(x + 5)^2 = 4 \Rightarrow x + 5 = \pm 2$ . This gives  $x = -3, -7$ .
12. Use the labels on the display first; they tell you which count or total belongs in the answer.  $(x - 6)^2 = 9 \Rightarrow x - 6 = \pm 3$ . This gives  $x = 3, 9$ .
13. Use the labels on the display first; they tell you which count or total belongs in the answer.  $(x + 4)^2 = 9 \Rightarrow x + 4 = \pm 3$ . This gives  $x = -1, -7$ .
14. Read the table by matching the correct row and column first, then use the count or total that fits the question.  $(x - 7)^2 = 4 \Rightarrow x - 7 = \pm 2$ . This gives  $x = 5, 9$ .
15. Read the table by matching the correct row and column first, then use the count or total that fits the question.  $(x + 3)^2 = 7 \Rightarrow x + 3 = \pm \sqrt{7}$ . Irrational answer. This gives  $x = -3 \pm \sqrt{7}$ .
16. Focus on the main idea of the problem, then simplify carefully.  $(x - 2)^2 = 3$ . So the answer is  $x = 2 \pm \sqrt{3}$ .
17. Start with the definition the problem is testing, then apply it directly.  $(x + 1)^2 = -1$ . Can't take square root of a negative. So the answer is no real solution.
18. Use the labels on the display first; they tell you which count or total belongs in the answer. Divide by 2:  $x^2 + 4x - 5 = 0 \Rightarrow (x + 2)^2 = 9$ . This gives  $x = 1, -5$ .
19. Use the labels on the display first; they tell you which count or total belongs in the answer.  $(x - 3)^2 = 4 \Rightarrow x - 3 = \pm 2$ . This gives  $x = 1, 5$ .
20. Read the table by matching the correct row and column first, then use the count or total that fits the question.  $(x + 1)^2 = 9 \Rightarrow x + 1 = \pm 3$ . This gives  $x = 2, -4$ .
21. Work one inverse operation at a time and keep both sides balanced. Move 5:  $x^2 + 8x = -5$ . Add 16:  $(x + 4)^2 = 11$ . After simplifying, the answer is  $(x + 4)^2 = 11$ .
22. Use the labels on the display first; they tell you which count or total belongs in the answer. Add 25 to both sides:  $(x - 5)^2 = 16 \Rightarrow x - 5 = \pm 4$ . This gives  $x = 1, 9$ .
23.  $w(w + 4) = 80 \Rightarrow w^2 + 4w - 80 = 0$ . Complete:  $(w + 2)^2 = 84 \Rightarrow w = -2 + \sqrt{84} \approx 7.17$  m. The length is about 11.17 m.
24. Set up the model from the story, then calculate carefully.  $y = x^2 - 6x + 4 = (x^2 - 6x + 9) - 9 + 4 = (x - 3)^2 - 5$ .



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