

# Similarity and Dilations

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 24

## Q Quick Review

A **dilation** resizes a figure by a **scale factor**  $k$  from a center point. If  $k > 1$  the figure grows; if  $0 < k < 1$  it shrinks. Centered at the origin, a dilation follows the rule  $(x, y) \rightarrow (kx, ky)$ . Two figures are **similar** when one is a dilation (possibly with rigid motions) of the other: their matching *angles are equal* and their matching *sides are proportional*. Similar figures have the same shape but *not necessarily* the same size — think of a photo and its enlargement.

◇ **Example:** A point  $P(6, 9)$  is dilated by a scale factor of  $\frac{2}{3}$  centered at the origin. Find  $P'$ .  
 ⇒ A dilation centered at the origin just multiplies both coordinates by the scale factor. Here  $k = \frac{2}{3}$ , so multiply each coordinate by  $\frac{2}{3}$ : the  $x$ -value becomes  $\frac{2}{3} \cdot 6 = 4$ , and the  $y$ -value becomes  $\frac{2}{3} \cdot 9 = 6$ . Because  $k$  is less than 1, the point moved *closer* to the origin — the figure shrank.

**Answer:**  $P' = (4, 6)$

## PRACTICE

Find the image under the dilation, or the missing similar measure.

- |  |  |
|--|--|
| 1. $(2, 3)$ , $k = 2$ , center origin _____                    | 12. Similar: 6, 8, 10 scaled to 3, 4, ? _____                            |
| 2. $(5, 1)$ , $k = 3$ , center origin _____                    | 13. $\frac{x}{6} = \frac{10}{4}$ (similar sides) _____                   |
| 3. $(8, 4)$ , $k = \frac{1}{2}$ , center origin _____          | 14. $\frac{8}{x} = \frac{4}{9}$ (similar sides) _____                    |
| 4. $(9, 6)$ , $k = \frac{1}{3}$ , center origin _____          | 15. $\frac{x}{12} = \frac{5}{3}$ (similar sides) _____                   |
| 5. $(10, 15)$ , $k = \frac{2}{5}$ , center origin _____        | 16. Two similar triangles, scale 3. Perimeter 7 →? _____                 |
| 6. $(-4, 6)$ , $k = 2$ , center origin _____                   | 17. Similar rectangles, scale $\frac{1}{2}$ . Perimeter 24 →? _____      |
| 7. $(7, -3)$ , $k = 4$ , center origin _____                   | 18. In similar figures, matching angles are _____                        |
| 8. $(12, -8)$ , $k = \frac{3}{4}$ , center origin _____        | 19. $(6, 9)$ , $k = \frac{1}{3}$ , center origin _____                   |
| 9. Scale factor from a 4 cm side to an 12 cm side _____        | 20. A 5 ft model is $\frac{1}{20}$ of a building. Building height? _____ |
| 10. Scale factor from a 20 cm side to a 5 cm side _____        |  |
| 11. Similar: sides 3, 4, 5 scaled by 4. New longest side _____ |  |

### ◆ Word Problems

21. A photo that is 4 inches wide and 6 inches tall is enlarged by a scale factor of 2.5. What are the new width and height?  
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22. On a map, two cities are drawn 3 cm apart, and the map's scale factor to real life is 1 : 500,000. What is the real distance in kilometers?  
 \_\_\_\_\_
23. Triangle  $ABC$  is similar to triangle  $DEF$ . In  $\triangle ABC$  the sides are 9, 12, and 15 cm. The shortest side of  $\triangle DEF$  is 3 cm. Find the other two sides of  $\triangle DEF$ .  
 \_\_\_\_\_
24. A flagpole casts a 24 ft shadow at the same time a 5 ft student casts a 4 ft shadow. The triangles formed are similar. How tall is the flagpole?  
 \_\_\_\_\_



## Answer Keys

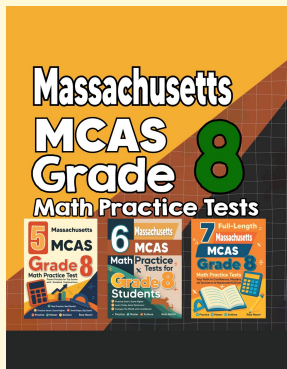
- |                   |                            |
|-------------------|----------------------------|
| 1. (4, 6)         | 13. $x = 15$               |
| 2. (15, 3)        | 14. $x = 18$               |
| 3. (4, 2)         | 15. $x = 20$               |
| 4. (3, 2)         | 16. 21                     |
| 5. (4, 6)         | 17. 12                     |
| 6. (-8, 12)       | 18. equal                  |
| 7. (28, -12)      | 19. (2, 3)                 |
| 8. (9, -6)        | 20. 100 ft                 |
| 9. 3              | 21. 10 in wide, 15 in tall |
| 10. $\frac{1}{4}$ | 22. 15 km                  |
| 11. 20            | 23. 4 cm and 5 cm          |
| 12. 5             | 24. 30 ft                  |

### Step-by-Step Explanations

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|--|--|
| <p>1. Multiply both coordinates by 2: <math>(2 \cdot 2, 3 \cdot 2) = (4, 6)</math>.</p> <p>2. Multiply both by 3: <math>(15, 3)</math>.</p> <p>3. Multiply both by <math>\frac{1}{2}</math>: <math>(4, 2)</math>. Since <math>k &lt; 1</math>, it shrinks.</p> <p>4. Multiply both by <math>\frac{1}{3}</math>: <math>(3, 2)</math>.</p> <p>5. <math>\frac{2}{5} \cdot 10 = 4</math> and <math>\frac{3}{5} \cdot 15 = 6</math>.</p> <p>6. Multiply both by 2: <math>(-8, 12)</math>.</p> <p>7. Multiply both by 4: <math>(28, -12)</math>.</p> <p>8. <math>\frac{3}{4} \cdot 12 = 9</math> and <math>\frac{3}{4} \cdot (-8) = -6</math>.</p> <p>9. Scale factor = <math>\frac{\text{new}}{\text{old}} = \frac{12}{4} = 3</math>.</p> <p>10. Scale factor = <math>\frac{5}{20} = \frac{1}{4}</math> — a shrink.</p> <p>11. Multiply the longest side by 4: <math>5 \cdot 4 = 20</math>.</p> <p>12. The scale factor is <math>\frac{3}{6} = \frac{1}{2}</math>, so <math>10 \cdot \frac{1}{2} = 5</math>.</p> <p>13. Cross multiply: <math>4x = 60</math>, so <math>x = 15</math>.</p> | <p>14. Cross multiply: <math>4x = 72</math>, so <math>x = 18</math>.</p> <p>15. Cross multiply: <math>3x = 60</math>, so <math>x = 20</math>.</p> <p>16. All sides triple, so the perimeter also triples: <math>7 \cdot 3 = 21</math>.</p> <p>17. Every side is halved, so the perimeter is halved: <math>24 \cdot \frac{1}{2} = 12</math>.</p> <p>18. Similarity keeps shape, so corresponding angles stay equal.</p> <p>19. Multiply both by <math>\frac{1}{3}</math>: <math>(2, 3)</math>.</p> <p>20. The real building is 20 times the model: <math>5 \cdot 20 = 100</math> ft.</p> <p>21. Multiply each dimension by 2.5: width <math>4 \times 2.5 = 10</math> in and height <math>6 \times 2.5 = 15</math> in.</p> <p>22. Real distance = <math>3 \text{ cm} \times 500,000 = 1,500,000 \text{ cm} = 15,000 \text{ m} = 15 \text{ km}</math>.</p> <p>23. The scale factor is <math>\frac{3}{9} = \frac{1}{3}</math>. So the sides shrink: <math>12 \times \frac{1}{3} = 4</math> cm and <math>15 \times \frac{1}{3} = 5</math> cm.</p> <p>24. Set up the proportion <math>\frac{h}{24} = \frac{5}{4}</math>. Cross multiplying gives <math>4h = 120</math>, so <math>h = 30</math> ft.</p> |
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