

Rational and Irrational Numbers

Name: _____ Date: _____ Score: _____ / 24

Q Quick Review

A **rational number** is any number you can write as a fraction $\frac{a}{b}$ of two integers (with $b \neq 0$). When you write a rational number as a decimal, it always either *stops* (like 0.75) or *repeats* a pattern forever (like 0.3333...). An **irrational number** cannot be written as such a fraction — its decimal goes on forever with *no repeating pattern*. Famous examples are π and most square roots, such as $\sqrt{2}$ and $\sqrt{10}$. A quick test: \sqrt{n} is rational only when n is a **perfect square** (like $\sqrt{49} = 7$); otherwise it is irrational.

◇ **Example:** Tell whether $\sqrt{45}$ is rational or irrational.

⇒ Start by asking the key question: is 45 a perfect square? The perfect squares near it are $36 = 6^2$ and $49 = 7^2$, and 45 is stuck *between* them — it is not a perfect square. That means $\sqrt{45}$ is not a whole number, and in fact its decimal 6.7082... never stops and never repeats. So $\sqrt{45}$ cannot be written as a fraction of integers — it is **irrational**.

Answer: $\sqrt{45}$ is irrational

PRACTICE

Classify each number as rational or irrational.

- | | | | |
|---------------------|-------|-----------------------|-------|
| 1. $\frac{3}{5}$ | _____ | 11. $2.\overline{18}$ | _____ |
| 2. $\sqrt{49}$ | _____ | 12. $\sqrt{100}$ | _____ |
| 3. $\sqrt{7}$ | _____ | 13. $-\sqrt{2}$ | _____ |
| 4. 0.25 | _____ | 14. 3.5 | _____ |
| 5. π | _____ | 15. $\sqrt{64}$ | _____ |
| 6. -8 | _____ | 16. $\sqrt{2} + 1$ | _____ |
| 7. $0.\overline{6}$ | _____ | 17. $\frac{22}{7}$ | _____ |
| 8. $\sqrt{16}$ | _____ | 18. 0.1010010001... | _____ |
| 9. $\sqrt{20}$ | _____ | 19. $\sqrt{81}$ | _____ |
| 10. $\frac{0}{9}$ | _____ | 20. $\sqrt{50}$ | _____ |

◆ Word Problems

21. A square garden has an area of 36 square feet. Is the side length a rational or an irrational number? Explain. _____
22. A different square tile has an area of 30 square inches. Is its side length rational or irrational? _____
23. Maria says every number that has a decimal point is irrational. Give one example that shows she is wrong. _____
24. A circle has a radius of 4 cm. Its circumference is $2\pi r = 8\pi$ cm. Is the exact circumference rational or irrational? _____



Answer Keys

- | | |
|---------------|---------------------------------------|
| 1. rational | 13. irrational |
| 2. rational | 14. rational |
| 3. irrational | 15. rational |
| 4. rational | 16. irrational |
| 5. irrational | 17. rational |
| 6. rational | 18. irrational |
| 7. rational | 19. rational |
| 8. rational | 20. irrational |
| 9. irrational | 21. rational; side = 6 ft |
| 10. rational | 22. irrational; side = $\sqrt{30}$ in |
| 11. rational | 23. $0.5 = \frac{1}{2}$ (rational) |
| 12. rational | 24. irrational |

Step-by-Step Explanations

- | | |
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| <p>1. It is already a fraction of two integers, so it is rational by definition.</p> <p>2. $49 = 7^2$ is a perfect square, so $\sqrt{49} = 7$, a whole number — rational.</p> <p>3. 7 is between the perfect squares 4 and 9, so $\sqrt{7}$ is not a whole number and does not repeat — irrational.</p> <p>4. This decimal stops, and $0.25 = \frac{1}{4}$, so it is rational.</p> <p>5. $\pi = 3.14159\dots$ goes on forever with no repeating pattern, so it is irrational.</p> <p>6. Every integer is rational: $-8 = \frac{-8}{1}$.</p> <p>7. A repeating decimal is rational; in fact $0.\overline{6} = \frac{2}{3}$.</p> <p>8. $16 = 4^2$ is a perfect square, so $\sqrt{16} = 4$ — rational.</p> <p>9. 20 sits between 16 and 25, so it is not a perfect square and $\sqrt{20}$ is irrational.</p> <p>10. $\frac{0}{9} = 0$, and 0 is an integer, so it is rational.</p> <p>11. The block 18 repeats forever, and any repeating decimal can be written as a fraction — rational.</p> <p>12. $100 = 10^2$, so $\sqrt{100} = 10$, a whole number — rational.</p> <p>13. $\sqrt{2}$ is irrational, and negating an irrational number keeps it irrational.</p> <p>14. This decimal stops: $3.5 = \frac{7}{2}$, so it is rational.</p> | <p>15. $64 = 8^2$, so $\sqrt{64} = 8$ — a perfect square root is rational.</p> <p>16. Adding the rational number 1 to the irrational $\sqrt{2}$ still gives an irrational number.</p> <p>17. It is a ratio of two integers — rational. (It is a famous <i>approximation</i> of π, but not equal to π.)</p> <p>18. The decimal grows forever but the digits never settle into a repeating block, so it is irrational.</p> <p>19. $81 = 9^2$, so $\sqrt{81} = 9$ — rational.</p> <p>20. 50 lies between 49 and 64, so it is not a perfect square and $\sqrt{50}$ is irrational.</p> <p>21. The side length is $\sqrt{36}$, and $36 = 6^2$ is a perfect square, so $\sqrt{36} = 6$ ft — a whole number, which is rational.</p> <p>22. The side is $\sqrt{30}$. Since 30 is between the perfect squares 25 and 36, it is not a perfect square, so $\sqrt{30}$ is irrational.</p> <p>23. A decimal that stops or repeats is rational. For example 0.5 has a decimal point but equals $\frac{1}{2}$, so Maria's rule is false.</p> <p>24. The circumference is 8π. Since π is irrational, multiplying it by the nonzero rational number 8 keeps the result irrational.</p> |
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