Probability of Compound Events

Name: ________ Date: ________ Score: ____ / 24

Quick Review

A **compound event** combines two or more simple events. To find its probability, first decide how the events relate. For **independent** events (one outcome does not affect the other, like separate coin flips), *multiply*: $P(A \text{ and } B) = P(A) \cdot P(B)$. For **dependent** events (the first outcome changes the second, like drawing without replacing), use the *updated* probability for the second event. You can also list the **sample space** — all possible outcomes — and count the favorable ones. A useful shortcut: $P(\text{at least one}) = 1 - P(\text{none})$.

◇ **Example:** A bag has 3 red and 5 blue marbles. Find the probability of drawing red, replacing it, then drawing red again.

⇒ Because we *replace* the first marble, the bag is exactly the same for the second draw — the two draws are **independent**. On each draw, $P(\text{red}) = \frac{3}{8}$ (3 red out of 8 marbles). Since the events are independent, we multiply: $P(\text{red then red}) = \frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$. If we had *not* replaced the first marble, the second probability would change to $\frac{2}{7}$ — so always check whether there is replacement!

Answer: $P(\text{red, then red}) = \frac{9}{64}$

PRACTICE

Find each compound probability. Write answers as reduced fractions.

1. $P(\text{heads and heads on two coin flips})$ ________
2. $P(\text{tails and a } 3 \text{ on a die})$ ________
3. $P(\text{a } 6 \text{ and then a } 6 \text{ on two die rolls})$ ________
4. $P(\text{heads, heads, heads on three flips})$ ________
5. $P(\text{even on a die and heads on a coin})$ ________
6. $P(\text{a } 1 \text{ then a } 2 \text{ on two die rolls})$ ________
7. $P(\text{spinner } 1\text{-}4 \text{ lands on } 3 \text{ twice})$ ________
8. $P(\text{odd on a die and tails on a coin})$ ________
9. $P(\text{two dice both show even})$ ________
10. $P(\text{drawing red twice WITH replacement: } 3 \text{ red, } 5 \text{ blue})$ ________
11. $P(\text{drawing blue twice WITH replacement: } 3 \text{ red, } 5 \text{ blue})$ ________
12. $P(\text{drawing red then blue WITH replacement: } 3 \text{ red, } 5 \text{ blue})$ ________
13. $P(\text{red then red WITHOUT replacement: } 2 \text{ red, } 3 \text{ blue})$ ________
14. $P(\text{two dice both show } 6)$ ________
15. $P(\text{rolling two dice with a sum of } 2)$ ________
16. $P(\text{rolling two dice with a sum of } 7)$ ________
17. $P(\text{at least one head on two coin flips})$ ________
18. $P(\text{coin heads and spinner } 1\text{-}4 \text{ on } 2)$ ________
19. $P(\text{drawing green twice WITHOUT replacement: } 3 \text{ green, } 2 \text{ red})$ ________
20. $P(\text{a } 5 \text{ on a die, then heads, then a } 5 \text{ again})$ ________

♦ Word Problems

21. A spinner has 4 equal sections and a coin is flipped. What is the probability of landing on section 1 *and* getting heads? ________
22. A drawer has 4 black socks and 6 white socks. You grab two socks without looking and without replacing the first. What is the probability both are black? ________
23. You roll two fair dice. What is the probability that the sum is 7? ________
24. A quiz has 3 true/false questions and you guess each one. What is the probability of guessing all 3 correctly? ________

Answer Keys

1. $\frac{1}{4}$
2. $\frac{1}{12}$
3. $\frac{1}{36}$
4. $\frac{1}{8}$
5. $\frac{1}{4}$
6. $\frac{1}{36}$
7. $\frac{1}{16}$
8. $\frac{1}{4}$
9. $\frac{1}{4}$
10. $\frac{9}{64}$
11. $\frac{25}{64}$
12. $\frac{15}{64}$
13. $\frac{1}{10}$
14. $\frac{1}{36}$
15. $\frac{1}{36}$
16. $\frac{1}{6}$
17. $\frac{3}{4}$
18. $\frac{1}{8}$
19. $\frac{3}{10}$
20. $\frac{1}{72}$
21. $\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$
22. $\frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15}$
23. $\frac{6}{36} = \frac{1}{6}$
24. $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Step-by-Step Explanations

1. Independent flips: $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.
2. Independent: $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$.
3. Independent rolls: $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$.
4. Three independent flips: $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.
5. Independent: $\frac{3}{6} \cdot \frac{1}{2} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.
6. Independent rolls: $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$.
7. Independent spins: $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$.
8. Independent: $\frac{3}{6} \cdot \frac{1}{2} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.
9. Each die: $P(\text{even}) = \frac{1}{2}$, so $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.
10. With replacement the draws are independent: $\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$.
11. Independent draws: $\frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}$.
12. Independent: $\frac{3}{8} \cdot \frac{5}{8} = \frac{15}{64}$.
13. First: $\frac{2}{5}$. After removing a red, $\frac{1}{4}$. Multiply: $\frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10}$.
14. Independent: $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$.
15. Only 1 outcome out of 36 gives a sum of 2: $(1, 1)$.
16. There are 6 ways to make 7 out of 36 outcomes: $\frac{6}{36} = \frac{1}{6}$.
17. Use the shortcut: $1 - P(\text{no heads}) = 1 - \frac{1}{4} = \frac{3}{4}$.
18. Independent: $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$.
19. First: $\frac{3}{5}$. After removing a green, $\frac{2}{4} = \frac{1}{2}$. Multiply: $\frac{3}{5} \cdot \frac{1}{2} = \frac{3}{10}$.
20. All independent: $\frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{72}$.
21. The spin and flip are independent. $P(\text{section } 1) = \frac{1}{4}$ and $P(\text{heads}) = \frac{1}{2}$, so multiply: $\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$.
22. First sock black: $\frac{4}{10}$. Now 3 black of 9 remain, so the second is $\frac{3}{9}$. Multiply: $\frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$.
23. There are 36 equally likely outcomes. The pairs that sum to 7 are $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)$ — 6 of them — so $\frac{6}{36} = \frac{1}{6}$.
24. Each guess is independent with probability $\frac{1}{2}$ of being right, so all three correct is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.