

Geometric Sequences

Name: _____ Date: _____ Score: _____ / 24

Q Quick Review

A **geometric sequence** is a list of numbers where you *multiply* by the same value each step. That fixed multiplier is the **common ratio** r — find it by dividing any term by the one before it. To reach the n th term, use $a_n = a_1 \cdot r^{n-1}$, where a_1 is the first term. For example, in 2, 6, 18, 54, ... the common ratio is $r = 3$, and the 5th term is $a_5 = 2 \cdot 3^4 = 2 \cdot 81 = 162$. Unlike arithmetic sequences, geometric ones grow (or shrink) faster and faster — they are nonlinear.

◊ **Example:** Find the 5th term of the geometric sequence 2, 6, 18, 54, ...
 ⇒ First find the common ratio by dividing: $6 \div 2 = 3$, and $18 \div 6 = 3$, so $r = 3$. The first term is $a_1 = 2$. Now use $a_n = a_1 \cdot r^{n-1}$ with $n = 5$: $a_5 = 2 \cdot 3^{5-1} = 2 \cdot 3^4 = 2 \cdot 81 = 162$. So the 5th term is 162. Notice how quickly it grows — that's the power of multiplying!

Answer: $a_5 = 162$

PRACTICE

Find the requested term of each geometric sequence.

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|--------------------------------|-------|--|-------|
| 1. 2, 6, 18, 54, ... ; a_5 | _____ | 11. 1, 3, 9, 27, ... ; a_6 | _____ |
| 2. 5, 10, 20, 40, ... ; a_6 | _____ | 12. 4, 8, 16, 32, ... ; a_7 | _____ |
| 3. 3, 6, 12, 24, ... ; a_8 | _____ | 13. 3, 15, 75, 375, ... ; a_4 | _____ |
| 4. 1, 4, 16, 64, ... ; a_5 | _____ | 14. 81, 27, 9, 3, ... ; a_6 | _____ |
| 5. 64, 32, 16, 8, ... ; a_7 | _____ | 15. 10, 20, 40, 80, ... ; a_6 | _____ |
| 6. 7, 21, 63, 189, ... ; a_4 | _____ | 16. 2, 4, 8, 16, ... ; a_8 | _____ |
| 7. 2, 10, 50, 250, ... ; a_5 | _____ | 17. 1, 5, 25, 125, ... ; a_5 | _____ |
| 8. 1, 2, 4, 8, ... ; a_{10} | _____ | 18. Find r for 3, 12, 48, 192, ... | _____ |
| 9. 100, 20, 4, ... ; a_4 | _____ | 19. Find a_1 if $a_3 = 45$ and $r = 3$ | _____ |
| 10. 6, 12, 24, 48, ... ; a_9 | _____ | 20. Is 64 a term of 1, 2, 4, 8, ... ? | _____ |

◆ Word Problems

21. A type of bacteria doubles every hour. A dish starts with 3 bacteria. How many are there at the start of the 8th hour (counting the start as hour 1)? _____
22. A ball is dropped and each bounce reaches half the previous height. The first bounce is 64 inches. How high is the 7th bounce? _____
23. An investment triples every year. It begins at \$2. What is it worth at the start of year 5 (counting the start as year 1)? _____
24. A rumor spreads so that each round it reaches 5 times as many new people. In round 1 it reaches 2 people. How many new people hear it in round 5? _____



Answer Keys

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|------------------|-------------------|
| 1. 162 | 13. 375 |
| 2. 160 | 14. $\frac{1}{3}$ |
| 3. 384 | 15. 320 |
| 4. 256 | 16. 256 |
| 5. 1 | 17. 625 |
| 6. 189 | 18. 4 |
| 7. 1250 | 19. 5 |
| 8. 512 | 20. yes |
| 9. $\frac{4}{5}$ | 21. 384 bacteria |
| 10. 1536 | 22. 1 inch |
| 11. 243 | 23. \$162 |
| 12. 256 | 24. 1250 people |

Step-by-Step Explanations

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|---|---|
| <p>1. Here $r = 3$ and $a_1 = 2$, so $a_5 = 2 \cdot 3^4 = 2 \cdot 81 = 162$.</p> <p>2. Here $r = 2$ and $a_1 = 5$, so $a_6 = 5 \cdot 2^5 = 5 \cdot 32 = 160$.</p> <p>3. Here $r = 2$ and $a_1 = 3$, so $a_8 = 3 \cdot 2^7 = 3 \cdot 128 = 384$.</p> <p>4. Here $r = 4$ and $a_1 = 1$, so $a_5 = 1 \cdot 4^4 = 256$.</p> <p>5. Here $r = \frac{1}{2}$ and $a_1 = 64$, so $a_7 = 64 \cdot \left(\frac{1}{2}\right)^6 = \frac{64}{64} = 1$.</p> <p>6. Here $r = 3$ and $a_1 = 7$, so $a_4 = 7 \cdot 3^3 = 7 \cdot 27 = 189$.</p> <p>7. Here $r = 5$ and $a_1 = 2$, so $a_5 = 2 \cdot 5^4 = 2 \cdot 625 = 1250$.</p> <p>8. Here $r = 2$ and $a_1 = 1$, so $a_{10} = 1 \cdot 2^9 = 512$.</p> <p>9. Here $r = \frac{1}{5}$ and $a_1 = 100$, so $a_4 = 100 \cdot \left(\frac{1}{5}\right)^3 = \frac{100}{125} = \frac{4}{5}$.</p> <p>10. Here $r = 2$ and $a_1 = 6$, so $a_9 = 6 \cdot 2^8 = 6 \cdot 256 = 1536$.</p> <p>11. Here $r = 3$ and $a_1 = 1$, so $a_6 = 1 \cdot 3^5 = 243$.</p> <p>12. Here $r = 2$ and $a_1 = 4$, so $a_7 = 4 \cdot 2^6 = 4 \cdot 64 = 256$.</p> <p>13. Here $r = 5$ and $a_1 = 3$, so $a_4 = 3 \cdot 5^3 = 3 \cdot 125 = 375$.</p> <p>14. Here $r = \frac{1}{3}$ and $a_1 = 81$, so $a_6 = 81 \cdot \left(\frac{1}{3}\right)^5 = \frac{81}{243} = \frac{1}{3}$.</p> | <p>15. Here $r = 2$ and $a_1 = 10$, so $a_6 = 10 \cdot 2^5 = 10 \cdot 32 = 320$.</p> <p>16. Here $r = 2$ and $a_1 = 2$, so $a_8 = 2 \cdot 2^7 = 2 \cdot 128 = 256$.</p> <p>17. Here $r = 5$ and $a_1 = 1$, so $a_5 = 1 \cdot 5^4 = 625$.</p> <p>18. Divide consecutive terms: $12 \div 3 = 4$, so the common ratio is $r = 4$.</p> <p>19. Use $a_3 = a_1 \cdot r^2$: $45 = a_1 \cdot 9$, so $a_1 = 5$.</p> <p>20. This sequence is the powers of 2, and $64 = 2^6$, so 64 is the 7th term — yes.</p> <p>21. This is geometric with $a_1 = 3$ and $r = 2$. At hour 8 there are $a_8 = 3 \cdot 2^7 = 3 \cdot 128 = 384$ bacteria.</p> <p>22. The heights are geometric with $a_1 = 64$ and $r = \frac{1}{2}$. The 7th bounce is $a_7 = 64 \cdot \left(\frac{1}{2}\right)^6 = \frac{64}{64} = 1$ inch.</p> <p>23. This is geometric with $a_1 = 2$ and $r = 3$. At year 5 the value is $a_5 = 2 \cdot 3^4 = 2 \cdot 81 = 162$ dollars.</p> <p>24. The counts are geometric with $a_1 = 2$ and $r = 5$. Round 5 reaches $a_5 = 2 \cdot 5^4 = 2 \cdot 625 = 1250$ new people.</p> |
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