Geometric Sequences

Name: ________________	Date: ____________	Score: ______ / 24

Quick Review

A **geometric sequence** is a list of numbers where you *multiply* by the same value each step. That fixed multiplier is the **common ratio** r — find it by dividing any term by the one before it. To reach the nth term, use $a_n = a_1 \cdot r^{n-1}$, where a_1 is the first term. For example, in $2, 6, 18, 54, \ldots$ the common ratio is $r = 3$, and the 5th term is $a_5 = 2 \cdot 3^4 = 2 \cdot 81 = 162$. Unlike arithmetic sequences, geometric ones grow (or shrink) faster and faster — they are nonlinear.

◇ **Example:** Find the 5th term of the geometric sequence $2, 6, 18, 54, \ldots$

⇒ First find the common ratio by dividing: $6 \div 2 = 3$, and $18 \div 6 = 3$, so $r = 3$. The first term is $a_1 = 2$. Now use $a_n = a_1 \cdot r^{n-1}$ with $n = 5$: $a_5 = 2 \cdot 3^{5-1} = 2 \cdot 3^4 = 2 \cdot 81 = 162$. So the 5th term is 162. Notice how quickly it grows — that's the power of multiplying!

Answer: $a_5 = 162$

PRACTICE

Find the requested term of each geometric sequence.

1. $2, 6, 18, 54, \ldots;\ a_5$ ________

2. $5, 10, 20, 40, \ldots;\ a_6$ ________

3. $3, 6, 12, 24, \ldots;\ a_8$ ________

4. $1, 4, 16, 64, \ldots;\ a_5$ ________

5. $64, 32, 16, 8, \ldots;\ a_7$ ________

6. $7, 21, 63, 189, \ldots;\ a_4$ ________

7. $2, 10, 50, 250, \ldots;\ a_5$ ________

8. $1, 2, 4, 8, \ldots;\ a_{10}$ ________

9. $100, 20, 4, \ldots;\ a_4$ ________

10. $6, 12, 24, 48, \ldots;\ a_9$ ________

11. $1, 3, 9, 27, \ldots;\ a_6$ ________

12. $4, 8, 16, 32, \ldots;\ a_7$ ________

13. $3, 15, 75, 375, \ldots;\ a_4$ ________

14. $81, 27, 9, 3, \ldots;\ a_6$ ________

15. $10, 20, 40, 80, \ldots;\ a_6$ ________

16. $2, 4, 8, 16, \ldots;\ a_8$ ________

17. $1, 5, 25, 125, \ldots;\ a_5$ ________

18. Find r for $3, 12, 48, 192, \ldots$ ________

19. Find a_1 if $a_3 = 45$ and $r = 3$ ________

20. Is 64 a term of $1, 2, 4, 8, \ldots$? ________

♦ Word Problems

21. A type of bacteria doubles every hour. A dish starts with 3 bacteria. How many are there at the start of the 8th hour (counting the start as hour 1)? __________

22. A ball is dropped and each bounce reaches half the previous height. The first bounce is 64 inches. How high is the 7th bounce? __________

23. An investment triples every year. It begins at \$2. What is it worth at the start of year 5 (counting the start as year 1)? __________

24. A rumor spreads so that each round it reaches 5 times as many new people. In round 1 it reaches 2 people. How many new people hear it in round 5? __________

Answer Keys

1. 162
2. 160
3. 384
4. 256
5. 1
6. 189
7. 1250
8. 512
9. $\frac{4}{5}$
10. 1536
11. 243
12. 256
13. 375
14. $\frac{1}{3}$
15. 320
16. 256
17. 625
18. 4
19. 5
20. yes
21. 384 bacteria
22. 1 inch
23. \$162
24. 1250 people

Step-by-Step Explanations

1. Here $r = 3$ and $a_1 = 2$, so $a_5 = 2 \cdot 3^4 = 2 \cdot 81 = 162$.
2. Here $r = 2$ and $a_1 = 5$, so $a_6 = 5 \cdot 2^5 = 5 \cdot 32 = 160$.
3. Here $r = 2$ and $a_1 = 3$, so $a_8 = 3 \cdot 2^7 = 3 \cdot 128 = 384$.
4. Here $r = 4$ and $a_1 = 1$, so $a_5 = 1 \cdot 4^4 = 256$.
5. Here $r = \frac{1}{2}$ and $a_1 = 64$, so $a_7 = 64 \cdot \left(\frac{1}{2}\right)^6 = \frac{64}{64} = 1$.
6. Here $r = 3$ and $a_1 = 7$, so $a_4 = 7 \cdot 3^3 = 7 \cdot 27 = 189$.
7. Here $r = 5$ and $a_1 = 2$, so $a_5 = 2 \cdot 5^4 = 2 \cdot 625 = 1250$.
8. Here $r = 2$ and $a_1 = 1$, so $a_{10} = 1 \cdot 2^9 = 512$.
9. Here $r = \frac{1}{5}$ and $a_1 = 100$, so $a_4 = 100 \cdot \left(\frac{1}{5}\right)^3 = \frac{100}{125} = \frac{4}{5}$.
10. Here $r = 2$ and $a_1 = 6$, so $a_9 = 6 \cdot 2^8 = 6 \cdot 256 = 1536$.
11. Here $r = 3$ and $a_1 = 1$, so $a_6 = 1 \cdot 3^5 = 243$.
12. Here $r = 2$ and $a_1 = 4$, so $a_7 = 4 \cdot 2^6 = 4 \cdot 64 = 256$.
13. Here $r = 5$ and $a_1 = 3$, so $a_4 = 3 \cdot 5^3 = 3 \cdot 125 = 375$.
14. Here $r = \frac{1}{3}$ and $a_1 = 81$, so $a_6 = 81 \cdot \left(\frac{1}{3}\right)^5 = \frac{81}{243} = \frac{1}{3}$.
15. Here $r = 2$ and $a_1 = 10$, so $a_6 = 10 \cdot 2^5 = 10 \cdot 32 = 320$.
16. Here $r = 2$ and $a_1 = 2$, so $a_8 = 2 \cdot 2^7 = 2 \cdot 128 = 256$.
17. Here $r = 5$ and $a_1 = 1$, so $a_5 = 1 \cdot 5^4 = 625$.
18. Divide consecutive terms: $12 \div 3 = 4$, so the common ratio is $r = 4$.
19. Use $a_3 = a_1 \cdot r^2$: $45 = a_1 \cdot 9$, so $a_1 = 5$.
20. This sequence is the powers of 2, and $64 = 2^6$, so 64 is the 7th term — yes.
21. This is geometric with $a_1 = 3$ and $r = 2$. At hour 8 there are $a_8 = 3 \cdot 2^7 = 3 \cdot 128 = 384$ bacteria.
22. The heights are geometric with $a_1 = 64$ and $r = \frac{1}{2}$. The 7th bounce is $a_7 = 64 \cdot \left(\frac{1}{2}\right)^6 = \frac{64}{64} = 1$ inch.
23. This is geometric with $a_1 = 2$ and $r = 3$. At year 5 the value is $a_5 = 2 \cdot 3^4 = 2 \cdot 81 = 162$ dollars.
24. The counts are geometric with $a_1 = 2$ and $r = 5$. Round 5 reaches $a_5 = 2 \cdot 5^4 = 2 \cdot 625 = 1250$ new people.