Estimating Irrational Numbers

Name: _____	Date: _____	Score: _____ / 24

Quick Review

You can pin down an irrational square root *without* a calculator by trapping it between two whole numbers. To estimate $\sqrt{n}$, find the **perfect square just below** n and the **perfect square just above** it. The square root of n lands between their roots. For a closer guess, check which perfect square n is *nearer* to — that tells you which whole number $\sqrt{n}$ is closer to. You can even test a decimal: square it and see if the result is just under or just over n. This same idea lets you place $\pi \approx 3.14$ on a number line too.

◇ **Example:** Between which two consecutive whole numbers does $\sqrt{52}$ lie?
⇒ List the perfect squares around 52: we have $49 = 7^2$ and $64 = 8^2$, and 52 sits between them. Taking square roots keeps the order, so $7 < \sqrt{52} < 8$. Want to go further? Since 52 is only 3 away from 49 but 12 away from 64, $\sqrt{52}$ is *much closer to* 7 — a good estimate is about 7.2.

Answer: $7 < \sqrt{52} < 8$

PRACTICE

Estimate each value or place it between two consecutive whole numbers.

1. $\sqrt{10}$ _____
2. $\sqrt{5}$ _____
3. $\sqrt{30}$ _____
4. $\sqrt{2}$ _____
5. $\sqrt{40}$ _____
6. $\sqrt{70}$ _____
7. $\sqrt{99}$ _____
8. $\sqrt{3}$ _____
9. $\sqrt{55}$ _____
10. $\sqrt{8}$ _____
11. $\sqrt{17}$ (nearest whole number) _____
12. $\sqrt{83}$ (nearest whole number) _____
13. $\sqrt{38}$ (nearest whole number) _____
14. $\sqrt{24}$ (nearest whole number) _____
15. $\sqrt{90}$ (nearest whole number) _____
16. $\sqrt{12}$ (nearest tenth) _____
17. $\sqrt{20}$ (nearest tenth) _____
18. $\sqrt{7}$ (nearest tenth) _____
19. π (between two whole numbers) _____
20. 2π (nearest whole number) _____

♦ Word Problems

21. A square rug has an area of 45 square feet. Between which two whole numbers is the length of one side? _____
22. A ladder leans so its base is $\sqrt{60}$ feet from a wall. Estimate this distance to the nearest whole number of feet. _____
23. A circular pond has a radius of 5 m, so its circumference is 10π m. Estimate the circumference to the nearest whole meter. _____
24. On a number line, would $\sqrt{50}$ sit to the left or right of 7? Explain. _____

Answer Keys

1. $3 < \sqrt{10} < 4$
2. $2 < \sqrt{5} < 3$
3. $5 < \sqrt{30} < 6$
4. $1 < \sqrt{2} < 2$
5. $6 < \sqrt{40} < 7$
6. $8 < \sqrt{70} < 9$
7. $9 < \sqrt{99} < 10$
8. $1 < \sqrt{3} < 2$
9. $7 < \sqrt{55} < 8$
10. $2 < \sqrt{8} < 3$
11. 4
12. 9
13. 6
14. 5
15. 9
16. 3.5
17. 4.5
18. 2.6
19. $3 < \pi < 4$
20. 6
21. between 6 and 7 feet
22. about 8 feet
23. about 31 meters
24. to the right of 7

Step-by-Step Explanations

1. 10 is between $9 = 3^2$ and $16 = 4^2$, so $\sqrt{10}$ is between 3 and 4.
2. 5 sits between $4 = 2^2$ and $9 = 3^2$, so $2 < \sqrt{5} < 3$.
3. 30 is between $25 = 5^2$ and $36 = 6^2$, giving $5 < \sqrt{30} < 6$.
4. 2 is between $1 = 1^2$ and $4 = 2^2$, so $\sqrt{2}$ is between 1 and 2.
5. 40 falls between $36 = 6^2$ and $49 = 7^2$, so $6 < \sqrt{40} < 7$.
6. 70 is between $64 = 8^2$ and $81 = 9^2$, giving $8 < \sqrt{70} < 9$.
7. 99 sits between $81 = 9^2$ and $100 = 10^2$, so $9 < \sqrt{99} < 10$.
8. 3 is between 1 and 4, so $\sqrt{3}$ is between 1 and 2.
9. 55 is between $49 = 7^2$ and $64 = 8^2$, so $7 < \sqrt{55} < 8$.
10. 8 is between 4 and 9, so $\sqrt{8}$ lies between 2 and 3.
11. 17 is between $16 = 4^2$ and $25 = 5^2$ but only 1 away from 16, so $\sqrt{17} \approx 4$.
12. 83 is between 81 and 100; it is much closer to $81 = 9^2$, so $\sqrt{83} \approx 9$.
13. 38 is between $36 = 6^2$ and 49, and only 2 away from 36, so $\sqrt{38} \approx 6$.
14. 24 is between 16 and 25; it is just 1 away from $25 = 5^2$, so $\sqrt{24} \approx 5$.
15. 90 is between $81 = 9^2$ and 100, and closer to 81, so $\sqrt{90} \approx 9$.
16. $3.4^2 = 11.56$ and $3.5^2 = 12.25$; since 12 is closer to 12.25, $\sqrt{12} \approx 3.5$.
17. $4.4^2 = 19.36$ and $4.5^2 = 20.25$; 20 is closer to 20.25, so $\sqrt{20} \approx 4.5$.
18. $2.6^2 = 6.76$ and $2.7^2 = 7.29$; 7 is closer to 6.76, so $\sqrt{7} \approx 2.6$.
19. $\pi = 3.14159\ldots$, which sits between the whole numbers 3 and 4.
20. $2\pi \approx 2 \times 3.14 = 6.28$, which rounds to 6.
21. The side is $\sqrt{45}$. Since 45 is between $36 = 6^2$ and $49 = 7^2$, the side length is between 6 and 7 feet.
22. 60 is between $49 = 7^2$ and $64 = 8^2$, and it is much closer to 64, so $\sqrt{60} \approx 8$ feet.
23. Using $\pi \approx 3.14$, the circumference $10\pi \approx 10 \times 3.14 = 31.4$ m, which rounds to 31 m.
24. $7 = \sqrt{49}$, and $50 > 49$, so $\sqrt{50} > 7$. That places $\sqrt{50}$ just to the right of 7 on the number line.

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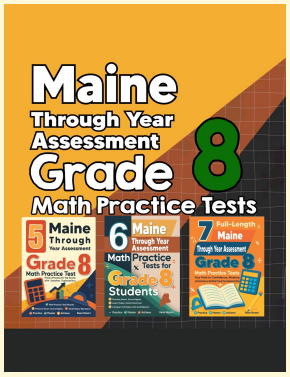

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