

# Estimating Irrational Numbers

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 24

## Q Quick Review

You can pin down an irrational square root *without* a calculator by trapping it between two whole numbers. To estimate  $\sqrt{n}$ , find the **perfect square just below**  $n$  and the **perfect square just above** it. The square root of  $n$  lands between their roots. For a closer guess, check which perfect square  $n$  is *nearer* to — that tells you which whole number  $\sqrt{n}$  is closer to. You can even test a decimal: square it and see if the result is just under or just over  $n$ . This same idea lets you place  $\pi \approx 3.14$  on a number line too.

◇ **Example:** Between which two consecutive whole numbers does  $\sqrt{52}$  lie?  
 ⇒ List the perfect squares around 52: we have  $49 = 7^2$  and  $64 = 8^2$ , and 52 sits between them. Taking square roots keeps the order, so  $7 < \sqrt{52} < 8$ . Want to go further? Since 52 is only 3 away from 49 but 12 away from 64,  $\sqrt{52}$  is *much closer* to 7 — a good estimate is about 7.2.

**Answer:**  $7 < \sqrt{52} < 8$

## PRACTICE

Estimate each value or place it between two consecutive whole numbers.

- |                |       |  |       |
|----------------|-------|--|-------|
| 1. $\sqrt{10}$ | _____ | 11. $\sqrt{17}$ (nearest whole number) | _____ |
| 2. $\sqrt{5}$  | _____ | 12. $\sqrt{83}$ (nearest whole number) | _____ |
| 3. $\sqrt{30}$ | _____ | 13. $\sqrt{38}$ (nearest whole number) | _____ |
| 4. $\sqrt{2}$  | _____ | 14. $\sqrt{24}$ (nearest whole number) | _____ |
| 5. $\sqrt{40}$ | _____ | 15. $\sqrt{90}$ (nearest whole number) | _____ |
| 6. $\sqrt{70}$ | _____ | 16. $\sqrt{12}$ (nearest tenth)        | _____ |
| 7. $\sqrt{99}$ | _____ | 17. $\sqrt{20}$ (nearest tenth)        | _____ |
| 8. $\sqrt{3}$  | _____ | 18. $\sqrt{7}$ (nearest tenth)         | _____ |
| 9. $\sqrt{55}$ | _____ | 19. $\pi$ (between two whole numbers)  | _____ |
| 10. $\sqrt{8}$ | _____ | 20. $2\pi$ (nearest whole number)      | _____ |

## ◆ Word Problems

21. A square rug has an area of 45 square feet. Between which two whole numbers is the length of one side? \_\_\_\_\_
22. A ladder leans so its base is  $\sqrt{60}$  feet from a wall. Estimate this distance to the nearest whole number of feet. \_\_\_\_\_
23. A circular pond has a radius of 5 m, so its circumference is  $10\pi$  m. Estimate the circumference to the nearest whole meter. \_\_\_\_\_
24. On a number line, would  $\sqrt{50}$  sit to the left or right of 7? Explain. \_\_\_\_\_



## Answer Keys

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|---|--|
| <p>1. <math>3 &lt; \sqrt{10} &lt; 4</math></p> <p>2. <math>2 &lt; \sqrt{5} &lt; 3</math></p> <p>3. <math>5 &lt; \sqrt{30} &lt; 6</math></p> <p>4. <math>1 &lt; \sqrt{2} &lt; 2</math></p> <p>5. <math>6 &lt; \sqrt{40} &lt; 7</math></p> <p>6. <math>8 &lt; \sqrt{70} &lt; 9</math></p> <p>7. <math>9 &lt; \sqrt{99} &lt; 10</math></p> <p>8. <math>1 &lt; \sqrt{3} &lt; 2</math></p> <p>9. <math>7 &lt; \sqrt{55} &lt; 8</math></p> <p>10. <math>2 &lt; \sqrt{8} &lt; 3</math></p> <p>11. 4</p> <p>12. 9</p> | <p>13. 6</p> <p>14. 5</p> <p>15. 9</p> <p>16. 3.5</p> <p>17. 4.5</p> <p>18. 2.6</p> <p>19. <math>3 &lt; \pi &lt; 4</math></p> <p>20. 6</p> <p>21. between 6 and 7 feet</p> <p>22. about 8 feet</p> <p>23. about 31 meters</p> <p>24. to the right of 7</p> |
|---|--|

### Step-by-Step Explanations

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|--|--|
| <p>1. 10 is between <math>9 = 3^2</math> and <math>16 = 4^2</math>, so <math>\sqrt{10}</math> is between 3 and 4.</p> <p>2. 5 sits between <math>4 = 2^2</math> and <math>9 = 3^2</math>, so <math>2 &lt; \sqrt{5} &lt; 3</math>.</p> <p>3. 30 is between <math>25 = 5^2</math> and <math>36 = 6^2</math>, giving <math>5 &lt; \sqrt{30} &lt; 6</math>.</p> <p>4. 2 is between <math>1 = 1^2</math> and <math>4 = 2^2</math>, so <math>\sqrt{2}</math> is between 1 and 2.</p> <p>5. 40 falls between <math>36 = 6^2</math> and <math>49 = 7^2</math>, so <math>6 &lt; \sqrt{40} &lt; 7</math>.</p> <p>6. 70 is between <math>64 = 8^2</math> and <math>81 = 9^2</math>, giving <math>8 &lt; \sqrt{70} &lt; 9</math>.</p> <p>7. 99 sits between <math>81 = 9^2</math> and <math>100 = 10^2</math>, so <math>9 &lt; \sqrt{99} &lt; 10</math>.</p> <p>8. 3 is between 1 and 4, so <math>\sqrt{3}</math> is between 1 and 2.</p> <p>9. 55 is between <math>49 = 7^2</math> and <math>64 = 8^2</math>, so <math>7 &lt; \sqrt{55} &lt; 8</math>.</p> <p>10. 8 is between 4 and 9, so <math>\sqrt{8}</math> lies between 2 and 3.</p> <p>11. 17 is between <math>16 = 4^2</math> and <math>25 = 5^2</math> but only 1 away from 16, so <math>\sqrt{17} \approx 4</math>.</p> <p>12. 83 is between 81 and 100; it is much closer to <math>81 = 9^2</math>, so <math>\sqrt{83} \approx 9</math>.</p> <p>13. 38 is between <math>36 = 6^2</math> and 49, and only 2 away from 36, so <math>\sqrt{38} \approx 6</math>.</p> <p>14. 24 is between 16 and 25; it is just 1 away from <math>25 = 5^2</math>, so <math>\sqrt{24} \approx 5</math>.</p> | <p>15. 90 is between <math>81 = 9^2</math> and 100, and closer to 81, so <math>\sqrt{90} \approx 9</math>.</p> <p>16. <math>3.4^2 = 11.56</math> and <math>3.5^2 = 12.25</math>; since 12 is closer to 12.25, <math>\sqrt{12} \approx 3.5</math>.</p> <p>17. <math>4.4^2 = 19.36</math> and <math>4.5^2 = 20.25</math>; 20 is closer to 20.25, so <math>\sqrt{20} \approx 4.5</math>.</p> <p>18. <math>2.6^2 = 6.76</math> and <math>2.7^2 = 7.29</math>; 7 is closer to 6.76, so <math>\sqrt{7} \approx 2.6</math>.</p> <p>19. <math>\pi = 3.14159\dots</math>, which sits between the whole numbers 3 and 4.</p> <p>20. <math>2\pi \approx 2 \times 3.14 = 6.28</math>, which rounds to 6.</p> <p>21. The side is <math>\sqrt{45}</math>. Since 45 is between <math>36 = 6^2</math> and <math>49 = 7^2</math>, the side length is between 6 and 7 feet.</p> <p>22. 60 is between <math>49 = 7^2</math> and <math>64 = 8^2</math>, and it is much closer to 64, so <math>\sqrt{60} \approx 8</math> feet.</p> <p>23. Using <math>\pi \approx 3.14</math>, the circumference <math>10\pi \approx 10 \times 3.14 = 31.4</math> m, which rounds to 31 m.</p> <p>24. <math>7 = \sqrt{49}</math>, and <math>50 &gt; 49</math>, so <math>\sqrt{50} &gt; 7</math>. That places <math>\sqrt{50}</math> just to the right of 7 on the number line.</p> |
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