Estimating Expressions with Irrational Numbers

Name: _____	Date: _____	Score: _____ / 24

Quick Review

When an expression mixes irrational numbers with ordinary arithmetic, the plan is simple: **estimate each irrational piece first**, then do the adding, subtracting, or multiplying. Replace a square root with a close decimal — for instance $\sqrt{2} \approx 1.4$ or $\sqrt{8} \approx 2.8$ — and use $\pi \approx 3.14$. Then carry out the operation as usual and round at the end. You can also *compare* two irrational expressions by estimating both. A handy check: a bigger number under the radical always gives a bigger square root, so $\sqrt{20} > \sqrt{15}$ without any calculation at all.

◇ **Example:** Estimate $\sqrt{10} + \sqrt{5}$ to the nearest tenth.
⇒ Tackle each root on its own first. For $\sqrt{10}$: since $3.1^2 = 9.61$ and $3.2^2 = 10.24$, we get $\sqrt{10} \approx 3.2$. For $\sqrt{5}$: since $2.2^2 = 4.84$ and $2.3^2 = 5.29$, we get $\sqrt{5} \approx 2.2$. Now just add the estimates: $3.2 + 2.2 = 5.4$. So the sum is about 5.4 — estimate the parts, then combine!

Answer: $\sqrt{10} + \sqrt{5} \approx 5.4$

PRACTICE

Estimate each expression, or compare the two values.

1. $\sqrt{4} + \sqrt{9}$ _____
2. $\sqrt{16} - \sqrt{1}$ _____
3. $\sqrt{2} + \sqrt{3}$ (nearest tenth) _____
4. $\sqrt{8} + \sqrt{2}$ (nearest tenth) _____
5. $\sqrt{10} - \sqrt{2}$ (nearest tenth) _____
6. $2\sqrt{9}$ _____
7. $3\sqrt{16}$ _____
8. $\sqrt{2} \cdot \sqrt{8}$ _____
9. $\pi + 2$ (nearest tenth) _____
10. $\pi - 1$ (nearest tenth) _____
11. 2π (nearest tenth) _____
12. $\sqrt{15} + \sqrt{15}$ (nearest tenth) _____
13. $\sqrt{50} - \sqrt{2}$ (nearest tenth) _____
14. $\sqrt{36} + \pi$ (nearest tenth) _____
15. Compare: $\sqrt{20} \square \sqrt{15}$ _____
16. Compare: $\sqrt{2} \square 1.5$ _____
17. Compare: $\pi \square \sqrt{10}$ _____
18. Compare: $\sqrt{7} + 1 \square \sqrt{7} + 2$ _____
19. $\sqrt{3} \cdot \sqrt{12}$ _____
20. $\dfrac{\sqrt{49}}{\sqrt{4}}$ _____

Word Problems

21. A rectangle has a length of $\sqrt{20}$ cm and a width of $\sqrt{5}$ cm. Estimate its perimeter to the nearest tenth of a centimeter. _____

22. Two square tiles have areas 18 in^2 and 8 in^2. Estimate how much longer one side of the bigger tile is than one side of the smaller tile, to the nearest tenth of an inch. _____

23. A circular tabletop has a radius of $\sqrt{2}$ ft. Using $\pi \approx 3.14$, estimate its circumference $2\pi r$ to the nearest tenth of a foot. _____

24. Carlos walks $\sqrt{45}$ blocks and Dana walks 7 blocks. Without a calculator, who walks farther, and how do you know? _____

Answer Keys

1. $\boxed{5}$
2. $\boxed{3}$
3. $\boxed{3.1}$
4. $\boxed{4.2}$
5. $\boxed{1.7}$
6. $\boxed{6}$
7. $\boxed{12}$
8. $\boxed{4}$
9. $\boxed{5.1}$
10. $\boxed{2.1}$
11. $\boxed{6.3}$
12. $\boxed{7.7}$
13. $\boxed{5.7}$
14. $\boxed{9.1}$
15. $\boxed{\sqrt{20} > \sqrt{15}}$
16. $\boxed{\sqrt{2} < 1.5}$
17. $\boxed{\pi < \sqrt{10}}$
18. $\boxed{\sqrt{7}+1 < \sqrt{7}+2}$
19. $\boxed{6}$
20. $\boxed{\frac{7}{2}}$
21. $\boxed{\text{about } 13.4 \text{ cm}}$
22. $\boxed{\text{about } 1.4 \text{ in}}$
23. $\boxed{\text{about } 8.8 \text{ ft}}$
24. $\boxed{\text{Dana walks farther}}$

Step-by-Step Explanations

1. Both are perfect squares: $\sqrt{4} = 2$ and $\sqrt{9} = 3$, so the sum is exactly 5.
2. $\sqrt{16} = 4$ and $\sqrt{1} = 1$, so $4 - 1 = 3$.
3. $\sqrt{2} \approx 1.4$ and $\sqrt{3} \approx 1.7$, so the sum is about $1.4 + 1.7 = 3.1$.
4. $\sqrt{8} \approx 2.8$ and $\sqrt{2} \approx 1.4$, so $2.8 + 1.4 = 4.2$.
5. $\sqrt{10} \approx 3.16$ and $\sqrt{2} \approx 1.41$, so $3.16 - 1.41 \approx 1.7$.
6. $\sqrt{9} = 3$, so $2 \times 3 = 6$.
7. $\sqrt{16} = 4$, so $3 \times 4 = 12$.
8. Multiplying under one radical, $\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$ exactly.
9. Using $\pi \approx 3.1$, we get $3.1 + 2 = 5.1$.
10. With $\pi \approx 3.1$, $3.1 - 1 = 2.1$.
11. $2 \times 3.14 = 6.28$, which rounds to 6.3.
12. $\sqrt{15} \approx 3.87$, so $3.87 + 3.87 \approx 7.7$.
13. $\sqrt{50} \approx 7.1$ and $\sqrt{2} \approx 1.4$, so $7.1 - 1.4 = 5.7$.
14. $\sqrt{36} = 6$ and $\pi \approx 3.1$, so $6 + 3.1 = 9.1$.
15. A larger number under the radical gives a larger root, and $20 > 15$, so $\sqrt{20} > \sqrt{15}$.
16. $1.5^2 = 2.25$, which is more than 2, so $1.5 > \sqrt{2}$.
17. $\pi \approx 3.14$ and $\sqrt{10} \approx 3.16$, so π is slightly less than $\sqrt{10}$.
18. Both share $\sqrt{7}$, so the one with the larger added number is bigger.
19. $\sqrt{3} \cdot \sqrt{12} = \sqrt{36} = 6$ exactly.
20. $\sqrt{49} = 7$ and $\sqrt{4} = 2$, so the quotient is $\frac{7}{2}$.
21. $\sqrt{20} \approx 4.5$ and $\sqrt{5} \approx 2.2$. The perimeter is $2(4.5) + 2(2.2) = 9.0 + 4.4 = 13.4$ cm.
22. The sides are $\sqrt{18} \approx 4.2$ in and $\sqrt{8} \approx 2.8$ in. The difference is $4.2 - 2.8 = 1.4$ in.
23. With $\sqrt{2} \approx 1.4$, the circumference is $2 \times 3.14 \times 1.4 \approx 8.8$ ft.
24. $7 = \sqrt{49}$, and $45 < 49$, so $\sqrt{45} < 7$. That means Carlos walks less than Dana.

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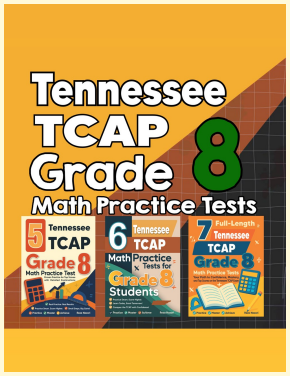

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