

# Equations with Special Solutions

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 24

## Q Quick Review

Most linear equations have **exactly one solution**, but two special things can happen. If your steps lead to a statement that is *always true*, like  $5 = 5$ , the equation has **infinitely many solutions** — every number works, because both sides are really the same expression. If your steps lead to a statement that is *never true*, like  $3 = 7$ , the equation has **no solution**. The way to tell: simplify both sides, gather the variable terms, and see whether the variable *survives* or *cancels out*.

◇ **Example:** Solve  $2(x + 3) = 2x + 6$ .

⇒ Let's just simplify and see what happens. Distribute the 2 on the left side:  $2x + 6 = 2x + 6$ . Now we notice both sides are *identical*. If we try to gather the variable terms by subtracting  $2x$  from both sides, the  $x$ 's vanish completely and we're left with  $6 = 6$  — a statement that is always true. That tells us *every* value of  $x$  makes the equation work, so there are infinitely many solutions.

**Answer:** infinitely many solutions

## PRACTICE

Solve each equation. Some have one solution, no solution, or infinitely many.

- |                         |       |                          |       |
|-------------------------|-------|--------------------------|-------|
| 1. $x + 4 = x + 4$      | _____ | 11. $5x + 3 = 5x$        | _____ |
| 2. $x + 2 = x + 9$      | _____ | 12. $6x - 2 = 2(3x - 1)$ | _____ |
| 3. $2x + 1 = 2x + 1$    | _____ | 13. $3x + 8 = 20$        | _____ |
| 4. $3x = 3x + 5$        | _____ | 14. $7x + 1 = 7x + 1$    | _____ |
| 5. $5x - 2 = 5x - 2$    | _____ | 15. $2(x + 4) = 2x + 7$  | _____ |
| 6. $4x + 7 = 4x - 1$    | _____ | 16. $5(x - 2) = 5x - 10$ | _____ |
| 7. $2(x + 1) = 2x + 2$  | _____ | 17. $4x - 9 = 4x + 3$    | _____ |
| 8. $3(x + 2) = 3x + 5$  | _____ | 18. $8x = 8x$            | _____ |
| 9. $2x + 5 = 11$        | _____ | 19. $6x + 2 = 3x + 17$   | _____ |
| 10. $4(x - 1) = 4x - 4$ | _____ | 20. $3(2x + 1) = 6x + 3$ | _____ |

## ◆ Word Problems

21. Sam claims that doubling the sum of a number and 4 always equals twice the number plus 8. Is he right? Solve  $2(x + 4) = 2x + 8$ . \_\_\_\_\_
22. A puzzle says: "A number plus 5 equals the same number plus 2." Solve  $x + 5 = x + 2$  and explain what it means. \_\_\_\_\_
23. Two phone plans cost the same: plan A is  $3x + 10$  dollars and plan B is  $3x + 25$  dollars for  $x$  months. For what  $x$  are they equal? \_\_\_\_\_
24. A teacher writes  $4(x + 3) = 4x + 12$  and asks which values of  $x$  make it true. What is the answer? \_\_\_\_\_



## Answer Keys

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|--|--|
| <ol style="list-style-type: none"> <li>1. infinitely many</li> <li>2. no solution</li> <li>3. infinitely many</li> <li>4. no solution</li> <li>5. infinitely many</li> <li>6. no solution</li> <li>7. infinitely many</li> <li>8. no solution</li> <li>9. <math>x = 3</math></li> <li>10. infinitely many</li> <li>11. no solution</li> <li>12. infinitely many</li> </ol> | <ol style="list-style-type: none"> <li>13. <math>x = 4</math></li> <li>14. infinitely many</li> <li>15. no solution</li> <li>16. infinitely many</li> <li>17. no solution</li> <li>18. infinitely many</li> <li>19. <math>x = 5</math></li> <li>20. infinitely many</li> <li>21. infinitely many solutions; Sam is right</li> <li>22. no solution</li> <li>23. no solution; never equal</li> <li>24. all real numbers</li> </ol> |
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### Step-by-Step Explanations

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| <ol style="list-style-type: none"> <li>1. Both sides are identical, so every value of <math>x</math> works.</li> <li>2. Subtract <math>x</math>: <math>2 = 9</math> is never true, so there is no solution.</li> <li>3. The two sides match exactly, so all numbers are solutions.</li> <li>4. Subtract <math>3x</math>: <math>0 = 5</math> is false, so no solution.</li> <li>5. Identical sides means infinitely many solutions.</li> <li>6. Subtract <math>4x</math>: <math>7 = -1</math> is false, so no solution.</li> <li>7. Distribute: <math>2x + 2 = 2x + 2</math> — identical, so infinitely many.</li> <li>8. Distribute: <math>3x + 6 = 3x + 5</math>; subtract <math>3x</math>: <math>6 = 5</math> is false.</li> <li>9. Subtract 5: <math>2x = 6</math>, then divide by 2: <math>x = 3</math> (one solution).</li> <li>10. Distribute: <math>4x - 4 = 4x - 4</math> — identical, infinitely many.</li> <li>11. Subtract <math>5x</math>: <math>3 = 0</math> is false, so no solution.</li> <li>12. Right side is <math>6x - 2</math>, matching the left, so infinitely many.</li> <li>13. Subtract 8: <math>3x = 12</math>, then divide by 3: <math>x = 4</math>.</li> <li>14. The sides are identical, so all values work.</li> </ol> | <ol style="list-style-type: none"> <li>15. Distribute: <math>2x + 8 = 2x + 7</math>; subtract <math>2x</math>: <math>8 = 7</math> is false.</li> <li>16. Distribute: <math>5x - 10 = 5x - 10</math> — identical sides.</li> <li>17. Subtract <math>4x</math>: <math>-9 = 3</math> is false, so no solution.</li> <li>18. Both sides are exactly the same, so every number is a solution.</li> <li>19. Subtract <math>3x</math>: <math>3x + 2 = 17</math>, subtract 2: <math>3x = 15</math>, so <math>x = 5</math>.</li> <li>20. Distribute: <math>6x + 3 = 6x + 3</math> — identical, infinitely many.</li> <li>21. Distribute the left side: <math>2x + 8 = 2x + 8</math>. The sides match for every number, so Sam's claim is always true.</li> <li>22. Subtract <math>x</math> from both sides: <math>5 = 2</math>, which is never true. No number can make this work.</li> <li>23. Set them equal: <math>3x + 10 = 3x + 25</math>. Subtract <math>3x</math>: <math>10 = 25</math> is false, so the plans are never the same cost.</li> <li>24. Distribute the left side: <math>4x + 12 = 4x + 12</math>. Both sides are identical, so every value of <math>x</math> is a solution.</li> </ol> |
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