

Equations with Special Solutions

Name: _____

Date: _____

Score: _____ / 24

Q Quick Review

Most linear equations have **exactly one solution**, but two special things can happen. If your steps lead to a statement that is *always true*, like $5 = 5$, the equation has **infinitely many solutions** — every number works, because both sides are really the same expression. If your steps lead to a statement that is *never true*, like $3 = 7$, the equation has **no solution**. The way to tell: simplify both sides, gather the variable terms, and see whether the variable *survives* or *cancels out*.

◇ **Example:** Solve $2(x + 3) = 2x + 6$.

⇒ Let's just simplify and see what happens. Distribute the 2 on the left side: $2x + 6 = 2x + 6$. Now we notice both sides are *identical*. If we try to gather the variable terms by subtracting $2x$ from both sides, the x 's vanish completely and we're left with $6 = 6$ — a statement that is always true. That tells us *every* value of x makes the equation work, so there are infinitely many solutions.

Answer: infinitely many solutions

PRACTICE

Solve each equation. Some have one solution, no solution, or infinitely many.

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|-------------------------|-------|--------------------------|-------|
| 1. $x + 4 = x + 4$ | _____ | 11. $5x + 3 = 5x$ | _____ |
| 2. $x + 2 = x + 9$ | _____ | 12. $6x - 2 = 2(3x - 1)$ | _____ |
| 3. $2x + 1 = 2x + 1$ | _____ | 13. $3x + 8 = 20$ | _____ |
| 4. $3x = 3x + 5$ | _____ | 14. $7x + 1 = 7x + 1$ | _____ |
| 5. $5x - 2 = 5x - 2$ | _____ | 15. $2(x + 4) = 2x + 7$ | _____ |
| 6. $4x + 7 = 4x - 1$ | _____ | 16. $5(x - 2) = 5x - 10$ | _____ |
| 7. $2(x + 1) = 2x + 2$ | _____ | 17. $4x - 9 = 4x + 3$ | _____ |
| 8. $3(x + 2) = 3x + 5$ | _____ | 18. $8x = 8x$ | _____ |
| 9. $2x + 5 = 11$ | _____ | 19. $6x + 2 = 3x + 17$ | _____ |
| 10. $4(x - 1) = 4x - 4$ | _____ | 20. $3(2x + 1) = 6x + 3$ | _____ |

◆ Word Problems

21. Sam claims that doubling the sum of a number and 4 always equals twice the number plus 8. Is he right? Solve $2(x + 4) = 2x + 8$. _____
22. A puzzle says: "A number plus 5 equals the same number plus 2." Solve $x + 5 = x + 2$ and explain what it means. _____
23. Two phone plans cost the same: plan A is $3x + 10$ dollars and plan B is $3x + 25$ dollars for x months. For what x are they equal? _____
24. A teacher writes $4(x + 3) = 4x + 12$ and asks which values of x make it true. What is the answer? _____



Answer Keys

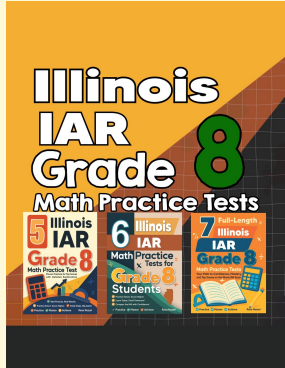
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|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ol style="list-style-type: none"> 1. infinitely many 2. no solution 3. infinitely many 4. no solution 5. infinitely many 6. no solution 7. infinitely many 8. no solution 9. $x = 3$ 10. infinitely many 11. no solution 12. infinitely many | <ol style="list-style-type: none"> 13. $x = 4$ 14. infinitely many 15. no solution 16. infinitely many 17. no solution 18. infinitely many 19. $x = 5$ 20. infinitely many 21. infinitely many solutions; Sam is right 22. no solution 23. no solution; never equal 24. all real numbers |
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Step-by-Step Explanations

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| <ol style="list-style-type: none"> 1. Both sides are identical, so every value of x works. 2. Subtract x: $2 = 9$ is never true, so there is no solution. 3. The two sides match exactly, so all numbers are solutions. 4. Subtract $3x$: $0 = 5$ is false, so no solution. 5. Identical sides means infinitely many solutions. 6. Subtract $4x$: $7 = -1$ is false, so no solution. 7. Distribute: $2x + 2 = 2x + 2$ — identical, so infinitely many. 8. Distribute: $3x + 6 = 3x + 5$; subtract $3x$: $6 = 5$ is false. 9. Subtract 5: $2x = 6$, then divide by 2: $x = 3$ (one solution). 10. Distribute: $4x - 4 = 4x - 4$ — identical, infinitely many. 11. Subtract $5x$: $3 = 0$ is false, so no solution. 12. Right side is $6x - 2$, matching the left, so infinitely many. 13. Subtract 8: $3x = 12$, then divide by 3: $x = 4$. 14. The sides are identical, so all values work. | <ol style="list-style-type: none"> 15. Distribute: $2x + 8 = 2x + 7$; subtract $2x$: $8 = 7$ is false. 16. Distribute: $5x - 10 = 5x - 10$ — identical sides. 17. Subtract $4x$: $-9 = 3$ is false, so no solution. 18. Both sides are exactly the same, so every number is a solution. 19. Subtract $3x$: $3x + 2 = 17$, subtract 2: $3x = 15$, so $x = 5$. 20. Distribute: $6x + 3 = 6x + 3$ — identical, infinitely many. 21. Distribute the left side: $2x + 8 = 2x + 8$. The sides match for every number, so Sam's claim is always true. 22. Subtract x from both sides: $5 = 2$, which is never true. No number can make this work. 23. Set them equal: $3x + 10 = 3x + 25$. Subtract $3x$: $10 = 25$ is false, so the plans are never the same cost. 24. Distribute the left side: $4x + 12 = 4x + 12$. Both sides are identical, so every value of x is a solution. |
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