

# Density of Real Numbers

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 24

## Q Quick Review

The real numbers are **dense**, which means that between *any* two different numbers — no matter how close together — there is always another number. In fact there are *infinitely many*! A reliable way to find one is to take the **average** (the midpoint): add the two numbers and divide by 2. You can also write both numbers as decimals and pick any value that fits between them. This is true for rationals and irrationals alike — there is never a “gap” on the real number line where no number lives.

◇ **Example:** Find a number between  $\frac{1}{4}$  and  $\frac{1}{3}$ .

⇒ The surest method is to find the midpoint — the average of the two numbers. First give them a common denominator:  $\frac{1}{4} = \frac{3}{12}$  and  $\frac{1}{3} = \frac{4}{12}$ . Now average them: add to get  $\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$ , then divide by 2 to get  $\frac{7}{24}$ . Since  $\frac{7}{24}$  sits exactly halfway between, it is guaranteed to be between the two numbers.

**Answer:**  $\frac{7}{24}$

## PRACTICE

Find a number between the two given numbers (the midpoint works well).

- |   |       |  |       |
|---|-------|--|-------|
| 1. between 2 and 3                          | _____ | 11. between $\frac{1}{5}$ and $\frac{2}{5}$  | _____ |
| 2. between 5 and 6                          | _____ | 12. between $\frac{2}{3}$ and $\frac{3}{4}$  | _____ |
| 3. between 0 and 1                          | _____ | 13. between 1.99 and 2                       | _____ |
| 4. between -1 and 0                         | _____ | 14. between 0.33 and 0.34                    | _____ |
| 5. between 1.2 and 1.3                      | _____ | 15. between $\sqrt{2}$ and $\sqrt{3}$        | _____ |
| 6. between 0.5 and 0.6                      | _____ | 16. between 3 and $\pi$                      | _____ |
| 7. between 3.7 and 3.8                      | _____ | 17. between $\pi$ and 4                      | _____ |
| 8. between -2 and -1                        | _____ | 18. between $-\frac{1}{2}$ and $\frac{1}{2}$ | _____ |
| 9. between $\frac{1}{2}$ and 1              | _____ | 19. between $\frac{5}{6}$ and 1              | _____ |
| 10. between $\frac{1}{3}$ and $\frac{2}{3}$ | _____ | 20. between 7.4 and 7.41                     | _____ |

## ◆ Word Problems

21. Two runners finish a race with times of 12.4 seconds and 12.5 seconds. Name a possible time for a runner who finished between them. \_\_\_\_\_
22. Sam claims there is no number between  $\frac{3}{8}$  and  $\frac{4}{8}$ . Find a number that proves him wrong. \_\_\_\_\_
23. A thermometer reads between 98.6°F and 98.7°F. Give one temperature it could be showing. \_\_\_\_\_
24. On a number line, explain why there must be a number between 0.999 and 1. \_\_\_\_\_



## Answer Keys

- |                            |  |
|----------------------------|--|
| 1. $\boxed{2.5}$           | 12. $\boxed{\frac{17}{24}}$            |
| 2. $\boxed{5.5}$           | 13. $\boxed{1.995}$                    |
| 3. $\boxed{0.5}$           | 14. $\boxed{0.335}$                    |
| 4. $\boxed{-0.5}$          | 15. $\boxed{1.5}$                      |
| 5. $\boxed{1.25}$          | 16. $\boxed{3.1}$                      |
| 6. $\boxed{0.55}$          | 17. $\boxed{3.5}$                      |
| 7. $\boxed{3.75}$          | 18. $\boxed{0}$                        |
| 8. $\boxed{-1.5}$          | 19. $\boxed{\frac{11}{12}}$            |
| 9. $\boxed{\frac{3}{4}}$   | 20. $\boxed{7.405}$                    |
| 10. $\boxed{\frac{1}{2}}$  | 21. $\boxed{12.45 \text{ seconds}}$    |
| 11. $\boxed{\frac{3}{10}}$ | 22. $\boxed{\frac{7}{16}}$             |
|                            | 23. $\boxed{98.65^\circ\text{F}}$      |
|                            | 24. $\boxed{\text{yes; e.g. } 0.9995}$ |

### Step-by-Step Explanations

- |   |  |
|---|--|
| <p>1. The midpoint is <math>\frac{2+3}{2} = 2.5</math>, which lies between them.</p> <p>2. Average them: <math>\frac{5+6}{2} = 5.5</math>.</p> <p>3. The midpoint <math>\frac{0+1}{2} = 0.5</math> is between 0 and 1.</p> <p>4. Average them: <math>\frac{-1+0}{2} = -0.5</math>.</p> <p>5. The midpoint is <math>\frac{1.2+1.3}{2} = 1.25</math>.</p> <p>6. Average them: <math>\frac{0.5+0.6}{2} = 0.55</math>.</p> <p>7. The midpoint is <math>\frac{3.7+3.8}{2} = 3.75</math>.</p> <p>8. Average them: <math>\frac{-2+(-1)}{2} = -1.5</math>.</p> <p>9. Average: <math>\frac{1}{2} + 1 = \frac{3}{2}</math>, then divide by 2 to get <math>\frac{3}{4}</math>.</p> <p>10. Average: <math>\frac{1}{3} + \frac{2}{3} = 1</math>, then divide by 2 to get <math>\frac{1}{2}</math>.</p> <p>11. Average: <math>\frac{1}{5} + \frac{2}{5} = \frac{3}{5}</math>, divided by 2 is <math>\frac{3}{10}</math>.</p> <p>12. Common denominator: <math>\frac{8}{12} + \frac{9}{12} = \frac{17}{12}</math>, divided by 2 is <math>\frac{17}{24}</math>.</p> <p>13. The midpoint is <math>\frac{1.99+2}{2} = 1.995</math>.</p> | <p>14. Average them: <math>\frac{0.33+0.34}{2} = 0.335</math>.</p> <p>15. <math>\sqrt{2} \approx 1.41</math> and <math>\sqrt{3} \approx 1.73</math>, and 1.5 fits neatly between them.</p> <p>16. <math>\pi \approx 3.14</math>, so 3.1 lies between 3 and <math>\pi</math>.</p> <p>17. <math>\pi \approx 3.14</math>, so 3.5 is between <math>\pi</math> and 4.</p> <p>18. The midpoint of <math>-\frac{1}{2}</math> and <math>\frac{1}{2}</math> is 0.</p> <p>19. Average: <math>\frac{5}{6} + 1 = \frac{11}{6}</math>, divided by 2 is <math>\frac{11}{12}</math>.</p> <p>20. The midpoint is <math>\frac{7.4+7.41}{2} = 7.405</math>.</p> <p>21. The midpoint of the two times is <math>\frac{12.4+12.5}{2} = 12.45</math> seconds, which is guaranteed to be between them.</p> <p>22. Average the two: <math>\frac{3}{8} + \frac{4}{8} = \frac{7}{8}</math>, then divide by 2 to get <math>\frac{7}{16}</math>, which is between them.</p> <p>23. The midpoint <math>\frac{98.6+98.7}{2} = 98.65^\circ\text{F}</math> lies between the two readings, so it is a valid possibility.</p> <p>24. The real numbers are dense, so between any two different numbers there is another. The midpoint <math>\frac{0.999+1}{2} = 0.9995</math> is one such number.</p> |
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