Counting Principle and Permutations

Name: ______________ Date: ______________ Score: ______ / 24

Quick Review

The **Fundamental Counting Principle** says: if one choice can happen in m ways and a second *independent* choice in n ways, then together they can happen in $m \times n$ ways — just multiply the options at each step. A **permutation** is an arrangement where *order matters*. The number of ways to arrange all n different items is $n! = n \times (n-1) \times \cdots \times 2 \times 1$ (read "n factorial"). To arrange r of n items, use $P(n, r) = \frac{n!}{(n-r)!}$, which is the product of the first r factors counting down from n. Remember $0! = 1$.

◇ **Example:** A deli offers 3 breads, 4 fillings, and 2 sauces. How many different sandwiches are possible? Also, in how many orders can 5 friends line up?

⇒ For the sandwich, use the counting principle: multiply the choices at each step. 3 breads $\times$ 4 fillings $\times$ 2 sauces $= 24$ sandwiches. For the line-up, order matters and we are arranging all 5 friends, so we use a factorial: $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ different orders. Notice how fast the factorial grows — arrangements pile up quickly!

Answer: 24 sandwiches, 120 line-ups

PRACTICE

Use the counting principle or permutations. Give a whole-number answer.

1. 3×4 ________
2. $2 \times 3 \times 5$ ________
3. $3 \times 4 \times 2$ ________
4. $4 \times 4 \times 4$ ________
5. 5×6 ________
6. $3!$ ________
7. $4!$ ________
8. $5!$ ________
9. $6!$ ________
10. $7!$ ________
11. $P(4, 2)$ ________
12. $P(5, 2)$ ________
13. $P(5, 3)$ ________
14. $P(6, 2)$ ________
15. $P(6, 3)$ ________
16. $P(7, 2)$ ________
17. $P(7, 3)$ ________
18. $P(8, 2)$ ________
19. $P(8, 3)$ ________
20. $P(10, 3)$ ________

♦ Word Problems

21. A school uniform has 3 shirt colors, 2 pant colors, and 4 shoe styles. How many different uniform combinations are possible? ________
22. Six runners finish a race. In how many different orders can they cross the finish line (no ties)? ________
23. A club of 8 members must choose a president and a vice president (different people). How many ways can this be done? ________
24. A lock code uses 4 digits, and each digit can be 0 through 9, with repeats allowed. How many codes are possible? ________

Answer Keys

1. 12
2. 30
3. 24
4. 64
5. 30
6. 6
7. 24
8. 120
9. 720
10. 5040
11. 12
12. 20
13. 60
14. 30
15. 120
16. 42
17. 210
18. 56
19. 336
20. 720
21. $3 \times 2 \times 4 = 24$ combinations
22. $6! = 720$ orders
23. $P(8, 2) = 8 \times 7 = 56$ ways
24. $10 \times 10 \times 10 \times 10 = 10{,}000$ codes

Step-by-Step Explanations

1. Counting principle: 3 choices then 4 choices give $3 \times 4 = 12$.
2. Multiply the options at each step: $2 \times 3 \times 5 = 30$.
3. Multiply across all three stages: $3 \times 4 \times 2 = 24$.
4. Four options at each of three stages: $4 \times 4 \times 4 = 64$.
5. Two independent choices: $5 \times 6 = 30$ combinations.
6. $3! = 3 \times 2 \times 1 = 6$.
7. $4! = 4 \times 3 \times 2 \times 1 = 24$.
8. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
9. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.
10. $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$.
11. $P(4, 2) = 4 \times 3 = 12$ (the first 2 factors counting down from 4).
12. $P(5, 2) = 5 \times 4 = 20$.
13. $P(5, 3) = 5 \times 4 \times 3 = 60$.
14. $P(6, 2) = 6 \times 5 = 30$.
15. $P(6, 3) = 6 \times 5 \times 4 = 120$.
16. $P(7, 2) = 7 \times 6 = 42$.
17. $P(7, 3) = 7 \times 6 \times 5 = 210$.
18. $P(8, 2) = 8 \times 7 = 56$.
19. $P(8, 3) = 8 \times 7 \times 6 = 336$.
20. $P(10, 3) = 10 \times 9 \times 8 = 720$.
21. By the counting principle, multiply the choices at each step: $3 \times 2 \times 4 = 24$ different uniforms.
22. All 6 runners are being arranged and order matters, so it is $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ possible finishing orders.
23. Order matters — president and vice president are different roles — so use $P(8, 2) = 8 \times 7 = 56$ ways.
24. Each of the 4 positions has 10 choices and repeats are allowed, so by the counting principle there are $10^4 = 10{,}000$ codes.

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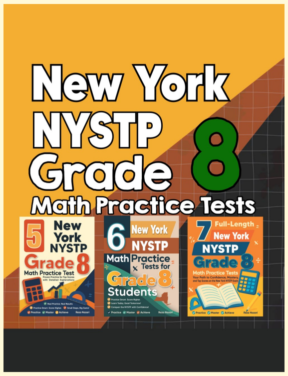

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