

Counting Principle and Permutations

Name: _____ Date: _____ Score: _____ / 24

Q Quick Review

The **Fundamental Counting Principle** says: if one choice can happen in m ways and a second *independent* choice in n ways, then together they can happen in $m \times n$ ways — just multiply the options at each step. A **permutation** is an arrangement where *order matters*. The number of ways to arrange all n different items is $n! = n \times (n - 1) \times \dots \times 2 \times 1$ (read “ n factorial”). To arrange r of n items, use $P(n, r) = \frac{n!}{(n - r)!}$, which is the product of the first r factors counting down from n . Remember $0! = 1$.

◇ **Example:** A deli offers 3 breads, 4 fillings, and 2 sauces. How many different sandwiches are possible? Also, in how many orders can 5 friends line up?

⇒ For the sandwich, use the counting principle: multiply the choices at each step. $3 \text{ breads} \times 4 \text{ fillings} \times 2 \text{ sauces} = 24$ sandwiches. For the line-up, order matters and we are arranging all 5 friends, so we use a factorial: $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ different orders. Notice how fast the factorial grows — arrangements pile up quickly!

Answer: 24 sandwiches, 120 line-ups

PRACTICE

Use the counting principle or permutations. Give a whole-number answer.

- | | | | |
|--------------------------|-------|----------------|-------|
| 1. 3×4 | _____ | 11. $P(4, 2)$ | _____ |
| 2. $2 \times 3 \times 5$ | _____ | 12. $P(5, 2)$ | _____ |
| 3. $3 \times 4 \times 2$ | _____ | 13. $P(5, 3)$ | _____ |
| 4. $4 \times 4 \times 4$ | _____ | 14. $P(6, 2)$ | _____ |
| 5. 5×6 | _____ | 15. $P(6, 3)$ | _____ |
| 6. $3!$ | _____ | 16. $P(7, 2)$ | _____ |
| 7. $4!$ | _____ | 17. $P(7, 3)$ | _____ |
| 8. $5!$ | _____ | 18. $P(8, 2)$ | _____ |
| 9. $6!$ | _____ | 19. $P(8, 3)$ | _____ |
| 10. $7!$ | _____ | 20. $P(10, 3)$ | _____ |

◆ Word Problems

21. A school uniform has 3 shirt colors, 2 pant colors, and 4 shoe styles. How many different uniform combinations are possible?

22. Six runners finish a race. In how many different orders can they cross the finish line (no ties)? _____
23. A club of 8 members must choose a president and a vice president (different people). How many ways can this be done?

24. A lock code uses 4 digits, and each digit can be 0 through 9, with repeats allowed. How many codes are possible? _____



Answer Keys

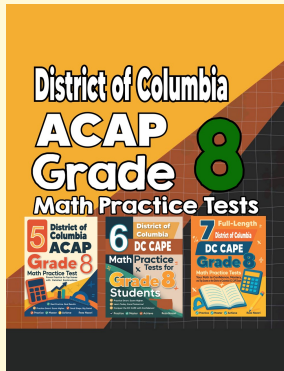
- | | |
|----------|---|
| 1. 12 | 13. 60 |
| 2. 30 | 14. 30 |
| 3. 24 | 15. 120 |
| 4. 64 | 16. 42 |
| 5. 30 | 17. 210 |
| 6. 6 | 18. 56 |
| 7. 24 | 19. 336 |
| 8. 120 | 20. 720 |
| 9. 720 | 21. $3 \times 2 \times 4 = 24$ combinations |
| 10. 5040 | 22. $6! = 720$ orders |
| 11. 12 | 23. $P(8, 2) = 8 \times 7 = 56$ ways |
| 12. 20 | 24. $10 \times 10 \times 10 \times 10 = 10,000$ codes |

Step-by-Step Explanations

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| <p>1. Counting principle: 3 choices then 4 choices give $3 \times 4 = 12$.</p> <p>2. Multiply the options at each step: $2 \times 3 \times 5 = 30$.</p> <p>3. Multiply across all three stages: $3 \times 4 \times 2 = 24$.</p> <p>4. Four options at each of three stages: $4 \times 4 \times 4 = 64$.</p> <p>5. Two independent choices: $5 \times 6 = 30$ combinations.</p> <p>6. $3! = 3 \times 2 \times 1 = 6$.</p> <p>7. $4! = 4 \times 3 \times 2 \times 1 = 24$.</p> <p>8. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.</p> <p>9. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.</p> <p>10. $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$.</p> <p>11. $P(4, 2) = 4 \times 3 = 12$ (the first 2 factors counting down from 4).</p> <p>12. $P(5, 2) = 5 \times 4 = 20$.</p> <p>13. $P(5, 3) = 5 \times 4 \times 3 = 60$.</p> <p>14. $P(6, 2) = 6 \times 5 = 30$.</p> | <p>15. $P(6, 3) = 6 \times 5 \times 4 = 120$.</p> <p>16. $P(7, 2) = 7 \times 6 = 42$.</p> <p>17. $P(7, 3) = 7 \times 6 \times 5 = 210$.</p> <p>18. $P(8, 2) = 8 \times 7 = 56$.</p> <p>19. $P(8, 3) = 8 \times 7 \times 6 = 336$.</p> <p>20. $P(10, 3) = 10 \times 9 \times 8 = 720$.</p> <p>21. By the counting principle, multiply the choices at each step: $3 \times 2 \times 4 = 24$ different uniforms.</p> <p>22. All 6 runners are being arranged and order matters, so it is $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ possible finishing orders.</p> <p>23. Order matters — president and vice president are different roles — so use $P(8, 2) = 8 \times 7 = 56$ ways.</p> <p>24. Each of the 4 positions has 10 choices and repeats are allowed, so by the counting principle there are $10^4 = 10,000$ codes.</p> |
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