

# Volume of Pyramids

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: \_\_\_\_\_ / 17

Here is a cool fact: a pyramid holds exactly *one third* the volume of a prism with the same base and height—and that is why the formula is  $V_{\text{pyramid}} = \frac{1}{3}Bh$ ! The height in this formula is the **vertical** height, not the slant height along a face, so watch out for that common mixup. Once the one-third relationship makes sense to you, the formula stops feeling like something to memorize and starts feeling like something you *understand*.



## Key Concepts & Quick Review

**Any pyramid:**  $V = \frac{1}{3} \times B \times h$  ( $B$  = base area,  $h$  = **vertical** height, not slant height  $\ell$ ).

**Square pyramid:**  $V = \frac{1}{3}b^2h$ . **Triangular pyramid:**  $V = \frac{1}{3} \cdot \frac{1}{2}b_{\Delta}h_{\Delta} \cdot H$ .

Three pyramids of the same base and height = one prism of the same base and height.

## Examples

① Find the volume of a square pyramid: base  $9\text{ cm}$ , height  $12\text{ cm}$ .

**Think It Through:** Use the pyramid volume formula  $V = \frac{1}{3}Bh$ . For a square base of side  $9\text{ cm}$ , the base area is  $B = 9^2 = 81\text{ cm}^2$ . Then multiply by the vertical height  $12$  and take one third:  $V = \frac{1}{3}(81)(12) = 324\text{ cm}^3$ . The  $\frac{1}{3}$  factor is what makes a pyramid smaller than a prism with the same base and height.

**Answer:**  $324\text{ cm}^3$

② A rectangular-base pyramid has base  $10\text{ m} \times 6\text{ m}$  and height  $9\text{ m}$ . Find its volume. How does it compare to the prism with the same base and height?

**Think It Through:** Start with the base area:  $B = 10 \times 6 = 60\text{ m}^2$ . Then use  $V_{\text{pyr}} = \frac{1}{3}Bh = \frac{1}{3}(60)(9) = 180\text{ m}^3$ . A prism with the same base and height would have volume  $Bh = 60 \times 9 = 540\text{ m}^3$ . That is exactly three times the pyramid volume, which matches the standard prism-pyramid relationship.

**Answer:**  $V = 180\text{ m}^3$ ; *prism is 3× larger*



**Practice Problems**

Find the volume of each pyramid.

1. A square pyramid has base side length 6 units and vertical height 4 units. Find its volume. \_\_\_\_\_
2. A square pyramid has base side length 9 units and vertical height 10 units. Find its volume. \_\_\_\_\_
3. A square pyramid has base side length 5 units and vertical height 12 units. Find its volume. \_\_\_\_\_
4. A square pyramid has base side length 8 units and vertical height 9 units. Find its volume. \_\_\_\_\_
5. A square pyramid has base side length 3 units and vertical height 7 units. Find its volume. \_\_\_\_\_
6. A pyramid has a rectangular base measuring 8 by 6 units and vertical height 5 units. Find its volume. \_\_\_\_\_
7. A pyramid has a rectangular base measuring 10 by 4 units and vertical height 9 units. Find its volume. \_\_\_\_\_
8. A pyramid has a triangular base with base 6 units and height 4 units. The pyramid's vertical height is 10 units. Find its volume. \_\_\_\_\_
9. A square pyramid has base side length 12 units and vertical height 8 units. Find its volume. \_\_\_\_\_
10. A square pyramid has base side length 2 units and vertical height 15 units. Find its volume. \_\_\_\_\_
11. A square pyramid has base side length  $x$  units and vertical height 6 units. If  $x = 5$ , find the volume. \_\_\_\_\_



12. A square pyramid has volume 192 cubic units and base side length 8 units. Find its vertical height. \_\_\_\_\_



13. A square pyramid has volume 75 cubic units and vertical height 9 units. Find its base side length. \_\_\_\_\_



14. A pyramid has base area 36 square units and vertical height 15 units. Find its volume. \_\_\_\_\_



15. A square pyramid has base side length 10 units and vertical height 3 units. Find its volume. \_\_\_\_\_



**Study Tips**

- ✎ Always use the **vertical height**  $h$  (from base to apex, perpendicular to base) — not the slant height  $l$ .
- ✎ The  $\frac{1}{3}$  factor is not optional — it's the essential difference between a pyramid and a prism. Forgetting it triples your answer.
- ✎ **Sanity check:** pyramid volume must always be *less than* the prism with the same base and height. If it's not, recheck.

**Word Problems**

16. An ancient Egyptian stone pyramid has a square base of  $200\text{ m}$  and a vertical height of  $120\text{ m}$ . Find its volume in cubic meters. If the stone has a density of  $2,500\text{ kg/m}^3$ , estimate the mass of the pyramid in kg. Express it in scientific notation. \_\_\_\_\_
17. A sand hopper on a construction site drops sand into a pile shaped like a square pyramid. The base is  $3\text{ m} \times 3\text{ m}$  and the pile grows at a rate of  $0.5\text{ m}$  in height per minute. Find the volume after  $4\text{ min}$  and after  $10\text{ min}$ . At what height will the volume reach  $18\text{ m}^3$ ? \_\_\_\_\_



## Answer Keys

- |   |   |
|---|---|
| <p>1) 48<br/>2) 270<br/>3) 100<br/>4) 192<br/>5) 21<br/>6) 80<br/>7) 120<br/>8) 40<br/>9) 384</p> | <p>10) 20<br/>11) 50<br/>12) 9<br/>13) 5<br/>14) 180<br/>15) 100<br/>16) <math>1,600,000 m^3</math>; mass <math>4 \times 10^9 kg</math><br/>17) After 4 min: <math>6 m^3</math>; after 10 min: <math>15 m^3</math>; for <math>18 m^3</math>: <math>h = 6 m</math></p> |
|---|---|

### Step-by-Step Explanations

**Strategy:** For Circumference of Circles, decide whether the problem gives radius or diameter before choosing the circumference formula. Circle problems become predictable once radius and diameter are separated.

**Practice 1:** Find the circumference of a circle with radius  $5 cm$ . **Answer:**  $31.4 cm$

For the first sample, use diameter with  $C$  equals pi  $d$  or radius with  $C$  equals two pi  $r$ .

**Practice 15:** Find the circumference of a circle with diameter  $4.2 cm$ . **Answer:**  $13.19 cm$

Late in the set, use diameter with  $C$  equals pi  $d$  or radius with  $C$  equals two pi  $r$ .

**Word-problem notes:**

**16. Answer:**  $C = 2\pi(40) \approx 251.3 m$ ; laps =  $5,000/251.3 \approx 19.9$ , so 20 laps; time =  $5/8 \times 60 = 37.5 min$ .

Use the radius to find the track circumference:  $C = 2\pi r = 2\pi(40) \approx 251.3 m$ . The athlete wants to run  $5 km$ , which is  $5,000 m$ , so divide by one lap length:  $5,000 \div 251.3 \approx 19.9$ . Since she needs complete laps to reach at least  $5 km$ , she must run 20 laps. For the running time, use time =  $\frac{\text{distance}}{\text{speed}} = \frac{5}{8}$  hour, which is  $37.5 min$ .

**17. Answer:**  $C = \pi(36) \approx 113.1 cm$ ; border area  $\approx 113.1 \times 2 \approx 226.2 cm^2$ .

The crust border goes all the way around the pizza, so its length is the pizza's circumference. Since the diameter is  $36 cm$ , use  $C = \pi d = \pi(36) \approx 113.1 cm$ . The border is about  $2 cm$  wide, so its area can be estimated by treating it like a long thin strip: length times width, or  $113.1 \times 2 \approx 226.2 cm^2$ .



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