

# Translations, Reflections, and Rotations

Name: \_\_\_\_\_

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A **transformation** moves or changes a figure on the coordinate plane, and in this topic you will meet three important ones! **Translations** (slides) shift every point the same distance in the same direction; **reflections** (flips) mirror a figure across a line like the  $x$ -axis or  $y$ -axis; and **rotations** (turns) spin a figure around a fixed point, usually the origin. All three are *rigid motions*—they change position but never size or shape, so the original and the image are always congruent. Each one has a simple coordinate rule (for example, reflecting over the  $x$ -axis sends  $(x, y)$  to  $(x, -y)$ ), and learning those rules lets you move figures with precision and describe the motion with algebra!



## Key Concepts & Quick Review

**Translation**  $(x, y) \rightarrow (x + a, y + b)$ : every point shifts  $a$  units horizontally and  $b$  units vertically.

**Reflection** rules for common lines of reflection:

- Over the  $x$ -axis:  $(x, y) \rightarrow (x, -y)$
- Over the  $y$ -axis:  $(x, y) \rightarrow (-x, y)$
- Over  $y = x$ :  $(x, y) \rightarrow (y, x)$

**Rotation about the origin** (counter-clockwise):

- $90^\circ$ :  $(x, y) \rightarrow (-y, x)$
- $180^\circ$ :  $(x, y) \rightarrow (-x, -y)$
- $270^\circ$ :  $(x, y) \rightarrow (y, -x)$

All three transformations produce an image **congruent** to the original figure.

## Examples

① Translate the point  $(3, -2)$  by  $(x + 4, y - 1)$ .

**Think It Through:** A translation slides every point the same amount, so just add the shift to each coordinate. For  $x$ :  $3 + 4 = 7$ . For  $y$ :  $-2 - 1 = -3$ . The image lands at  $(7, -3)$ —the point moved 4 units



right and 1 unit down, which is exactly what the rule says.

**Answer:**  $(7, -3)$

② Rotate the point  $(2, 5)$  by  $90^\circ$  counter-clockwise about the origin.

**Think It Through:** For a  $90^\circ$  counter-clockwise rotation about the origin, the rule is  $(x, y) \rightarrow (-y, x)$ . Plug in: the new  $x$ -coordinate becomes  $-5$  (flip the sign of the old  $y$ ), and the new  $y$ -coordinate becomes  $2$  (the old  $x$  stays the same). The image is  $(-5, 2)$ .

**Answer:**  $(-5, 2)$

**Practice Problems**

Apply the transformation. Write the coordinates of the image point.

1. Translate the point  $(1, 4)$  using the rule  $(x, y) \rightarrow (x + 3, y - 2)$ . Write the image point. \_\_\_\_\_
2. Translate the point  $(-2, 5)$  using the rule  $(x, y) \rightarrow (x - 1, y + 4)$ . Write the image point. \_\_\_\_\_
3. Reflect the point  $(0, 3)$  over the  $x$ -axis. Write the image point. \_\_\_\_\_
4. Reflect the point  $(4, -1)$  over the  $y$ -axis. Write the image point. \_\_\_\_\_
5. Rotate the point  $(6, 2)$   $90^\circ$  counterclockwise about the origin. Write the image point. \_\_\_\_\_
6. Rotate the point  $(3, -4)$   $180^\circ$  about the origin. Write the image point. \_\_\_\_\_
7. Reflect the point  $(-5, 1)$  over the  $x$ -axis. Write the image point. \_\_\_\_\_
8. Translate the point  $(2, 7)$  using the rule  $(x, y) \rightarrow (x - 5, y - 3)$ . Write the image point. \_\_\_\_\_
9. Rotate the point  $(-3, -2)$   $270^\circ$  counterclockwise about the origin. Write the image point. \_\_\_\_\_
10. Reflect the point  $(1, -6)$  over the  $y$ -axis. Write the image point. \_\_\_\_\_

11. Rotate the point  $(4, 0)$   $90^\circ$  counterclockwise about the origin. Write the image point. \_\_\_\_\_



12. Reflect the point  $(5, 5)$  over the line  $y = x$ . Write the image point. \_\_\_\_\_



13. Rotate the point  $(-1, 3)$   $180^\circ$  about the origin. Write the image point. \_\_\_\_\_

14. Translate the point  $(0, -4)$  using the rule  $(x, y) \rightarrow (x + 6, y + 4)$ . Write the image point. \_\_\_\_\_



15. Reflect the point  $(7, 2)$  over the  $x$ -axis. Write the image point.



**Study Tips**

- 👉 For **reflections**, think of folding the paper along the line of reflection. The image is the same distance from the line as the original.
- 👉 Memorise the three rotation rules. A quick check: after a  $90^\circ$  CCW rotation, a point in Quadrant I moves to Quadrant II.
- 👉 Rigid motions do **not** change lengths or angles. If the image is a different size, the transformation is not a translation, reflection, or rotation.

**Word Problems**

16. A triangle has vertices  $A(1, 2)$ ,  $B(4, 2)$ , and  $C(4, 6)$ . It is translated 3 units left and 2 units up. What are the new vertices? \_\_\_\_\_
17. Point  $P(3, -1)$  is reflected over the  $x$ -axis and then rotated  $90^\circ$  counter-clockwise about the origin. What are the final coordinates? \_\_\_\_\_
18. This coordinate plane shows a pre-image triangle  $\triangle ABC$  and its image  $\triangle A'B'C'$ . Identify the transformation (translation, reflection, or rotation) that maps the pre-image to the image, list the coordinates of the three image vertices, and write the rule (e.g.,  $(x, y) \rightarrow (\dots)$ ). \_\_\_\_\_



## Answer Keys

- |  |  |
|--|--|
| <p>1) (4, 2)</p> <p>2) (-3, 9)</p> <p>3) (0, -3)</p> <p>4) (-4, -1)</p> <p>5) (-2, 6)</p> <p>6) (-3, 4)</p> <p>7) (-5, -1)</p> <p>8) (-3, 4)</p> <p>9) (-2, 3)</p> <p>10) (-1, -6)</p> | <p>11) (0, 4)</p> <p>12) (5, 5)</p> <p>13) (1, -3)</p> <p>14) (6, 0)</p> <p>15) (7, -2)</p> <p>16) <math>A'(-2, 4), B'(1, 4), C'(1, 8)</math>.</p> <p>17) <math>(-1, 3)</math>.</p> <p>18) Reflection over the <math>y</math>-axis; <math>A'(-1, 2), B'(-4, 2), C'(-4, 5)</math>; rule: <math>(x, y) \rightarrow (-x, y)</math>.</p> |
|--|--|

### Step-by-Step Explanations

**Strategy:** For Solving Multi-Step Problems with Rational Numbers, choose the operation that matches the story, then keep fractions and signs organized through every step. The context should tell whether the answer is a total, a change, a distance, or a rate.

**Practice 1:**  $\frac{x}{2} + 3 = 7$  **Answer:** 8

At the beginning of the practice, combine like terms on both sides before choosing the inverse operation.

**Practice 15:**  $2(0.5x-3)=4$  **Answer:** 10

For the second model problem, clear parentheses and constants step by step so the final variable term is visible.

**Word-problem notes:**

**16. Answer:**  $12.5 + 8.75w = 30 + 5.5w \Rightarrow w = \frac{70}{13} \approx 5.38$  weeks; each will have about \$59.62; Devon saves \$3.25/wk faster.

Write an expression for each friend's savings after  $w$  weeks. Devon has  $12.5 + 8.75w$ , and Kenji has  $30 + 5.5w$ . Set them equal because the question asks when the amounts match:  $12.5 + 8.75w = 30 + 5.5w$ . Subtract  $5.5w$  to get  $12.5 + 3.25w = 30$ , then subtract 12.5 and divide by 3.25. This gives  $w = \frac{70}{13} \approx 5.38$  weeks. Substituting that value back in shows each will have about \$59.62. Devon saves faster because  $8.75 - 5.50 = 3.25$ , so he saves \$3.25 more per week.

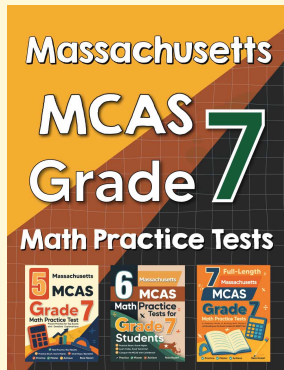
**17. Answer:**  $-\frac{3}{8}t = -4.5 \Rightarrow t = 12 \text{ min}$ ; total:  $12 + 15 = 27 < 30 \text{ min}$  — yes, possible.

The ranger is going downward, so the elevation change equation is  $-\frac{3}{8}t = -4.5$ . To solve for time, divide both sides by  $-\frac{3}{8}$ , or multiply by its reciprocal. That gives  $t = 12 \text{ min}$ . Now add the 15-minute setup time:  $12 + 15 = 27 \text{ min}$ . Since  $27 < 30$ , the full operation is possible within the required time limit.



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