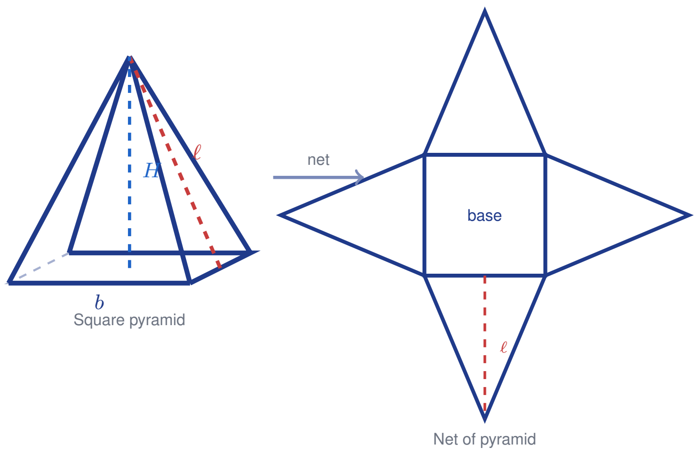
Surface Area of Pyramids

Name: ____________	Date: ____________	Score: ______ / 17

A pyramid has one base and triangular faces that meet at the top (the apex)—and to find its surface area, you add the base area to the areas of all those triangular sides. The key measurement is the **slant height**: the height *along* a triangular face, not the straight vertical height of the whole pyramid. Mixing up those two heights is the most common mistake in pyramid problems, so label them carefully! Once you keep them separate, pyramid surface area is a straightforward calculation.



Key Concepts & Quick Review

Square pyramid SA: $SA = b^2 + 4 \times \frac{1}{2} b\ell = b^2 + 2b\ell$

Any pyramid: $SA =$ Base area $+ \frac{1}{2} \times P_{\text{base}} \times \ell$ (P = base perimeter, ℓ = slant height).

Slant height $\ell \neq$ vertical height H. Use: $\ell = \sqrt{H^2 + (b/2)^2}$ (Pythagorean theorem).

Examples

① Find the surface area of a square pyramid with base $8\ cm$ and slant height $10\ cm$.

Think It Through: For a square pyramid, total surface area is the base area plus the four triangular faces. The shortcut formula is $SA = b^2 + 2b\ell$. Substitute $b = 8$ and $\ell = 10$: $8^2 + 2(8)(10) = 64 + 160$. The base contributes $64\ cm^2$, and the triangular faces together contribute $160\ cm^2$, giving $224\ cm^2$ in total.

Answer: $224\ cm^2$

② A square pyramid has base $12\ m$ and vertical height $8\ m$. Find the slant height and surface area.

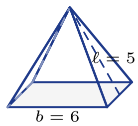
Think It Through: Because surface area uses slant height, find ℓ first. Draw the right triangle formed by the vertical height 8, half the base $12/2 = 6$, and slant height ℓ. Then use the Pythagorean theorem: $\ell = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = 10\ m$. Now use the square pyramid formula $SA = b^2 + 2b\ell = 12^2 + 2(12)(10) = 144 + 240 = 384\ m^2$.

Answer: $\ell = 10\ m;\ SA = 384\ m^2$

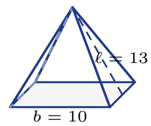
Practice Problems

Find the surface area of each pyramid. Find ℓ first if only H is given.

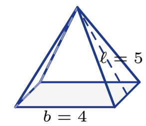
1. Find the surface area of the square pyramid.



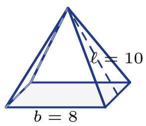
2. Find the surface area of the square pyramid.



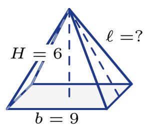
3. Find the surface area of the square pyramid.



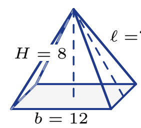
4. Find the surface area of the square pyramid.



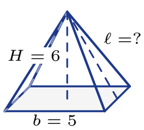
5. Find the slant height, then find the surface area.



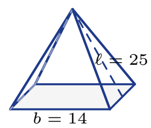
6. Find the slant height, then find the surface area.



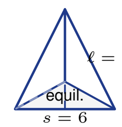
7. Find the slant height, then find the surface area.



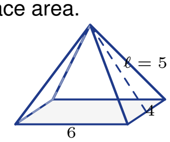
8. Find the surface area of the square pyramid.



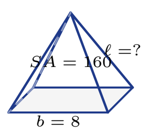
9. Find the surface area of the pyramid with a triangular base.



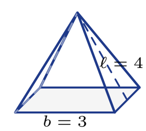
10. Use $B = 24$, $P = 20$, and $\ell = 5$ to find the surface area.



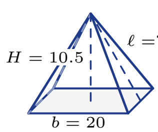
11. Find the slant height of the square pyramid.



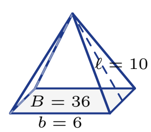
12. Find the surface area of the square pyramid.



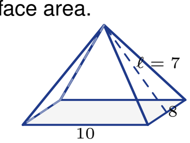
13. Find the slant height, then find the surface area.



14. Use the given base area to find the surface area.



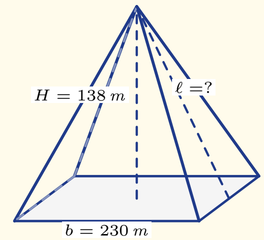
15. Use $B = 80$, $P = 36$, and $\ell = 7$ to find the surface area.



Study Tips

- **Slant height $\ell \neq$ vertical height H.** Always check which one is given. Draw a right triangle cross-section to find the other.
- The **lateral surface area** is $\frac{1}{2}P\ell$ (half the perimeter times slant height). Adding the base gives total SA.
- For a square pyramid: $SA = b^2 + 2b\ell$. Remember it as base squared plus the four triangle areas.

✍ Word Problems



lateral area only

16. The Pyramid of Giza (simplified) has a square base of $230\ m$ and a vertical height of $138\ m$. Calculate the slant height of each triangular face. Then find the total lateral surface area (the four triangular faces only). If one archaeologist can survey $500\ m^2$ per day, how many days for the full lateral surface?

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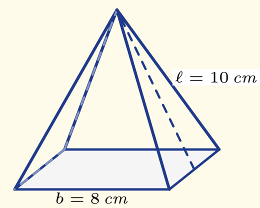

include base

17. A decorative chocolate pyramid has a square base of $8\ cm$ and slant height of $10\ cm$. The entire outside (including base) is dipped in gold foil. Find the total surface area of foil needed for one pyramid. A factory produces 200 pyramids per batch; how many cm^2 of foil per batch?

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Answer Keys

1) 96
2) 360
3) 56
4) 224
5) $\ell = 7.5$; $SA = 216$
6) $\ell = 10$; $SA = 384$
7) $\ell = 6.5$; $SA = 90$
8) 896
9) about 87.6
10) 74
11) 6
12) 33
13) $\ell = 14.5$; $SA = 980$
14) 156
15) 206
16) $\ell \approx 179.6\,m$; lateral $SA \approx 82{,}616\,m^2$; about 165 days
17) $224\,cm^2$; batch $44{,}800\,cm^2$

Step-by-Step Explanations

Strategy: For Area of Quadrilaterals, identify the shape first so the correct formula is doing the work: rectangle, parallelogram, trapezoid, rhombus, or kite. This page is about choosing the formula because the shape has been identified.

Practice 1: Find the area of a rectangle with base 9 and height 7. **Answer:** 63
At the beginning of this area practice, choose the quadrilateral formula from the labeled shape before substituting values.

Practice 15: Find the area of a trapezoid with bases $\frac{3}{2}$ and $\frac{5}{2}$ and height 4. **Answer:** 8
For the later quadrilateral-area model, match the dimensions to the formula first; the names of the parts matter as much as the numbers.

Word-problem notes:

16. Answer: $A = \frac{1}{2}(24+36)(15) = 450\,m^2$; cost $= 450 \times 4.20 = \$1{,}890$.
Add the two parallel sides first: $24 + 36 = 60$. Then use the trapezoid formula $A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(60)(15) = 450\,m^2$. Once the area is known, multiply by the price per square metre: $450 \times 4.20 = \$1{,}890$. The height must be the perpendicular distance between the parallel sides, not a slanted edge.

17. Answer: One tile: $\frac{1}{2}(18)(24) = 216\,cm^2$; total: $32{,}400\,cm^2$; boxes needed: $\lceil 150/12 \rceil = 13$; cost: $\$364$.
The area of one rhombus tile is found from its diagonals: $A = \frac{1}{2}(18)(24) = 216\,cm^2$. For 150 tiles, multiply: $150 \times 216 = 32{,}400\,cm^2$. To buy the tiles, divide by 12 tiles per box. Since $150 \div 12 = 12.5$, you must round up to 13 boxes because you cannot buy half a box. Finally, $13 \times 28 = \$364$.