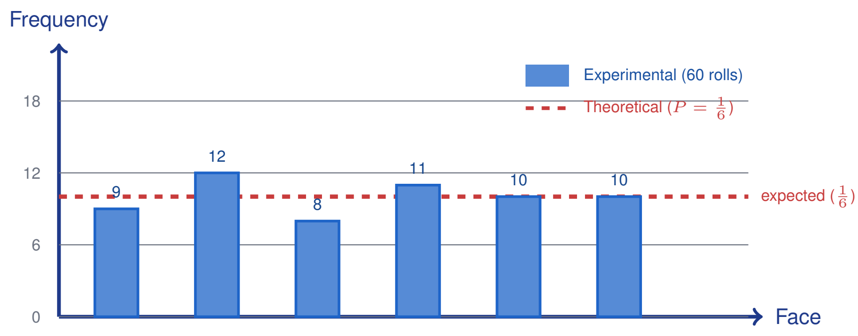
Simulations and Experimental Probability

Name: ____________________	Date: ______________	Score: _____ / 17

Probability can come from math *or* from running actual experiments—and it is fascinating to see how the two compare! **Theoretical probability** is based on what *should* happen in a perfectly fair situation, while **experimental probability** is based on what *actually* happened in your trials. In small samples the two may be quite different, but as you run more and more trials they tend to get closer and closer. This idea bridges the gap between formulas and real-world data!



With more trials, bars converge toward the dashed line — Law of Large Numbers

Key Concepts & Quick Review

$P_{\text{exp}}(E) = \dfrac{\text{favourable trials}}{\text{total trials}}$ vs. $P_{\text{theo}}(E) = \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$

Law of Large Numbers: the more trials run, the closer experimental probability gets to theoretical. Small samples are noisy — never trust 5 coin flips to estimate a fair coin.

Examples

① A coin is flipped 80 times; heads appears 44 times. (a) Find the experimental probability of heads. (b) How far is this from theoretical? Is the coin likely fair?

Think It Through: Experimental probability uses the actual results: $\frac{44}{80} = 0.55 = 55\%$. The theoretical probability for a fair coin is 50%. The difference is therefore $55\% - 50\% = 5\%$. With only 80 flips, a difference like that is not unusual, so the results still look consistent with a fair coin.

Answer: *(a)* $\frac{44}{80} = 0.55$*; (b)* 5% *off; likely fair*

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② Design a simulation to find the probability that a basketball player who makes $\frac{2}{3}$ of free throws scores at least 2 out of 3 attempts. Use a die: 1–4 = make, 5–6 = miss. Run 10 trials mentally.

Think It Through: A fair die can model a $\frac{2}{3}$ success rate by letting 1–4 mean make and 5–6 mean miss. One simulation trial is three die rolls, one for each free throw. Count the number of makes; the trial is a success if there are at least 2 makes. Repeating this many times estimates the probability. The theoretical value comes from adding the probability of exactly 2 makes and exactly 3 makes, which gives

$\frac{20}{27} \approx 74\%$.

Answer: *Simulation method described;* $P \approx 74\%$ *theoretically*

Practice Problems

Compute experimental probability or compare to theoretical.

1. A coin is flipped 40 times and lands on heads 18 times. Find the experimental probability of heads. ______
2. A die is rolled 50 times and lands on six 12 times. Find the experimental probability of rolling a six. ______
3. A spinner is spun 100 times and lands on red 38 times. Find the experimental probability of red. ______
4. A simulation has 84 successful outcomes in 200 trials. Find the experimental probability of success. ______
5. A coin is flipped 30 times and lands on heads 16 times. Compare the experimental probability of heads to the theoretical probability. ______
6. A die is rolled 60 times and lands on one 10 times. Is this far from the theoretical probability $\frac{1}{6}$? Explain briefly. ______
7. A fair coin is flipped 500 times. Find the expected number of heads. ______
8. A fair six-sided die is rolled 300 times. Find the expected number of fives. ______
9. In a 60-trial simulation with a fair six-sided die, success means rolling greater than 4. Find the expected success count. ______
10. A simulation uses a die where 1–3 means W and 4–6 means L. Find the theoretical probability of getting WW in two rolls. ______
11. A bag has 25 marbles, including 10 red marbles. If 50 draws are made with replacement, find the expected number of red draws. ______
12. An experiment has experimental probability 0.45 after 200 trials. Find the number of favorable trials. ______
13. An experiment has experimental probability $\frac{3}{8}$ after 240 trials. Find the number of favorable trials. ______
14. An event has theoretical probability 0.3. Over 500 trials, find the expected count. ______
15. An experiment has 52 successes in 80 trials. Find the experimental probability as a percent. ______

Study Tips

- Experimental probability requires **counting actual outcomes**, not theoretical ones. Don't mix the two formulae.
- When designing a simulation, map the probability to a tool: $P = \frac{1}{2} \to$ coin; $P = \frac{1}{6} \to$ one face of

a six-sided die; $P = \frac{3}{10} \to$ digits 0–2 out of 0–9.

☞ A large gap between experimental and theoretical probability suggests either a **biased experiment** or simply **too few trials** — more trials always helps.

✍ Word Problems

16. A student claims a spinner is unfair. She spins it 120 times and records: Red $= 52$, Blue $= 38$, Green $= 30$. The spinner has three equal sectors. (a) Calculate the experimental probability for each color. (b) What is the theoretical probability for each? (c) Does the data suggest the spinner is unfair? How many more trials would you recommend? ________

17. A cereal company puts a toy in 1 out of every 5 boxes. Design a simulation using random digits 0–9 to estimate the probability that a shopper must buy at least 3 boxes to get a toy. Describe your simulation setup and explain how to interpret one trial. Then state the theoretical probability of needing exactly 3 boxes.

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Answer Keys

1) $\frac{9}{20}$
2) $\frac{6}{25}$
3) $\frac{19}{50}$
4) $\frac{21}{50}$
5) 0.533 vs. 0.5
6) no; exactly theoretical
7) 250
8) 50
9) 20
10) $\frac{1}{4}$
11) 20
12) 90
13) 90
14) 150
15) 65%
16) (a) R about 43%, B about 32%, G 25%; (b) about 33.3% each; (c) red is high; at least 500 more trials
17) Simulation: digits 0–1 toy, 2–9 no toy; record boxes until first toy; exactly 3 boxes: 12.8%

Step-by-Step Explanations

Strategy: For Comparing Two Data Distributions, compare both center and spread; a higher typical value and a wider spread tell different stories. A short comparison table helps keep center and spread from getting mixed together.

Practice 1: A: $10, 20, 30, 40, 50$; B: $20, 25, 30, 35, 40$. Compare medians and IQRs. **Answer:** same median 30; A IQR 30, B IQR 15

For the first worked item, compare center and spread separately so the statement is complete.

Practice 15: A: mean $= 25$, MAD $= 5$. B: mean $= 30$, MAD $= 2$. Write a full 2-sentence comparison. **Answer:** B higher and more consistent

Near the end of this topic, compare center and spread separately so the statement is complete.

Word-problem notes:

16. Answer: Palace: med $= 28$, IQR $= 10$; Speedy: med $= 25$, IQR $= 18$; Speedy is slightly faster on average but far less consistent — Palace is more reliable.

Pizza Palace has median 28 and IQR 10, while Speedy Slice has median 25 and IQR 18. That means Speedy Slice is usually a little faster, because its center is lower, but its delivery times vary much more from order to order. For reliability, Pizza Palace is the better choice because the smaller IQR shows more consistent performance.

17. Answer: Beginners: mean ≈ 73.9, MAD ≈ 6.1; Advanced: mean ≈ 56.3, MAD ≈ 1.8; Advanced runners are significantly faster on average and strikingly more consistent.

The advanced group has the lower mean time, about $56.3\ s$ compared with about $73.9\ s$ for beginners, so the advanced runners are faster on average. Their MAD is also much smaller, about 1.8 versus 6.1, which means their times are clustered much more tightly around the mean. So the advanced group is both faster and more consistent.

18. Answer: Shop A: min 30, Q1 42, med 50, Q3 58, max 70, IQR $= 16$; Shop B: min 20, Q1 38, med 55, Q3 78, max 95, IQR $= 40$. Shop B has a slightly higher median (more customers on a typical day), but Shop A is far more consistent (smaller IQR).

Read each five-number summary directly from the plot. Shop A: $30, 42, 50, 58, 70$, so $\text{IQR}_A = 58 - 42 = 16$. Shop B: $20, 38, 55, 78, 95$, so $\text{IQR}_B = 78 - 38 = 40$. Comparing centers, Shop B's median (55) is a bit higher than Shop A's (50), meaning Shop B sees more customers on a typical day. Comparing spreads, Shop A's much smaller IQR shows that Shop A has steadier daily customer counts.

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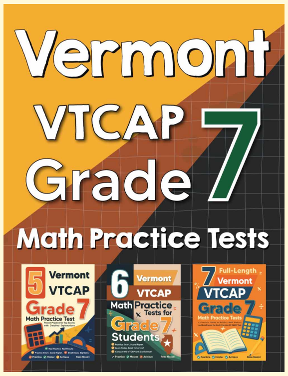

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