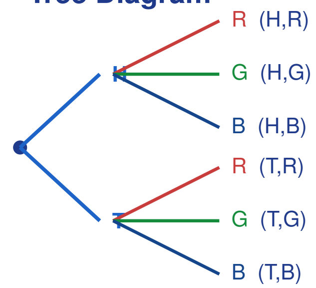
Sample Spaces and Counting Outcomes

Name: ______________________	Date: _______________	Score: ______ / 18

Before you can find a probability, you need to know *all* the possible outcomes—and that complete list is called the **sample space**. Tree diagrams, organized lists, tables, and the **Counting Principle** are your tools for listing outcomes without missing any. Once your sample space is neatly organized, probability questions become much easier to solve. Think of it as building the map before you start the treasure hunt!

Tree Diagram



$2 \times 3 = 6$ outcomes

Outcome Grid

two dice

Die 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Yellow cells: sum $= 7$ $(6$ of $36)$

Key Concepts & Quick Review

Counting Principle: if Event 1 has m outcomes and Event 2 has n outcomes, together they have $m \times n$ outcomes.

Tools: Tree diagram (shows all branches); outcome grid/table (rows $\times$ columns); organised list (write every pair systematically). Always verify total count.

Examples

① A restaurant has 3 soup choices and 4 sandwich choices. How many different soup-and-sandwich lunches are possible?

Think It Through: Each soup can be paired with any of the 4 sandwiches. That means for every one of the 3 soup choices, there are 4 possible lunches. The Counting Principle says to multiply the number of choices at each stage, so $3 \times 4 = 12$ total lunches.

Answer: 12 *different lunches*

② Two six-sided dice are rolled. Using the grid above, find: (a) the total number of outcomes, (b) $P(\text{sum} = 7)$, (c) $P(\text{sum} > 9)$.

Think It Through: For two dice, each die has 6 possible results, so the total number of ordered outcomes is $6 \times 6 = 36$. To get a sum of 7, list the ordered pairs: $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$. That gives 6 favourable outcomes, so the probability is $\frac{6}{36} = \frac{1}{6}$. For sums greater than 9, count sums of 10, 11, and 12. There are 3, 2, and 1 such outcomes respectively, again giving 6 favourable outcomes out of 36.

Answer: *(a) 36; (b) $\frac{1}{6}$; (c) $\frac{1}{6}$*

Practice Problems

Count outcomes using the Counting Principle, or find the probability using a sample space.

1. A coin is flipped and a four-sided die is rolled. How many total outcomes are possible? ______
2. Two coins are flipped. How many total outcomes are possible? ______
3. Three coins are flipped. How many total outcomes are possible? ______
4. A six-sided die and a four-sided die are rolled. How many total outcomes are possible? ______
5. There are 3 shirt choices and 4 pant choices. How many different outfits are possible? ______
6. A pizza menu has 4 sizes and 6 topping choices. How many size-and-topping combinations are possible? ______
7. An ice cream shop offers 5 flavors and 3 cone types. How many choices are possible? ______
8. A lock has 3 dials, and each dial can show any digit from 0 to 9. How many combinations are possible? ______
9. A coin is flipped and a six-sided die is rolled. Find the probability of heads and an even number. ______
10. A coin is flipped and a six-sided die is rolled. Find the probability of tails and a prime number. ______
11. Two six-sided dice are rolled. Find the probability that the sum is 6. ______
12. Two six-sided dice are rolled. Find the probability that the sum is 2. ______
13. Two six-sided dice are rolled. Find the probability that the sum is at most 4. ______
14. Two six-sided dice are rolled. Find the probability that both dice match. ______
15. Two six-sided dice are rolled. Find the probability that the sum is even. ______

Study Tips

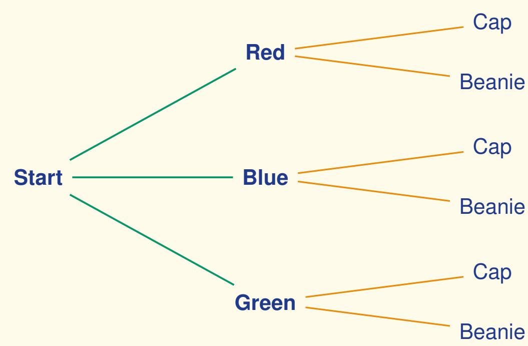
- Use a **tree diagram** when there are few stages and you need to see every outcome. Use the **Counting Principle** when stages are many and you only need the total count.
- An **outcome grid** is ideal for two dice or two spinners — it's a 6×6 (or similar) table that puts every outcome in a cell.
- After building the sample space, count favourable outcomes by highlighting or circling them before dividing — don't try to count in your head.

✍ Word Problems

16. A frozen yogurt shop offers 5 flavours, 4 toppings, and 2 sizes. A customer picks one of each. (a) How many distinct frozen yogurt combinations are possible? (b) If the customer chooses randomly, what is the probability they get chocolate flavour, sprinkles topping, and large size? __________

17. Two fair spinners are spun simultaneously. Spinner A has sections labelled $1, 2, 3$ and Spinner B has sections labelled A, B, C, D. (a) Draw or list the full sample space. (b) How many outcomes are there in total? (c) Find $P($number is odd AND letter is a vowel$)$. (d) Find $P($number $> 1)$. __________

18. This tree diagram shows the outcomes of choosing a shirt (Red, Blue, or Green) and then a hat (Cap or Beanie). Use the diagram to find (a) the total number of outcomes, (b) the probability of choosing a Blue shirt and a Cap, and (c) the probability of choosing a Beanie (any shirt color).



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Answer Keys

1) 8
2) 4
3) 8
4) 24
5) 12
6) 24
7) 15
8) 1000
9) $\frac{1}{4}$
10) $\frac{1}{4}$
11) $\frac{5}{36}$
12) $\frac{1}{36}$
13) $\frac{1}{6}$
14) $\frac{1}{6}$
15) $\frac{1}{2}$
16) (a) 40 combinations; (b) $\frac{1}{40}$
17) (a) 12 pairs; (b) 12; (c) $\frac{1}{6}$; (d) $\frac{2}{3}$
18) (a) 6 outcomes; (b) $\frac{1}{6}$; (c) $\frac{1}{2}$

Step-by-Step Explanations

Strategy: For Making Predictions from Samples, turn the sample result into a rate and scale that rate to the full population. The prediction is strongest when the sample rate and population size are both shown.

Practice 1: In a sample, 3 of 25 like jazz. Predict the count in a population of 500. **Answer:** 60
In the opening prediction example, turn the sample result into a rate before scaling it to the population.

Practice 15: In a sample, 3 of 15 prefer e-books. Predict the count out of $3{,}000$. **Answer:** 600
For the end-of-set prediction, multiply the sample rate by the population size and round only if the context needs a whole person or item.

Word-problem notes:

16. Answer: $\frac{12}{60} = \frac{80}{N} \Rightarrow N = 400$; % error $= \frac{|400-420|}{420} \times 100 \approx 4.8\%$.
The idea is that the fraction of tagged deer in the second capture should be about the same as the fraction of tagged deer in the whole population. So write $\frac{12}{60} = \frac{80}{N}$. Solving gives $N = 400$, so the estimate is 400 deer. To find percent error, compare the estimate to the actual population: $\frac{|400-420|}{420} \times 100 \approx 4.8\%$. A small percent error means the estimate was fairly close.

17. Answer: $\frac{3}{75} = \frac{x}{15000} \Rightarrow x = 600$ defective; $600/15000 = 4\% > 1\%$ — does **not** meet the standard.
Use proportional reasoning to scale the sample result up to the full production: $\frac{3}{75} = \frac{x}{15000}$. Solving gives $x = 600$, so about 600 bulbs are predicted to be defective. Now compare that to the full output: $\frac{600}{15000} = 0.04$ or 4%. Because 4% is greater than the allowed 1%, this batch does not meet the standard.

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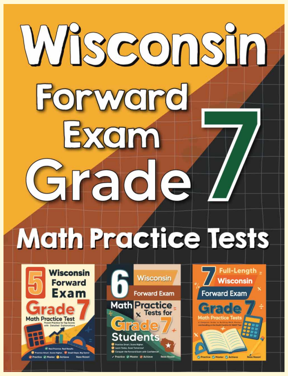

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