

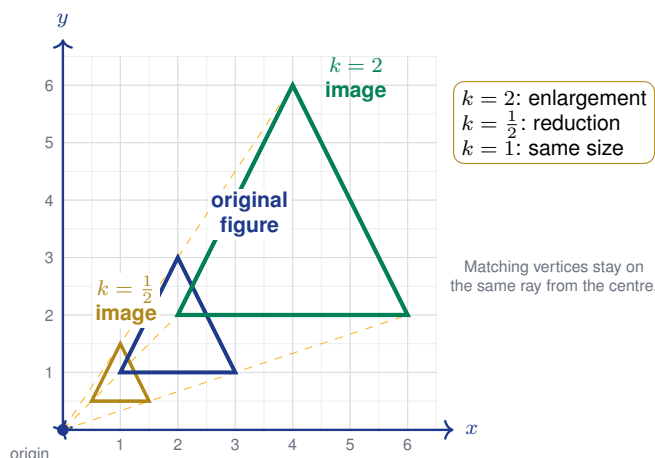
# Dilations and Scale Factors on the Coordinate Plane

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: \_\_\_\_\_ / 18

A **dilation** is a transformation that resizes a figure by multiplying every coordinate by a **scale factor**  $k$ , using the rule  $(x, y) \rightarrow (kx, ky)$  when the centre is the origin. Unlike slides, flips, and turns, a dilation changes the *size* of a figure while keeping its shape exactly the same—so the image and the original are *similar* but not congruent. When  $k > 1$  the figure grows larger (enlargement), and when  $0 < k < 1$  it shrinks (reduction). Learning to spot the scale factor and apply dilations on the coordinate plane connects directly to everything you have studied about similarity, scale drawings, and proportional reasoning!



## Key Concepts & Quick Review

**Dilation centered at the origin with scale factor  $k$ :**

$$(x, y) \rightarrow (kx, ky)$$

- $k > 1$ : the image is **larger** (enlargement).
- $0 < k < 1$ : the image is **smaller** (reduction).
- $k = 1$ : the image is the **same size**.

**Key properties preserved:** Angle measures stay the same. Side lengths are multiplied by  $|k|$ . The image is **similar** to the pre-image.

**Scale factor from two figures:**  $k = \frac{\text{image length}}{\text{original length}}$

### Examples

① Dilate the point  $(4, -6)$  by a scale factor of  $\frac{1}{2}$  centred at the origin.


**Think It Through:** A dilation from the origin means you multiply every coordinate by the scale factor. Here the factor is  $\frac{1}{2}$ , so the figure shrinks to half its size:  $x$ -coordinate:  $4 \times \frac{1}{2} = 2$ ;  $y$ -coordinate:



$-6 \times \frac{1}{2} = -3$ . The image is  $(2, -3)$ .

 **Answer:**  $(2, -3)$

② A triangle has vertices  $A(1, 2)$ ,  $B(3, 2)$ ,  $C(3, 5)$ . After a dilation the image is  $A'(3, 6)$ ,  $B'(9, 6)$ ,  $C'(9, 15)$ . What is the scale factor?

 **Think It Through:** To find the scale factor, compare any corresponding coordinate from the image to the original. From  $A(1, 2)$  to  $A'(3, 6)$ :  $\frac{3}{1} = 3$ . Double-check with  $B$ :  $\frac{9}{3} = 3$ . Both give  $k = 3$ , confirming the triangle was enlarged to three times its size.



 **Answer:**  $k = 3$

### Practice Problems

Dilate each point by the given scale factor  $k$  (centred at the origin). Write the image coordinates.

1. Dilate the point  $(2, 4)$  by scale factor  $k = 3$  from the origin. Write the image point. \_\_\_\_\_
2. Dilate the point  $(6, -3)$  by scale factor  $k = 2$  from the origin. Write the image point. \_\_\_\_\_
3. Dilate the point  $(8, 10)$  by scale factor  $k = \frac{1}{2}$  from the origin. Write the image point. \_\_\_\_\_
4. Dilate the point  $(-4, 2)$  by scale factor  $k = 3$  from the origin. Write the image point. \_\_\_\_\_
5. Dilate the point  $(5, 0)$  by scale factor  $k = 4$  from the origin. Write the image point. \_\_\_\_\_
6. Dilate the point  $(9, -6)$  by scale factor  $k = \frac{1}{3}$  from the origin. Write the image point. \_\_\_\_\_
7. Dilate the point  $(-2, -8)$  by scale factor  $k = \frac{1}{2}$  from the origin. Write the image point. \_\_\_\_\_
8. Dilate the point  $(1, 7)$  by scale factor  $k = 5$  from the origin. Write the image point. \_\_\_\_\_
9. Dilate the point  $(10, 4)$  by scale factor  $k = 0.1$  from the origin. Write the image point. \_\_\_\_\_
10. Dilate the point  $(-3, 9)$  by scale factor  $k = 2$  from the origin. Write the image point. \_\_\_\_\_
11. A dilation maps pre-image point  $(2, 3)$  to image point  $(6, 9)$ . Find the scale factor  $k$ . \_\_\_\_\_
12. A dilation maps pre-image point  $(8, 4)$  to image point  $(4, 2)$ . Find the scale factor  $k$ . \_\_\_\_\_
13. A dilation maps pre-image point  $(5, 10)$  to image point  $(15, 30)$ . Find the scale factor  $k$ . \_\_\_\_\_
14. A dilation maps pre-image point  $(12, 6)$  to image point  $(4, 2)$ . Find the scale factor  $k$ . \_\_\_\_\_
15. Dilate the point  $(0, -5)$  by scale factor  $k = 6$  from the origin. Write the image point. \_\_\_\_\_

### Study Tips

-  Dilations centred at the origin are simple: just **multiply both coordinates** by  $k$ .
-  A dilation does **not** change angles, so the image is *similar* to the original. Use proportions to find missing side lengths.



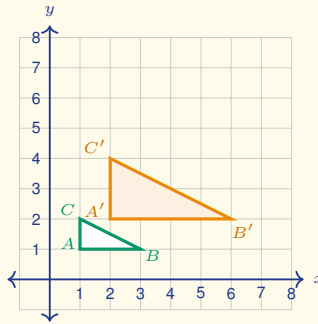
👉 If  $k$  is a fraction less than 1, the image shrinks; if  $k > 1$ , the image grows.  $k = 1$  means no change.

### 📖 Word Problems

16. A rectangle has vertices  $P(2, 1)$ ,  $Q(6, 1)$ ,  $R(6, 3)$ ,  $S(2, 3)$ . It is dilated by a factor of  $\frac{3}{2}$  from the origin. What are the vertices of the image? \_\_\_\_\_

17. A map uses a scale factor of  $\frac{1}{50,000}$ . Two towns are  $4 \text{ cm}$  apart on the map. What is the real distance in kilometers? \_\_\_\_\_

18. The pre-image  $\triangle ABC$  and the image  $\triangle A'B'C'$  shown here are related by a dilation centered at the origin. Read the coordinates from the graph, find the scale factor  $k$ , and decide whether the dilation is an enlargement or a reduction.



## Answer Keys

- |  |   |
|--|---|
| <p>1) (6, 12)<br/>           2) (12, -6)<br/>           3) (4, 5)<br/>           4) (-12, 6)<br/>           5) (20, 0)<br/>           6) (3, -2)<br/>           7) (-1, -4)<br/>           8) (5, 35)<br/>           9) (1, 0.4)<br/>           10) (-6, 18)</p> | <p>11) 3<br/>           12) <math>\frac{1}{2}</math><br/>           13) 3<br/>           14) <math>\frac{1}{3}</math><br/>           15) (0, -30)<br/>           16) <math>P'(3, 1.5), Q'(9, 1.5), R'(9, 4.5), S'(3, 4.5)</math>.<br/>           17) 2 km.<br/>           18) Pre-image <math>A(1, 1), B(3, 1), C(1, 2)</math>; image <math>A'(2, 2), B'(6, 2), C'(2, 4)</math>; <math>k = 2</math>; enlargement.</p> |
|--|---|

### Step-by-Step Explanations

**Strategy:** For Introduction to Slope and Linear Relationships, start from clean points or a clear equation: slope is change in  $y$  over change in  $x$ , and the intercept is where the graph meets the  $y$ -axis. A slope estimate from the graph should agree with the rise-over-run calculation.

**Practice 1:** Find the slope of the line through (2, 3) and (5, 9). **Answer:** 2

In the first example, use change in  $y$  over change in  $x$  for slope, or read the coefficient and intercept from slope-intercept form.

**Practice 15:** Find the  $y$ -intercept  $b$  of the line  $y = \frac{2}{3}x - 1$ . **Answer:** -1

Toward the end, use change in  $y$  over change in  $x$  for slope, or read the coefficient and intercept from slope-intercept form.

**Word-problem notes:**

**16. Answer:**  $m = \frac{15-3}{7-1} = \frac{12}{6} = 2$  cm per day.

Rise =  $15 - 3 = 12$  cm; run =  $7 - 1 = 6$  days. Slope =  $\frac{12}{6} = 2$ .

**17. Answer:**  $y = 0.05x + 10$ ; for 200 texts the cost is \$20.

$y = 0.05x + 10$ . Substitute  $x = 200$ :  $y = 0.05(200) + 10 = 10 + 10 = \$20$ .

**18. Answer:** Slope  $m = \frac{1}{2}$ ;  $y$ -intercept  $b = 1$ ; equation  $y = \frac{1}{2}x + 1$ .

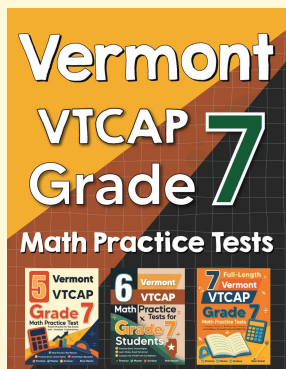
Pick two clear lattice points on the line: (0, 1) and (4, 3). The rise is  $3 - 1 = 2$  and the run is  $4 - 0 = 4$ , so  $m = \frac{2}{4} = \frac{1}{2}$ . The  $y$ -intercept is the point where the line crosses the  $y$ -axis, which is (0, 1), so  $b = 1$ .

The equation is  $y = \frac{1}{2}x + 1$ .



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