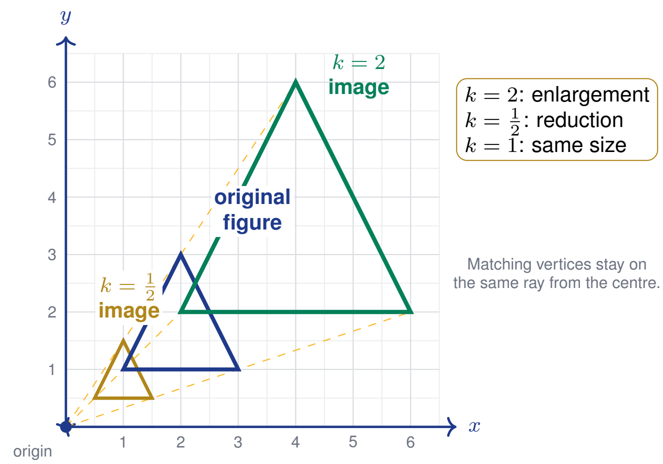
Dilations and Scale Factors on the Coordinate Plane

Name: ______________	Date: ______________	Score: ______ / 18

A **dilation** is a transformation that resizes a figure by multiplying every coordinate by a **scale factor** k, using the rule $(x, y) \to (kx, ky)$ when the centre is the origin. Unlike slides, flips, and turns, a dilation changes the *size* of a figure while keeping its shape exactly the same—so the image and the original are *similar* but not congruent. When $k > 1$ the figure grows larger (enlargement), and when $0 < k < 1$ it shrinks (reduction). Learning to spot the scale factor and apply dilations on the coordinate plane connects directly to everything you have studied about similarity, scale drawings, and proportional reasoning!



Key Concepts & Quick Review

Dilation centered at the origin with scale factor k:

$$(x,\ y) \longrightarrow (kx,\ ky)$$

- $k > 1$: the image is **larger** (enlargement).
- $0 < k < 1$: the image is **smaller** (reduction).
- $k = 1$: the image is the **same size**.

Key properties preserved: Angle measures stay the same. Side lengths are multiplied by $|k|$. The image is **similar** to the pre-image.

Scale factor from two figures: $k = \dfrac{\text{image length}}{\text{original length}}$

Examples

① Dilate the point $(4, -6)$ by a scale factor of $\frac{1}{2}$ centred at the origin.

Think It Through: A dilation from the origin means you multiply every coordinate by the scale factor. Here the factor is $\frac{1}{2}$, so the figure shrinks to half its size: x-coordinate: $4 \times \frac{1}{2} = 2$; y-coordinate:

$-6 \times \frac{1}{2} = -3$. The image is $(2, -3)$.

Answer: $(2, -3)$

② A triangle has vertices $A(1,2)$, $B(3,2)$, $C(3,5)$. After a dilation the image is $A'(3,6)$, $B'(9,6)$, $C'(9,15)$. What is the scale factor?

Think It Through: To find the scale factor, compare any corresponding coordinate from the image to the original. From $A(1,2)$ to $A'(3,6)$: $\frac{3}{1} = 3$. Double-check with B: $\frac{9}{3} = 3$. Both give $k = 3$, confirming the triangle was enlarged to three times its size.

Answer: $k = 3$

Practice Problems

Dilate each point by the given scale factor k (centred at the origin). Write the image coordinates.

1. Dilate the point $(2,4)$ by scale factor $k = 3$ from the origin. Write the image point. ______
2. Dilate the point $(6,-3)$ by scale factor $k = 2$ from the origin. Write the image point. ______
3. Dilate the point $(8,10)$ by scale factor $k = \frac{1}{2}$ from the origin. Write the image point. ______
4. Dilate the point $(-4,2)$ by scale factor $k = 3$ from the origin. Write the image point. ______
5. Dilate the point $(5,0)$ by scale factor $k = 4$ from the origin. Write the image point. ______
6. Dilate the point $(9,-6)$ by scale factor $k = \frac{1}{3}$ from the origin. Write the image point. ______
7. Dilate the point $(-2,-8)$ by scale factor $k = \frac{1}{2}$ from the origin. Write the image point. ______
8. Dilate the point $(1,7)$ by scale factor $k = 5$ from the origin. Write the image point. ______
9. Dilate the point $(10,4)$ by scale factor $k = 0.1$ from the origin. Write the image point. ______
10. Dilate the point $(-3,9)$ by scale factor $k = 2$ from the origin. Write the image point. ______
11. A dilation maps pre-image point $(2,3)$ to image point $(6,9)$. Find the scale factor k. ______
12. A dilation maps pre-image point $(8,4)$ to image point $(4,2)$. Find the scale factor k. ______
13. A dilation maps pre-image point $(5,10)$ to image point $(15,30)$. Find the scale factor k. ______
14. A dilation maps pre-image point $(12,6)$ to image point $(4,2)$. Find the scale factor k. ______
15. Dilate the point $(0,-5)$ by scale factor $k = 6$ from the origin. Write the image point. ______

Study Tips

- Dilations centred at the origin are simple: just **multiply both coordinates** by k.
- A dilation does **not** change angles, so the image is *similar* to the original. Use proportions to find missing side lengths.

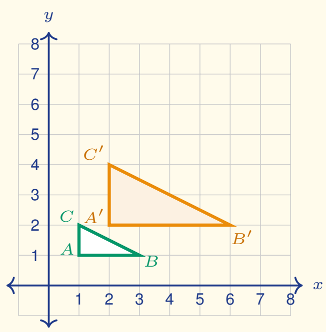
☞ If k is a fraction less than 1, the image shrinks; if $k > 1$, the image grows. $k = 1$ means no change.

✍ Word Problems

16. A rectangle has vertices $P(2,1)$, $Q(6,1)$, $R(6,3)$, $S(2,3)$. It is dilated by a factor of $\frac{3}{2}$ from the origin. What are the vertices of the image? ________

17. A map uses a scale factor of $\frac{1}{50{,}000}$. Two towns are $4\ cm$ apart on the map. What is the real distance in kilometers? ________

18. The pre-image $\triangle ABC$ and the image $\triangle A'B'C'$ shown here are related by a dilation centered at the origin. Read the coordinates from the graph, find the scale factor k, and decide whether the dilation is an enlargement or a reduction.



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Answer Keys

1) (6, 12)
2) (12, -6)
3) (4, 5)
4) (-12, 6)
5) (20, 0)
6) (3, -2)
7) (-1, -4)
8) (5, 35)
9) (1, 0.4)
10) (-6, 18)
11) 3
12) $\frac{1}{2}$
13) 3
14) $\frac{1}{3}$
15) (0, -30)
16) $P'(3, 1.5)$, $Q'(9, 1.5)$, $R'(9, 4.5)$, $S'(3, 4.5)$.
17) $2\,km$.
18) Pre-image $A(1,1)$, $B(3,1)$, $C(1,2)$; image $A'(2,2)$, $B'(6,2)$, $C'(2,4)$; $k = 2$; enlargement.

Step-by-Step Explanations

Strategy: For Introduction to Slope and Linear Relationships, start from clean points or a clear equation: slope is change in y over change in x, and the intercept is where the graph meets the y-axis. A slope estimate from the graph should agree with the rise-over-run calculation.

Practice 1: Find the slope of the line through $(2, 3)$ and $(5, 9)$. **Answer:** 2
In the first example, use change in y over change in x for slope, or read the coefficient and intercept from slope-intercept form.

Practice 15: Find the y-intercept b of the line $y = \frac{2}{3}x - 1$. **Answer:** -1
Toward the end, use change in y over change in x for slope, or read the coefficient and intercept from slope-intercept form.

Word-problem notes:

16. Answer: $m = \frac{15-3}{7-1} = \frac{12}{6} = 2\,cm$ per day.
Rise $= 15 - 3 = 12\,cm$; run $= 7 - 1 = 6$ days. Slope $= \frac{12}{6} = 2$.

17. Answer: $y = 0.05x + 10$; for 200 texts the cost is \$20.
$y = 0.05x + 10$. Substitute $x = 200$: $y = 0.05(200) + 10 = 10 + 10 = \$20$.

18. Answer: Slope $m = \frac{1}{2}$; y-intercept $b = 1$; equation $y = \frac{1}{2}x + 1$.
Pick two clear lattice points on the line: $(0, 1)$ and $(4, 3)$. The rise is $3 - 1 = 2$ and the run is $4 - 0 = 4$, so $m = \frac{2}{4} = \frac{1}{2}$. The y-intercept is the point where the line crosses the y-axis, which is $(0, 1)$, so $b = 1$. The equation is $y = \frac{1}{2}x + 1$.